Producer Behavior
## Introduction

### Chapter Outline

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We now turn to the supply side of the supply and demand model.

3 Key Questions:
1. How do firms decide whether and how much to produce?
2. How do firms choose between inputs such as capital and labor?
3. How does the timeframe of analysis affect firm decisions?

These questions are fundamental to understanding how supply responds to changing market conditions.
Production describes the process by which an entity turns raw inputs into a good or service.

- **Final goods** are purchased by consumers (e.g., bread).
- **Intermediate goods** are used as inputs in other production processes (e.g., wheat used to produce bread).

Start with a production function.

- Mathematical relationship between amount of output and various combinations of inputs
- Similar to a utility function for consumers, except more tangible
Simplifying Assumptions about Firms’ Production Behavior

1. The firm produces a single good.
2. The firm has already chosen which product to produce,
3. Firms minimize costs associated with every level of production.
   - Necessary condition for profit maximization
4. Only two inputs are used in production: capital and labor.
   - Capital: buildings, equipment, etc.
   - Labor: All human resources
5. In the short run, firms can choose the amount of labor employed, but capital is assumed to be fixed in total supply.
   - **Short run:** Period of time in which one more inputs used in production cannot be changed.
   - **Fixed inputs:** Inputs that cannot be changed in the short run.
   - **Variable inputs:** Inputs that can be changed in the short run.
   - **Long run:** Period of time when all inputs in production can be changed.

6. Output increases with inputs.

7. Inputs are characterized by **diminishing returns**.
   - If the amount of capital is held constant, each additional worker produces less incremental output than the last, and vice versa.

8. The firm can employ unlimited capital and labor at fixed prices, and

9. Capital markets are well functioning (the firm is not budget-constrained).
Production Functions

- Describe how output is made from different combinations of inputs,

\[ Q = f(K, L) \]

where \( Q \) is the quantity of output, \( K \) is the quantity of capital used, and \( L \) is quantity of labor used.

A common functional form used in economics is referred to as the Cobb–Douglas production function,

\[ Q = K^\alpha L^\beta \]

where the quantity of each input, each raised to a power (usually less than one), are multiplied together.
The **short run** refers to the case in which the level of capital is fixed, typically expressed as:

$$Q = f(K, L)$$

First, consider how production changes as we vary the amount of labor.

**Marginal product** refers to the additional output that a firm can produce using an additional unit of an input.
- Similar to marginal utility
- Generally assumed to fall as more of an input is used.
The **marginal product of labor**, \( MP_L \), is given as

\[
MP_L = \frac{\Delta Q}{\Delta L}
\]

Consider the production function

\[
Q = K^{0.5} L^{0.5}
\]

where capital is fixed at four units, so plug 4 into the function

\[
Q = 4^{0.5} L^{0.5} = 2L^{0.5}
\]

Table 6.1 calculates the marginal product of labor for this production function.
### Table 6.1: An Example of a Short-Run Production Function

<table>
<thead>
<tr>
<th>Capital, K</th>
<th>Labor, L</th>
<th>Output, Q</th>
<th>Marginal Product of Labor, $MP_L = \frac{\Delta Q}{\Delta L}$</th>
<th>Average Product of Labor, $AP_L = \frac{Q}{L}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>0</td>
<td>0.00</td>
<td>—</td>
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<td>1</td>
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<td>2.00</td>
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<td>0.83</td>
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<td>3</td>
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<td>0.63</td>
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<td>4</td>
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<td>1.00</td>
</tr>
<tr>
<td>4</td>
<td>5</td>
<td>4.47</td>
<td>0.47</td>
<td>0.89</td>
</tr>
</tbody>
</table>
Production in the Short Run

Figure 6.1 A Short-Run Production Function

Output ($Q$) vs Labor ($L$)

Short-run production function, $\bar{K} = 4$
Table 6.1 and Figure 6.1 reveal the common assumption of **diminishing marginal product** associated with production inputs.

