TELECOM ParisTech

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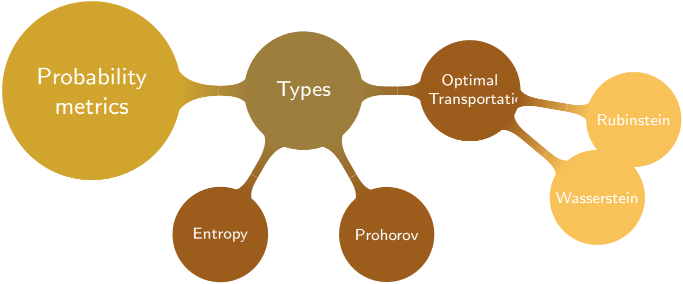
Functional Stein's method

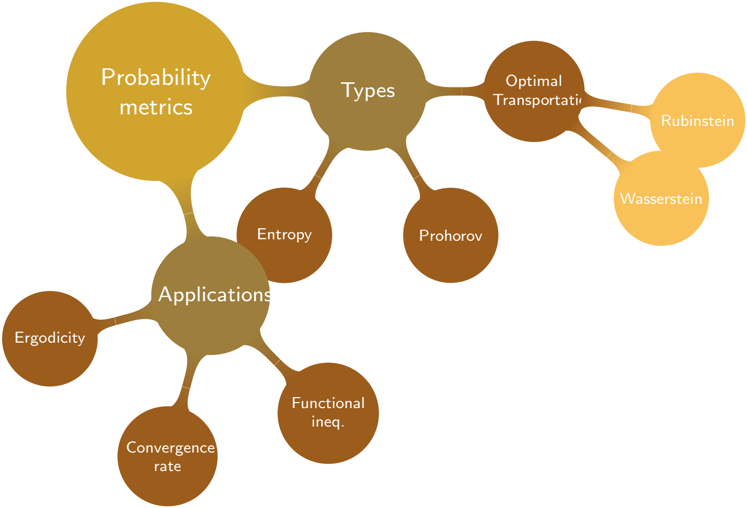
L. Decreusefond

Borchard symposium

Roadmap


Probability metrics
Types
Optimal Transportati
Rubinstein
Wasserstein
Entropy
Prohorov

Roadmap



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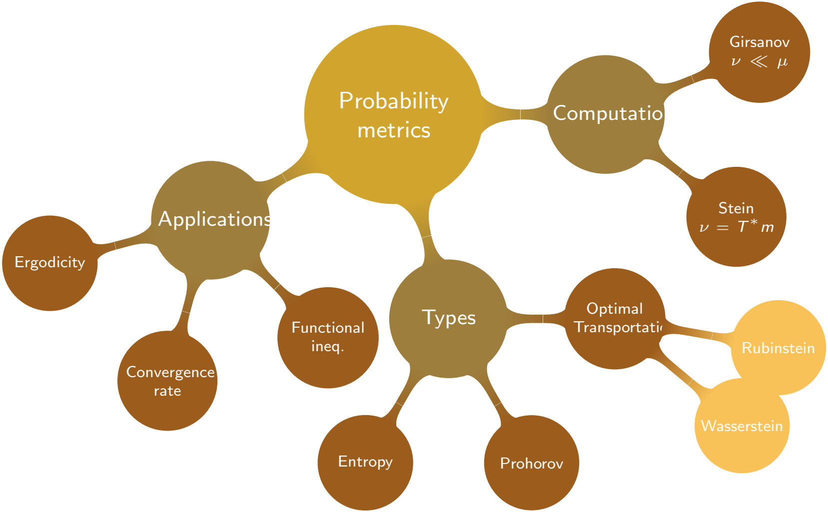
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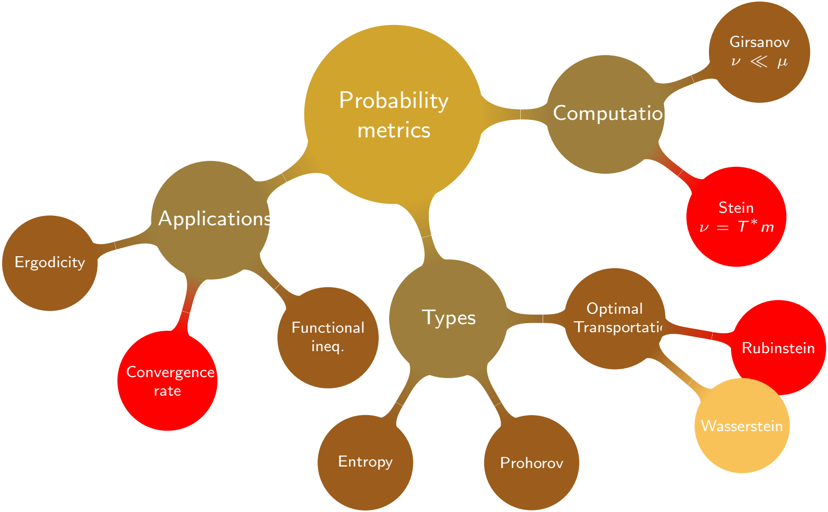