- As a firm employs more of one input, while holding all others fixed, the marginal product of that input will fall.
Returning to the mathematical representation of $MP_L$, 

$$MP_L = \frac{\Delta Q}{\Delta L} = \frac{f(K, L + \Delta L) - f(K, L)}{\Delta L}$$

and using the example production function $Q = 4^{0.5} L^{0.5} = 2L^{0.5}$

$$MP_L = \frac{2(L + \Delta L)^{0.5} - 2L^{0.5}}{\Delta L}$$

As $\Delta L$ becomes very small, we use calculus to arrive at the equation for $MP_L$

$$MP_L = \frac{df(K, L)}{dL} = \frac{1}{L^{0.5}} = L^{-0.5}$$

This is seen most easily using a graph.
Production in the Short Run

Figure 6.2 Deriving the Marginal Product of Labor

(a) Production function

Slope = 0.45
Slope = 0.5
Slope = 0.58
Slope = 0.71
Slope = 1

(b) \( MP_L = \frac{1}{L^{0.5}} \)
Another important production metric is average product. 

- Total output divided by the total amount of an input used.
- The average product of labor is given by the equation:

\[ AP_L = \frac{Q}{L} \]

✓ What is the difference between marginal and average product?
- The average product uses the total output, while the marginal product uses the additional output.
For our purposes, the **long run** is defined as a period of time during which all inputs into productions are fully adjustable.

Table 6.2 describes a long-run production function in which two inputs, capital and labor, are used to produce various quantities of a product.

- Columns represent different quantities of labor.
- Rows represent different quantities of capital.
  - Each cell in the table shows the quantity of output produced with the labor and capital represented by the column and row values.
Table 6.2: An Example of a Long-Run Production Function

<table>
<thead>
<tr>
<th>Units of Labor, $L$</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.00</td>
<td>1.41</td>
<td>1.73</td>
<td>2.00</td>
<td>2.24</td>
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<tr>
<td>2</td>
<td>1.41</td>
<td>2.00</td>
<td>2.45</td>
<td>2.83</td>
<td>3.16</td>
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<td>3</td>
<td>1.73</td>
<td>2.45</td>
<td>3.00</td>
<td>3.46</td>
<td>3.87</td>
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<tr>
<td>4</td>
<td>2.00</td>
<td>2.83</td>
<td>3.46</td>
<td>4.00</td>
<td>4.47</td>
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<tr>
<td>5</td>
<td>2.24</td>
<td>3.16</td>
<td>3.87</td>
<td>4.47</td>
<td>5.00</td>
</tr>
</tbody>
</table>

Units of Capital, $K$
Recall, the third assumption about production behavior:

Firms minimize the cost of production.

**Cost minimization** refers to the firm’s goal of producing a specific quantity of output at minimum cost.

- This is an example of a *constrained optimization* problem.
- The firm will minimize costs subject to a specific amount of output that must be produced.

The cost minimization model requires two concepts, **isoquants** and **isocost lines**.
An **isoquant** is a curve representing combinations of inputs that allow a firm to make a particular quantity of output

- Similar to **indifference** curves from consumer theory
The Firm’s Cost-Minimization Problem

Figure 6.3 Isoquants

- Output, $Q = 4$
- Output, $Q = 2$
- Output, $Q = 1$

Capital ($K$) vs. Labor ($L$)
An **isoquant** is a curve representing combinations of inputs that allow a firm to make a particular quantity of output.

The slope of an isoquant describes how inputs may be substituted to produce a fixed level of output.

This relationship is referred to as the **marginal rate of technical substitution**: The rate at which the firm can trade input $X$ for input $Y$, holding output constant ($MRTS_{XY}$).
The marginal product of labor ($MP_L$) is low relative to the marginal product of capital ($MP_K$).

The marginal product of Labor ($MP_L$) is low relative to the marginal product of capital ($MP_K$).
The Firm’s Cost-Minimization Problem

Mathematically, $MRTS_{LK}$ can be derived from the condition that, along an isoquant, quantity of output produced is held constant.

$$\Delta Q = MP_L \times \Delta L + MP_K \times \Delta K = 0$$

Rearranging to find the slope of the isoquant yields the $MRTS_{LK}$.

$$MP_K \times \Delta K = -MP_L \times \Delta L \rightarrow MRTS_{LK} = -\frac{\Delta K}{\Delta L} = \frac{MP_L}{MP_K}$$

Moving down an isoquant, the amount of capital used declines.

- $MRTS_{LK}$ describes the rate at which labor must be substituted for capital to hold the quantity produced constant.
- As you move down an isoquant, the slope gets smaller, meaning the firm has less capital and each unit is relatively more productive.
The Curvature of Isoquants: Substitutes and Complements

The shape of an isoquant reveals information about the relationship between inputs to production

1. Relatively **straight** isoquants imply that the inputs are *relatively substitutable*.
   - $MRTS_{LK}$ does NOT vary much along the curve.

2. Relatively **curved** isoquants imply the inputs are *relatively complementary*.
   - $MRTS_{LK}$ varies greatly along the curve.
Figure 6.5 The Shape of Isoquants Indicates the Substitutability of Inputs

(a) Inputs Are Close Substitutes
(b) Inputs Are Not Close Substitutes
The Curvature of Isoquants: Substitutes and Complements

To illustrate, consider extreme cases.

1. When inputs are perfect substitutes, they can be traded off in a constant ratio in a production process.
   - $MRTS$ is constant.
Consider production of electricity from either oil or natural gas \((Q)\) is kw-hours).

Assume the plant may switch between fuel sources at a relatively constant rate.

**MRTS** is constant.

**Numbers are examples**
The Firm’s Cost-Minimization Problem

6.4

The Curvature of Isoquants: Substitutes and Complements

To illustrate, consider extreme cases.

1. When inputs are **perfect substitutes**, they can be freely substituted without changing output.
2. When inputs are **perfect complements**, they must be used in a fixed ratio as part of a production process.
Consider the provision of bus services.

For a bus to operate, it requires one driver and one bus.

At point $A$, two buses are in service.

Adding another driver (point $B$) will not increase the number of buses in service.

For this, another bus is also needed (point $C$).
Isoquant maps show how quantities of inputs are related to output produced.

An isocost line shows all of the input combinations that yield the same cost.

- Similar to the budget constraint facing consumers, equation given by
  \[ C = rK + wL \]
  where \( C \) is total cost, \( R \) is the “rental rate” of capital, and \( W \) is the wage rate.
  
- Rearranging yields capital as a function of the rental rate, wage rate, and labor.
  \[ K = \frac{C}{r} - \frac{w}{r}L \]
  
- Or, graphically
Figure 6.7 Isocost Lines

Capital ($K$)

Labor ($L$)

$C = \$50$

$C = \$80$

$C = \$100$

Slope = $-\frac{W}{R} = -\frac{10}{20} = -0.5$
Identifying Minimum Cost: Combining Isoquants and Isocost Lines

Remember, the firm’s problem is one of constrained minimization.

- Firms minimize costs subject to a given amount of production.
- Cost minimization is achieved by adjusting the ratio of capital to labor.
  - Similar to expenditure minimization in Chapter 4

Graphically, cost minimization requires tangency between the isoquant associated with the chosen level of production, and the lowest cost isocost line.
The Firm’s Cost-Minimization Problem

Figure 6.10  Cost Minimization

Capital \( (K) \)

\[ Q = \bar{Q} \]

Labor \( (L) \)

- \( C_A \) can produce \( Q \) but is more expensive.
- \( C_B \) (cost-minimizing combination)
- \( C_C \) cannot produce \( Q \).
Identifying Minimum Cost: Combining Isoquants and Isocost Lines

Mathematically, tangency occurs where the slope of the isocost line is equal to the slope of the isoquant, or

\[
- \frac{w}{r} = - \frac{MP_L}{MP_K} \rightarrow \frac{MP_K}{r} = \frac{MP_L}{w}
\]

What does this condition imply?