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Rubinstein distance

Definition

- $(\mathfrak{F}, d)$ a Polish space

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- $F \in \text{Lip}_c$ iff $|F(x) - F(y)| \leq c(x, y)$

Rubinstein distance

Definition

- $(\mathfrak{F}, d)$ a Polish space
- c a distance on $\mathfrak{F}$
- $F \in \text{Lip}_c$ iff $|F(x) - F(y)| \leq c(x,y)$
- μ and ν' 2 proba. measures on F

$$\mathrm{d}_R(\mu, \nu') = \sup_{F \in \text{Lip}_c} \int F \mathrm{d}\mu - \int F \mathrm{d}\nu'$$

Some examples

- $\mathfrak{F} = \mathbf{R}^n$
- $d = c$=Euclidean distance
- Convergence in Rubinstein is equivalent to convergence in distribution (Dudley)

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Wiener space

- $\mathfrak{F}$= space of continuous functions on $[0, 1]$
- d=uniform distance
- c=distance in the Cameron-Martin space

$$c(f, g) = \left[\int_0^1 |f'(s) - g'(s)|^2 ds \right]^{1/2}$$

Configuration space

Definition

A configuration is a locally finite set of particles on a Polish space $\mathbb{Y}$

$$\int f \mathrm{d}\omega = \sum_{x \in \omega} f(x)$$

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Vague topology

$$\omega_n \xrightarrow{\text{vaguely}} \omega \iff \int f \mathrm{d}\omega_n \xrightarrow{n\to\infty} \int f \mathrm{d}\omega$$

for all f continuous with compact support from $\mathbb{Y}$ to $\mathbb{R}$

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d is the associated distance

Lipschitz functionals

Distance between configurations

$c(\omega, \eta) = \text{dist}_{\mathsf{TV}}(\omega, \eta) =$ number of different points

Lipschitz functionals

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Definition

$F : \mathrm{N}_{\mathbb{Y}} \to \mathbf{R}$ is TV–Lip$_1$ if

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$$|F(\omega) - F(\eta)| \le \text{dist}_{\text{TV}}(\omega, \eta)$$

Definition (Rubinstein distance)

$$\text{d}_{\text{R}}(\mathbf{P}, \mathbf{Q}) := \sup_{F \in \text{TV–Lip}_1} \big(\mathbf{E}_{\mathbf{P}} F - \mathbf{E}_{\mathbf{Q}} F\big),$$

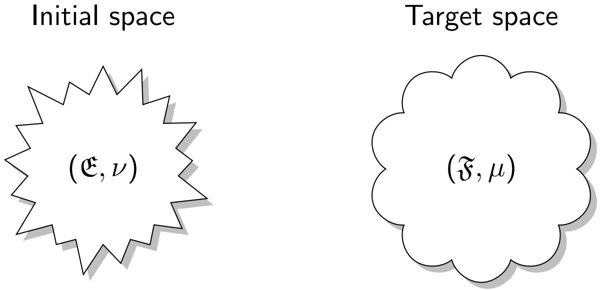
Convergence in configuration space

Theorem (D- Schulte-Thäle)

$$\mathrm{d}_{\mathrm{R}}(\mathbf{P}_n, \mathbf{Q}) \xrightarrow{n\to\infty} 0 \Longrightarrow \mathbf{P}_n \xrightarrow{distr.} \mathbf{Q}$$

Convergence in $\mathbb{N}_{\mathbb{Y}}$

Generic scheme



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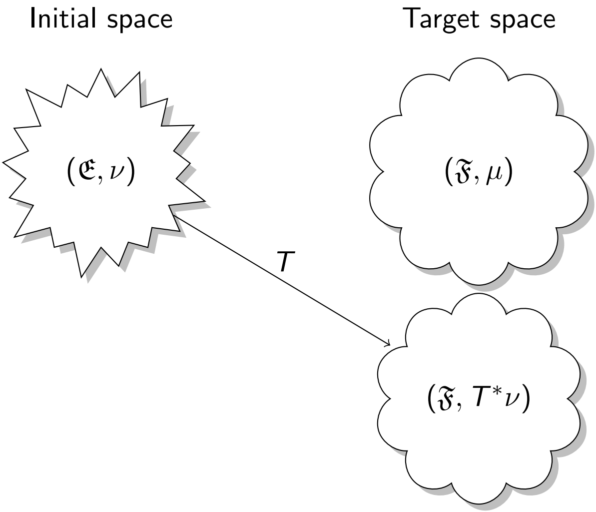

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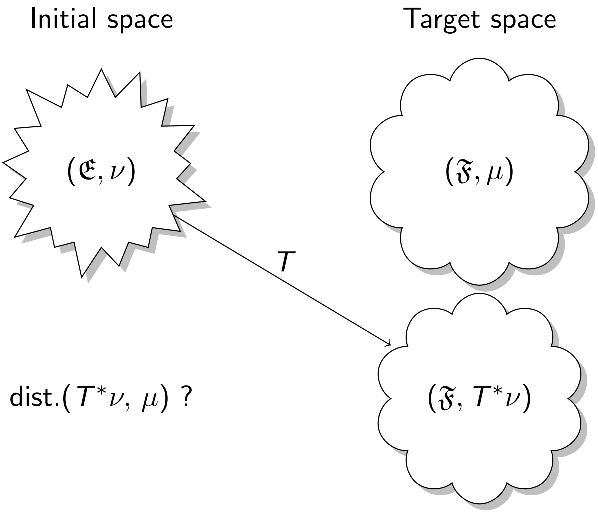
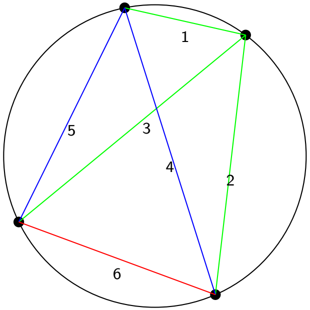
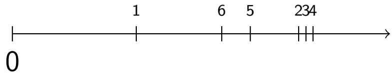


Motivation : Poisson polytopes



η



$\xi(\eta)$

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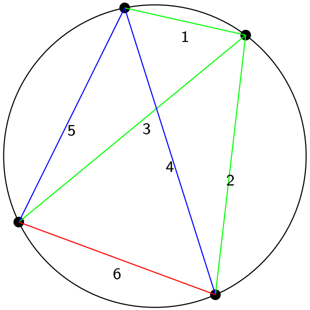

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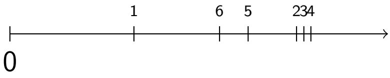


$\xi(\eta)$

Question

What happens when the number of points goes to infinity ?