- Costs are minimized when the marginal product per dollar spent is equalized across inputs.
Cost Minimizing Condition:

- Costs are minimized when the marginal product per dollar spent is equalized across inputs.

\[- \frac{w}{r} = - \frac{MP_L}{MP_K} \rightarrow \frac{MP_K}{r} = \frac{MP_L}{w}\]

**Question:** What if 1. \(\frac{MP_K}{r} > \frac{MP_L}{w}\) or 2. \(\frac{MP_K}{r} < \frac{MP_L}{w}\)

1. The marginal product per dollar spent on capital is higher than the marginal product per dollar spent on labor.
   - More capital and less labor should be used in production.
2. The marginal product per dollar spent on capital is less than the marginal product per dollar spent on labor.
   - More labor and less capital should be used in production.
Returns to Scale

Returns to scale refers to the change in output when all inputs are increased in the same proportion.

Returning to the Cobb–Douglas production function

\[ Q = K^\alpha L^\beta \]

If we assume \( \alpha = \beta = 0.5 \), then

\[ Q = K^{0.5} L^{0.5} \]

- If \( K = 2 \) and \( L = 2 \), then \( Q = 2 \).

What happens if the amount of capital and labor used both double?

\[ Q = 4^{0.5} 4^{0.5} = 2 \times 2 = 4 \]

Output Doubles!
This relationship, whereby production increases proportionally with inputs, is called **Constant Returns to Scale**.

- Double inputs $\rightarrow$ Output doubles
- Quadruple inputs $\rightarrow$ Output quadruples

**Increasing Returns to Scale:** Describes production for which changing all inputs by the same proportion changes output *more* than proportionally.

- Double inputs $\rightarrow$ Output quadruples

**Decreasing Returns to Scale:** Describes production for which changing all inputs by the same proportion changes output *less* than proportionally.

- Double inputs $\rightarrow$ Output increases by *less* than double.
- Quadruple inputs $\rightarrow$ Output only doubles.
QUESTION: Why might a firm experience *increasing returns to scale*?

- **Fixed costs** (e.g., webpage management, advertising contracts) do not vary with output.
- **Learning by doing** may occur, whereby a firm develops more efficient processes as it expands or produces more output.

Generally, firms should not experience *decreasing returns to scale*.

- When this phenomenon is observed in data, it often results from not accounting for all inputs (or attributes).
  - For instance, second manager may not be as competent as first.

QUESTION: Are there any examples of true decreasing returns?

- **Regulatory burden**: As firms grow larger, they are often subject to more regulations.
  - Compliance costs may be significant.
Returns to Scale

Figure 6.12 Returns to Scale

(a) Constant Returns to Scale
(b) Increasing Returns to Scale
(c) Decreasing Returns to Scale

Note: Labor and capital doubled between isoquants.
Technological Change

Examining firm-level production data over time reveals increasing output, even when input levels are held constant.

- The only way to explain this is by assuming some change to the production function.

This change is referred to as **Total Factor Productivity Growth**.

- An improvement in technology that changes the firm’s production function such that more output is obtained from the same amount of inputs.

Often assumed to enter multiplicatively with production,

\[ Q = Af(K, L) \]

where \( A \) is the level of total factor productivity.
With old technology, the production of $Q^*$ requires $L_1$ labor and $K_1$ capital.

When technology improves, the isoquant associated with $Q^*$ shifts inward, requiring less labor and capital ($L_2$ and $K_2$).
So far, we have only focused on how firms minimize costs, subject to a fixed quantity of output.

- We can use the cost minimization approach to describe how capital and labor change as output increases.