Poisson point process

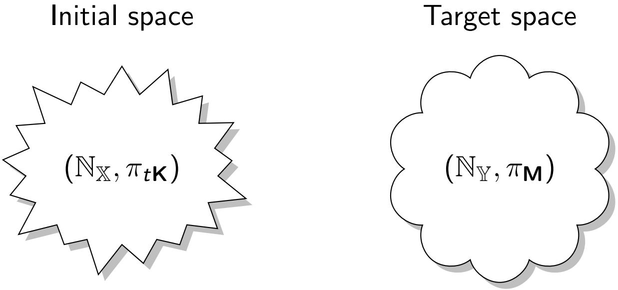
Hypothesis

Points are distributed to a Poisson process of control measure $t\mathbf{K}$

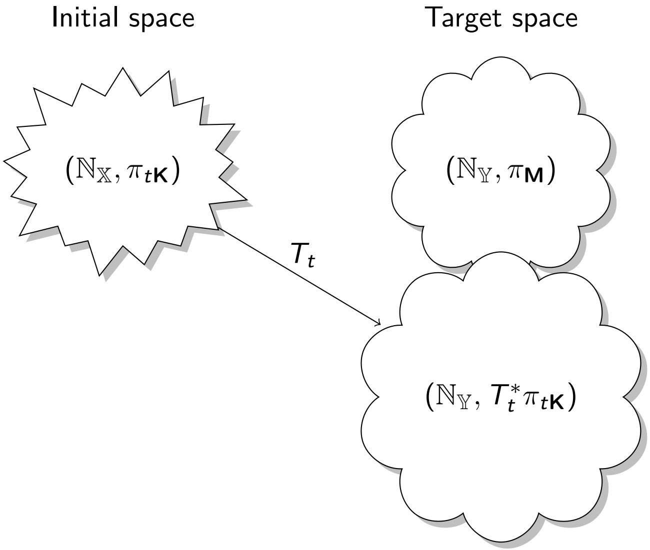
Definition

- The number of points is a Poisson rv $(t\,\mathbf{K}(\mathbb{S}^{d-1}))$
- Given the number of points, they are independently drawn with distribution $\mathbf{K}$

Framework


Initial space
Target space
$(\mathbb{N}_{\mathbb{X}}, \pi_{t\mathbf{K}})$
$(\mathbb{N}_{\mathbb{Y}}, \pi_{\mathbf{M}})$

Framework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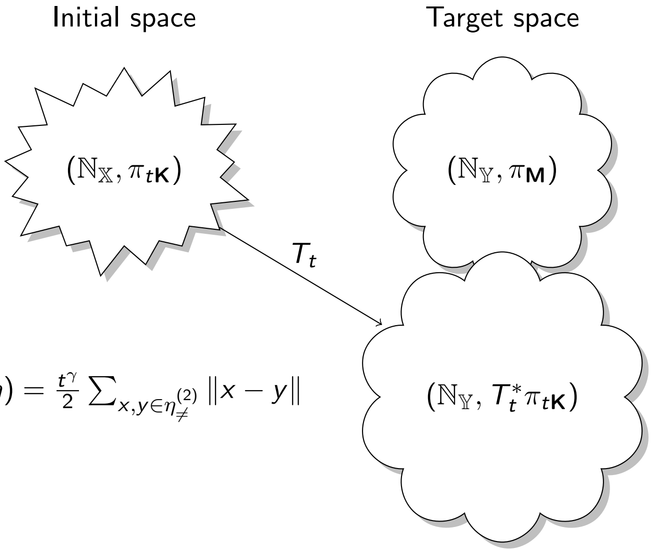
T_t
$(\mathbb{N}_{\mathbb{Y}}, T_t^* \pi_{t\mathbf{K}})$

Framework



$$T_t(\eta) = \frac{t^\gamma}{2} \sum_{x,y \in \eta_{\neq}^{(2)}} \|x - y\|$$

Rescaling : $\gamma = 2/(d-1)$

Campbell-Mecke formula

$$\mathbf{E} \sum_{x_1,\cdots,x_k \in \omega_{\neq}^{(k)}} f(x) = t^k \int f(x_1,\cdots,x_k)\mathbf{K}(\mathrm{d}x_1,\cdots,\mathrm{d}x_k)$$

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Mean number of points (after rescaling)

$$\frac{1}{2}\mathbf{E} \sum_{x \neq y \in \omega} \mathbf{1}_{\|t^\gamma x - t^\gamma y\| \leq \beta} = \frac{t^2}{2} \iint_{\mathbb{S}^{d-1} \otimes \mathbb{S}^{d-1}} \mathbf{1}_{\|x-y\| \leq t^{-\gamma}\beta} \mathrm{d}x\mathrm{d}y$$

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Geometry

$$V_{d-1}(\mathbb{S}^{d-1} \cap B^d_{\beta t^{-\gamma}}(y)) = \kappa_{d-1}(\beta t^{-\gamma})^{d-1} + \frac{(d-1)\kappa_{d-1}}{2}(\beta t^{-\gamma})^d + O(t^{-\gamma(}$$

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Schulte-Thäle (2012) based on Peccati (2011)

$$\left|\mathbf{P}(t^{2/(d-1)}T_m(\xi) > x) - e^{-\beta x^{(d-1)}} \sum_{i=0}^{m-1} \frac{(\beta x^{d-1})^i}{i!}\right| \le C_x t^{-\min(1/2, 2/(d-1))}$$

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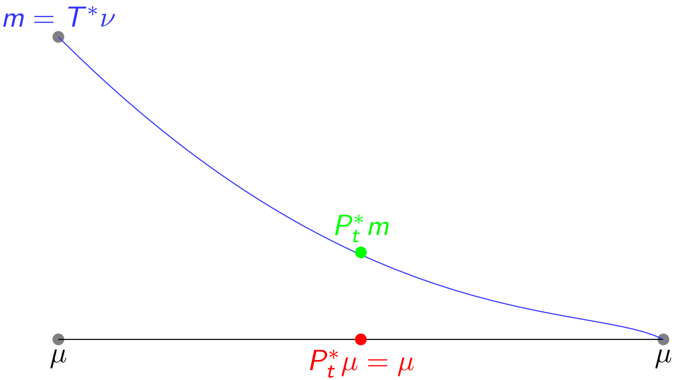
What about speed of convergence as a process ?

Stein method in one picture



Stein method

The main tool

Construct a Markov process $(X(s), s \geq 0)$

- with values in $\mathfrak{F}$
- ergodic with μ as invariant distribution

$$X(s) \xrightarrow{distr.} \mu$$

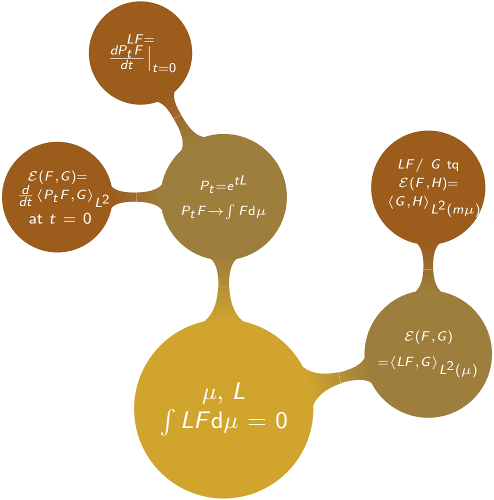
  for all initial condition $X(0)$

- for which μ is a stationary distribution

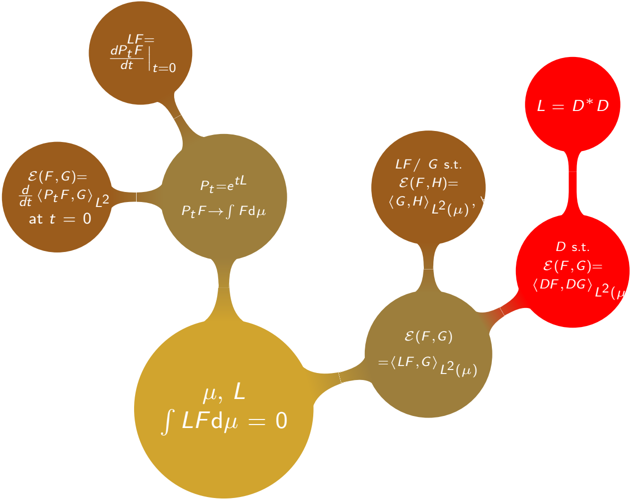
$$X(0) \overset{\text{distr.}}{=} \mu \Longrightarrow X(s) \overset{\text{distr.}}{=} \mu, \ \forall s > 0$$

- Equivalently $\int LF\mathrm{d}m = 0, \ \forall F$ iff $m = \mu$

Dirichlet structure



Dirichlet-Malliavin structure



Gaussian measure on $\mathbf{R}^n$

Standard Gaussian measure

- $\mathfrak{F} = \mathbf{R}^n$, $\mu = \mathcal{N}(0, \mathrm{Id})$
- $LF(u) = u.\nabla F(u) - \Delta F(u)$
- Semi-group