An **expansion path** is a curve that illustrates how the optimal mix of inputs varies with total output.

This allows construction of the **total cost curve**, which shows a firm’s cost of producing particular quantities.
The Firm's Expansion Path and Total Cost Curve

Figure 6.15 The Expansion Path and the Total Cost Curve

- **Expansion path**
  - Points: A, B, C
  - Levels: Q = 10, Q = 20, Q = 30

- **Total cost (TC)**
  - Points: A, B, C
  - Levels: $100, $180, $300

- **Axes**
  - Labor (L)
  - Capital (K)
  - Quantity of engines
This chapter looked closely at how firms make decisions.

- Firms are assumed to minimize costs at every level of production.
- The cost-minimizing combination of inputs occurs where the marginal rate of technical substitution is equal to the slope of the isocost line.

In Chapter 7 we delve deeper into the different costs facing firms, and how they change with the level of production.
The short-run production function for a firm that produces pizza

\[ Q = f(K, L) = 15K^{0.25}L^{0.75} \]

where \( Q \) is the number of pizzas produced per hour, is the number of ovens (fixed at 3 in the short run), and \( L \) is the number of workers employed.

**Answer the following:**

a. Write an equation for the short-run production function with output as a function of labor only,

b. Calculate total output per hour for \( L = 0, 1, 2, 3, 4, 5 \).

c. Calculate the \( MP_L \) for \( L = 1 \) to \( L = 5 \). Is \( MP_L \) diminishing?

d. Calculate the \( AP_L \) for the same labor levels as above.
a. Capital is fixed in the short run at 3. Substitute $K = 3$ into the production function.

$$Q = 15(3)^{0.25} L^{0.75} = 19.74 L^{0.75}$$

b. To calculate total output, simply substitute the different labor quantities into the production function above.

<table>
<thead>
<tr>
<th>Labor</th>
<th>Production</th>
</tr>
</thead>
<tbody>
<tr>
<td>$L = 0$</td>
<td>$Q = 19.74(0)^{0.75} = 0$</td>
</tr>
<tr>
<td>$L = 1$</td>
<td>$Q = 19.74(1)^{0.75} = 19.74$</td>
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<tr>
<td>$L = 2$</td>
<td>$Q = 19.74(2)^{0.75} = 33.2$</td>
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<tr>
<td>$L = 3$</td>
<td>$Q = 19.74(3)^{0.75} = 45.01$</td>
</tr>
<tr>
<td>$L = 4$</td>
<td>$Q = 19.74(4)^{0.75} = 55.82$</td>
</tr>
<tr>
<td>$L = 5$</td>
<td>$Q = 19.74(5)^{0.75} = 66.01$</td>
</tr>
</tbody>
</table>
c. The marginal product of labor is the additional amount of bread produced with one more unit of labor.

Are there diminishing returns to labor? How do you know?

Yes, because as more labor is added the additional output increases by less than previous unit.

d. Average product is simply total output divided by total labor used.

<table>
<thead>
<tr>
<th>Labor</th>
<th>Production</th>
<th>$MP_L$</th>
<th>$AP_L$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$L = 0$</td>
<td>$Q = 19.74(0)^{0.75} = 0$</td>
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<td>—</td>
</tr>
<tr>
<td>$L = 1$</td>
<td>$Q = 19.74(1)^{0.75} = 19.74$</td>
<td>19.74</td>
<td>19.74</td>
</tr>
<tr>
<td>$L = 2$</td>
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<td>16.60</td>
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<td>11.81</td>
<td>15.00</td>
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<tr>
<td>$L = 4$</td>
<td>$Q = 19.74(4)^{0.75} = 55.82$</td>
<td>10.81</td>
<td>13.96</td>
</tr>
<tr>
<td>$L = 5$</td>
<td>$Q = 19.74(5)^{0.75} = 66.01$</td>
<td>10.19</td>
<td>13.20</td>
</tr>
</tbody>
</table>
Short-run production function for a local bakery making loaves of bread

\[ Q = f(K, L) = 20K^{0.75}L^{0.25} \]

where \( Q \) is the number of loaves produced per hour, \( K \) is the number of ovens (fixed at 2), and \( L \) is the number of workers.