$$P_t F(x) = \int_{\mathbf{R}^n} F(e^{-t}u + \sqrt{1 - e^{-2t}}v)\, d\mu(v)$$

- $X = (X_1, \cdots, X_n)$ where X_k=Ornstein-Uhlenbeck process on $\mathbf{R}$

$$\mathrm{d}X_k(t) = -X_k(t)\mathrm{d}t + \sqrt{2}\mathrm{d}B_k(t)$$

- $D = \nabla$

Poisson distribution on N

Poisson

- $\mathfrak{F} = \mathbf{N}$, μ=Poisson $[\lambda]$
- $LF(n) = \lambda(F(n+1) - F(n)) + n(F(n-1) - F(n))$
- $X(t)$ = nb of occupied servers in M/M/∞
- Dist. $X(t)$ = Poisson$[\theta(t, X(0))]$ where

$$\theta(t,n) = e^{-t}n + (1-e^{-t})\lambda$$

- Semi-group

$$P_tF(n) = \sum_{k=0}^{\infty} F(k)e^{-\theta(t,n)}\frac{\theta(t,n)^k}{k!}$$

- $DF(n) = F(n+1) - F(n)$

Poisson point process

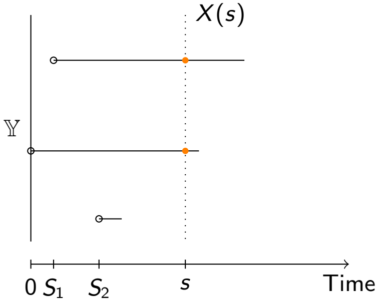
PPP over $\mathbb{Y}$

- $\mathfrak{F}$ = configuration space over $\mathbb{Y}$
- μ=dist. of PPP($\mathbf{M}$)
- Generator

$$LF(N) := \int_{\mathbf{R}^+} F(N+\epsilon_y) - F(\omega)\, \mathrm{d}\mathbf{M}(y) + \sum_{y\in N} F(N-\epsilon_y) - F(\omega)$$

- X : Glauber process
- Dist. $X(t)$=PPP($(1-e^{-t})\lambda$) + e^{-t}-thinning of the I.C.

Realization of a Glauber process



- $S_1, S_2, \cdots$: Poisson process of intensity $\mathbf{M}(\mathbb{Y})\ ds$
- Lifetimes : Exponential rv of param. 1
- Remark : Nb of particles $\sim$ M/M/∞

Discrete gradient

Definition

$$D_\tau F(N) = F(N+\epsilon_\tau) - F(N), \text{ for any } \tau \in \mathbb{Y}$$

Integration by parts

Definition

$\delta^\lambda = D^*$ defined by

$$\mathbf{E}_\mu\left[F\ \delta^\lambda(G)\right] = \mathbf{E}_\mu\left[\int_{\mathbb{Y}} D_\tau F\ G(\tau)\mathrm{d}\mathbf{M}(\tau)\right].$$

Theorem

For G deterministic,

$$\delta^\lambda G = \int_{\mathbb{Y}} G(\tau)(\delta N(\tau) - \mathrm{d}\mathbf{M}(\tau))$$

and $D\delta^\lambda G = G$.

Stein representation formula

Rubinstein distance between $T^*\nu$ and μ

$$P_\infty F(x) - P_0 F(x) = \int_0^\infty L P_t F(x) \mathrm{d}t$$

Stein representation formula

Rubinstein distance between $T^*\nu$ and μ

$$P_\infty F(x) - F(x) = \int_0^\infty LP_t F(x) \mathrm{d}t$$

Stein representation formula

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$$\int_{\mathfrak{F}} F \mathrm{d}\mu - F(x) = \int_0^\infty LP_t F(x) \mathrm{d}t$$

Stein representation formula

Rubinstein distance between $T^*\nu$ and μ

$$\int_{\mathfrak{F}} F\mathrm{d}\mu - \int_{\mathfrak{F}} F(x)\,\mathrm{d}(T^*\nu)(x) = \int_{\mathfrak{F}}\int_0^{\infty} LP_tF(x)\mathrm{d}t\,\mathrm{d}(T^*\nu)(x)$$

Stein representation formula

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IPP on initial space

$$\int_{\mathfrak{F}} F\mathrm{d}\mu - \int_{\mathfrak{E}} F \circ T\mathrm{d}\nu = \int_{\mathfrak{E}} \int_0^\infty (L^\mu P_t^\mu F) \circ T \ \mathrm{d}\nu \ \mathrm{d}t$$

Stein representation formula

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$$\int_{\mathfrak{F}} F \mathrm{d}\mu - \int_{\mathfrak{F}} F(x)\, \mathrm{d}(T^*\nu)(x) = \int_{\mathfrak{F}} \int_0^\infty LP_t F(x) \mathrm{d}t\, \mathrm{d}(T^*\nu)(x)$$

IPP on initial space

$$\int_{\mathfrak{F}} F \mathrm{d}\mu - \int_{\mathfrak{E}} F \circ T \mathrm{d}\nu = \int_{\mathfrak{E}} \int_0^\infty D^\nu((P_t^\mu F) \circ T)\, \mathrm{d}\nu\, \mathrm{d}t$$

$$+ \text{Remainder}$$

Last steps

Commutation of gradients

$$\int_{\mathfrak{F}} F \mathrm{d}\mu - \int_{\mathfrak{E}} F \circ T \mathrm{d}\nu = \int_{\mathfrak{E}} \int_0^\infty D^\mu (P_t^\mu F) \circ T \ \mathrm{d}\nu \ \mathrm{d}t + \text{Remainder} + \text{Remainder}'$$

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Intertwining

$$D^\mu P_t^\mu F = e^{-\Phi^\mu(t)} P_t^\mu D^\mu F$$

Example

Our settings

- $\mathbf{P}_t$: PPP of intensity $t\mathbf{K}$ on $C \subset \mathbb{X}$
- f : dom $f = C^2/\mathfrak{S}_2 \longrightarrow \mathbb{Y}$

Definition

$$T(\sum_{x\in\eta}\delta_x) = \sum_{(x_1,x_2)\in\eta^k_{\neq}} \delta_{t^{2/(d-1)}f(x_1,x_2)} := \xi(\eta)$$

- $\mathbf{L}$: image measure of $(t\mathbf{K})^2$ by f
- $\mathbf{M}$: intensity of the target Poisson PP

Example

Main result

Theorem (Two moments are sufficient)

$$\sup_{F \in \mathrm{TV\text{–}Lip}_1} \mathbf{E}\Big[F\big(PPP(\mathbf{M})\big)\Big] - \mathbf{E}\Big[F\big(T^*(PPP(t\mathbf{K}))\big)\Big]$$

Main result

Theorem (Two moments are sufficient)

$$\sup_{F \in \text{TV–Lip}_1} \mathbf{E}\Big[F\big(PPP(\mathbf{M})\big)\Big] - \mathbf{E}\Big[F\big(T^*(PPP(t\mathbf{K}))\big)\Big]$$

$$\leq \text{dist}_{TV}(\mathbf{M}, \mathbf{L}) + 2(var\,\xi(\mathbb{Y}) - \mathbf{E}\xi(\mathbb{Y}))$$

What we have to compute

Distance representation

$$
\begin{aligned}
&\mathrm{d}_{\mathrm{R}}(\mathrm{PPP}(\mathbf{M}), T^*(\mathrm{PPP}(t\mathbf{K}))) \\
&= \sup_{F \in \mathrm{TV\text{–}Lip}_1} \Bigg(\mathbf{E} \int_0^\infty \int_{\mathbb{Y}} [P_t F(\xi(\eta) + \delta_y) - P_t F(\xi(\eta))] \, \mathbf{M}(\mathrm{d}y) \, \mathrm{d}t \\
&\qquad + \mathbf{E} \int_0^\infty \sum_{y \in \xi(\eta)} [P_t F(\xi(\eta) - \delta_y) - P_t F(\xi(\eta))] \, \mathrm{d}t \Bigg)
\end{aligned}
$$

Transformation of the stochastic integral

Bis repetita

$$
\begin{aligned}
&d_{\mathrm{R}}(\mathrm{PPP}(\mathbf{M}), \xi(\eta)) \\
&= \sup_{F \in \mathrm{TV-Lip}_1} \Bigg(\mathbf{E} \int_0^\infty \int_{\mathbb{Y}} [P_t F(\xi(\eta) + \delta_y) - P_t F(\xi(\eta))] \, \mathbf{M}(dy) \, dt \\
&\qquad + \mathbf{E} \int_0^\infty \sum_{y \in \xi(\eta)} [P_t F(\xi(\eta) - \delta_y) - P_t F(\xi(\eta))] \, dt \Bigg)
\end{aligned}
$$