**Answer the following:**

a. Write an equation for the short-run production function with output as a function of labor only.

b. Calculate total output per hour for \( L = 0,1,2,3,4,5 \).

c. Calculate the \( MP_L \) for \( L = 1 \) to \( L = 5 \). Is \( MP_L \) diminishing?

d. Calculate the \( AP_L \) for the same labor levels above.
figure it out

a. Capital is fixed in the short run at 2.
Substitute $\bar{K} = 2$ into the production function

$$Q = 20(2)^{0.75} L^{0.25} = 33.64L^{0.25}$$

b. To calculate total output, simply substitute the different labor quantities into the production function above.

<table>
<thead>
<tr>
<th>Labor</th>
<th>Production</th>
</tr>
</thead>
<tbody>
<tr>
<td>$L = 0$</td>
<td>$Q = 33.64(0)^{0.25} = 0$</td>
</tr>
<tr>
<td>$L = 1$</td>
<td>$Q = 33.64(1)^{0.25} = 33.64$</td>
</tr>
<tr>
<td>$L = 2$</td>
<td>$Q = 33.64(2)^{0.25} = 40$</td>
</tr>
<tr>
<td>$L = 3$</td>
<td>$Q = 33.64(3)^{0.25} = 44.27$</td>
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<td>$Q = 33.64(4)^{0.25} = 47.57$</td>
</tr>
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<td>$L = 5$</td>
<td>$Q = 33.64(5)^{0.25} = 50.30$</td>
</tr>
</tbody>
</table>
c. The marginal product of labor is the additional amount of bread produced with one more unit of labor.

Are there diminishing returns to labor? How do you know?

- Yes, because as more labor is added the additional output increases by less than previous unit.

d. Average product is simply total output divided by total labor.

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<thead>
<tr>
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</tr>
</thead>
<tbody>
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<td>$Q = 33.64(0)^{0.25} = 0$</td>
<td>—</td>
<td>—</td>
</tr>
<tr>
<td>$L = 1$</td>
<td>$Q = 33.64(1)^{0.25} = 33.64$</td>
<td>33.64</td>
<td>33.64</td>
</tr>
<tr>
<td>$L = 2$</td>
<td>$Q = 33.64(2)^{0.25} = 40$</td>
<td>6.36</td>
<td>20</td>
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<td>14.76</td>
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<tr>
<td>$L = 4$</td>
<td>$Q = 33.64(4)^{0.25} = 47.57$</td>
<td>3.30</td>
<td>11.89</td>
</tr>
<tr>
<td>$L = 5$</td>
<td>$Q = 33.64(5)^{0.25} = 50.30$</td>
<td>2.73</td>
<td>10.06</td>
</tr>
</tbody>
</table>
Suppose the wage rate is $10 per hour \((w = 10/\text{hour})\) and the rental rate of capital is $25 per hour \((r = 25/\text{hour})\).

**Answer the following:**

a. Write an equation for the isocost line for a firm.

b. Draw a graph, with labor on the horizontal axis and capital on the vertical axis, showing the isocost line for \(C = 800\).
   - Indicate the horizontal and vertical intercepts along with the slope.

c. Suppose the price of capital falls to $20 per hour. Show what happens to the \(C = 800\) isocost line, including any changes in intercepts and the slope.
figure it out

a. The isocost line always takes the form of $C = rK + wL$.

Plugging in $r$ and $w$, $C = 25K + 10L$.

b. With $C = $800, the isocost line that will be plotted will be:

$800 = 25K + 10L$, the easiest way to plot it is to find the horizontal and vertical intercepts.

- **Horizontal Intercept**: The amount of labor the firm could hire for $800 if it only hired labor ($K = 0$).

  & \frac{$800}{w} \rightarrow \frac{$800}{$10} = 80 \text{ units} \\

- **Vertical Intercept**: The amount of capital the firm could hire for $800 if it only hired capital ($L = 0$).

  & \frac{$800}{r} \rightarrow \frac{$800}{$25} = 32 \text{ units} \\

Plot these 2 points and draw a line connecting them; this is the $C = $800 isocost line ($C_1$).
b. The slope of the isocost line is \( m = -\frac{w}{r} = -\frac{10}{25} \) or \(-0.4\).  

C. When \( r \) (the price of capital) falls to $20, only the vertical intercept is affected.  
- If the firm only used capital it could now employ \( \frac{800}{20} = 40 \) units.  
- The isocost line pivots up and becomes steeper (\( C_2 \)) with a new slope  
  \[ -\frac{w}{r} = -\frac{10}{20} \] or \(-0.5\).
Suppose the wage rate is $10 per hour \( (w = \$10/\text{hour}) \) and the rental rate of capital is $20 per hour \( (r = \$20/\text{hour}) \).

**Answer the following:**

a. Write an equation for the isocost line for the firm.

b. Draw a graph, with labor on the horizontal axis and capital on the vertical axis, showing the isocost line for \( C = \$400 \).
   - Indicate the horizontal and vertical intercepts along with the slope.

c. Suppose the price of capital falls to $20 per hour. Show what happens to the \( C = \$400 \) isocost line, including any changes in intercepts and the slope.
a. The isocost line always takes the form of $C = rK + wL$.

Plugging in $r$ and $w$, $C = 20K + 10L$

b. With $C = $400, the isocost line that will be plotted will be:

$400 = 20K + 10L$. The easiest way to plot it is to find the horizontal and vertical intercepts.

- **Horizontal Intercept**: The amount of labor the firm could hire for $400 if it only hired labor $(K=0)$.
  
  $\frac{400}{w} \rightarrow \frac{400}{10} = 40 \text{ units}$

- **Vertical Intercept**: The amount of capital the firm could hire for $400 if it only hired capital $(L=0)$.
  
  $\frac{400}{r} \rightarrow \frac{400}{20} = 20 \text{ units}$

Plot these 2 points and draw a line connecting them; this is the $C = $800 isocost line ($C_1$).
b. The slope of the isocost line is $m = -\frac{w}{r} = -\frac{10}{20}$ or $-0.5$

c. When $w$ (the price of labor) increases to $20$ an hour, only the horizontal intercept is affected.
- If the firm only used labor it could now employ $\frac{400}{20} = 20$ units.
- The isocost line pivots in and becomes steeper ($C_2$) with a new slope.
  \[ -\frac{w}{r} = -\frac{20}{20} \text{ or } -1 \]
A firm is employing 100 workers ($w = $15/hour) and 50 units of capital ($r = $30/hour). At these levels, the marginal product of labor ($MP_L$) is 45 and the marginal product of capital ($MP_K$) is 60.

**Answer the following:**

a. Is this firm minimizing costs?

b. If not, what changes should they make?

c. How does the answer to (2) depend on the timeframe of analysis?
Cost minimization occurs when \( \frac{MP_K}{r} = \frac{MP_L}{w} \) for this firm, we have

\[
\begin{align*}
\frac{MP_K}{r} &= \frac{60}{30} = 2 \\
\frac{MP_L}{w} &= \frac{45}{15} = 3
\end{align*}
\]

Since these two ratios are not equal, the firm is not minimizing costs.

As \( \frac{MP_L}{w} > \frac{MP_K}{r} \), changing the mix of capital and labor can lead to a lower cost of producing the same quantity of output.

- $1 spent on labor yields a greater marginal product (i.e. more output) than $1 spent on capital.
- The wages to labor and capital are fixed, so to equate these two, the quantity of labor employed must rise and/or the quantity of capital employed must fall.

c. Generally, the short run implies that only the amount of labor employed can be altered.
A firm is employing 25 workers \((w = \$10/\text{hour})\) and 5 units of capital \((r = \$20/\text{hour})\). At these levels, the marginal product of labor \((MP_L)\) is 25 and the marginal product of capital \((MP_K)\) is 30.

**Answer the following:**

a. Is this firm minimizing costs?

b. If not, what changes should they make?

c. How does the answer to (2) depend on the timeframe of analysis?
a. Cost minimization occurs when

\[
\frac{MP_K}{r} = \frac{MP_L}{w}
\]

For this firm, we have

\[
\frac{MP_K}{r} = \frac{30}{20} = 1.5
\]

\[
\frac{MP_L}{w} = \frac{25}{10} = 2.5
\]

Since these two ratios are not equal, the firm is not minimizing costs.

b. As \( \frac{MP_L}{w} > \frac{MP_K}{r} \), changing the mix of capital and labor can lead to a lower cost of producing the same quantity of output.

- $1 spent on labor yields a greater marginal product (i.e. more output) than $1 spent on capital.
- The wages to labor and capital are fixed, so to equate these two, the quantity of labor employed must rise and/or the quantity of capital employed must fall.

c. Generally, the short run implies that only the amount of labor...
For each of the following production functions, determine if they exhibit constant, decreasing, or increasing returns to scale.

a. \( Q = 2K + 15L \)

b. \( Q = \min(3K, 4L) \)

c. \( Q = 15K^{0.5}L^{0.4} \)
The simplest way to determine returns to scale is to plug in values for labor and capital, calculate output, then double the inputs and calculate output again.

a. Consider $K = L = 1$

$$Q = 2K + 15L = 2 + 15 = 17$$

Now, double the inputs ($K$ and $L$ now = 2).

$$Q = 2K + 15L = 2(2) + 15(2) = 34$$

Since output doubled when inputs doubled, we have **constant returns to scale**.
**figure it out**

b. Consider $K = L = 1$ again.

$$Q = \min(3K, 4L) = 3$$

Now, double the inputs ($K$ and $L$ now = 2).

$$Q = \min(3(2), 4(2)) = 6$$

Once again, we have **constant returns to scale.**

c. $K = L = 1$

$$Q = 15K^{0.5}L^{0.4} = 15$$

Now, double the inputs ($K$ and $L$ now = 2).

$$Q = 15(2)^{0.5}(2)^{0.4} = 27.99$$

Since the new output is *less* than twice the old output, we have **decreasing returns to scale.**
For each of the following production functions, determine if they exhibit constant, decreasing, or increasing returns to scale.

a. $Q = 3K + 5L$

b. $Q = \min (6K, 5L)$

c. $Q = 18K^{0.6}L^{0.3}$
figure it out

The simplest way to determine returns to scale is to plug in values for labor and capital, calculate output, then double the inputs and calculate output again.

a. Consider $K = L = 2$.

\[ Q = 3K + 5L = 6 + 10 = 16 \]

Now, double the inputs ($K$ and $L$ now = 4).

\[ Q = 3K + 5L = 3(4) + 5(4) = 32 \]

Since output doubled when inputs doubled, we have constant returns to scale.
b. Consider \( K = L = 2 \) again.

\[
Q = \min (6K, 5L) = 10
\]

Now, double the inputs (\( K \) and \( L \) now = 4).

\[
Q = \min (6(4), 5(4)) = 20
\]

Once again, we have **constant returns to scale**.

c. \( K = L = 2 \)

\[
Q = 18K^{0.6}L^{0.3} = 33.59
\]

Now, double the inputs (\( K \) and \( L \) now = 4).

\[
Q = 18(4)^{0.6}(4)^{0.3} = 62.68
\]

Since the new output is less than twice the old output, we have **decreasing returns to scale**.