Mecke formula

Mecke formula $\iff$ IPP

$$\mathbf{E}\left[\sum_{y\in\zeta} f(y,\zeta)\right] = \mathbf{E}\left[\int_{\mathbb{Y}} f(y,\zeta+\delta_y)\,\mathbf{M}(\mathrm{d}y)\right]$$

is equivalent to

$$\mathbf{E}\left[\int_{\mathbb{Y}} D_y U(\zeta) f(y,\zeta)\,\mathbf{M}(\mathrm{d}y)\right] = \mathbf{E}\left[U(\zeta)\int_{\mathbb{Y}} f(y,\zeta)(\mathrm{d}\zeta(y) - \mathbf{M}(\mathrm{d}y))\right]$$

where

$$D_y U(\zeta) = U(\zeta+\delta_y) - U(\zeta)$$

Consequence of the Mecke formula

Proof.

$$\mathbf{E}_\eta \sum_{y\in\xi(\eta)} P_tF(\xi(\eta)-\delta_y) - P_tF(\xi(\eta))$$

$$= \int_{\mathrm{dom} f} \mathbf{E}_\eta \Big[P_tF(\xi(\eta)) - P_tF(\xi(\eta)+\delta_{f(x_1,x_2)})\Big] \mathbf{K}^2(\mathrm{d}(x_1,x_2))$$

$$+ \text{Remainder}$$

□

Consequence of the Mecke formula

Proof.

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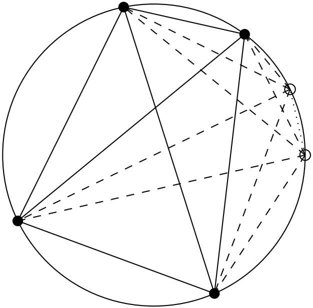
$$+ \text{Remainder}$$

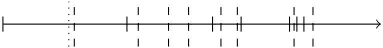
□

Problem

$$\xi(\eta)+\delta_{f(x_1,x_2)} \neq \xi(\eta+\delta_{x_1}+\delta_{x_2})$$

A bit of geometry





$$\eta + \delta_x + \delta_y$$

$$\xi(\eta + \delta_x + \delta_y)$$

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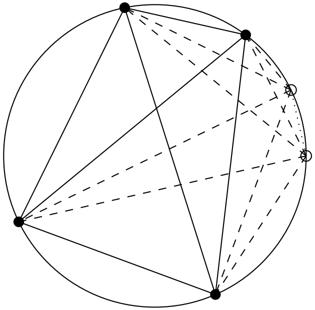

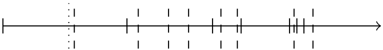

$$\eta + \delta_x + \delta_y \qquad\qquad \xi(\eta + \delta_x + \delta_y)$$

Conclusion

$$\xi(\eta + \delta_x + \delta_y) = \xi(\eta) + \xi(\delta_x + \delta_y) + \hat{\xi}(x, y; \eta)$$

Last step

$$\mathrm{d}_{\mathrm{R}}(\mathrm{PPP}(\mathbf{M}), \xi(\eta))$$
$$= \sup_{F \in \mathrm{TV-Lip}_1} \left(\int_0^\infty \int_{\mathbb{Y}} \mathbf{E}_\zeta \left[P_t F(\zeta + \delta_y) - P_t F(\zeta) \right] (\mathbf{M} - \mathbf{L})(\mathrm{d}y) \, \mathrm{d}t \right.$$
$$\left. + \mathrm{Remainder} \right)$$

A key property (on the target space)

Definition

$$D_x F(\zeta) = F(\zeta + \delta_x) - F(\zeta)$$

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Intertwining property

For the Glauber point process

$$D_x P_t F(\zeta) = e^{-t} P_t D_x F(\zeta)$$

Consequence

$$\int_0^\infty \int_{\mathbb{Y}} \mathbf{E}_\zeta \left[P_t F(\zeta + \delta_y) - P_t F(\zeta)\right] (\mathbf{M} - \mathbf{L})(\mathrm{d}y)\,\mathrm{d}t$$
$$= \int_0^\infty \int_{\mathbb{Y}} \mathbf{E}_\zeta[D_y P_t F(\zeta)]\,(\mathbf{M} - \mathbf{L})(\mathrm{d}y)\,\mathrm{d}t$$
$$= \int_0^\infty e^{-t} \int_{\mathbb{Y}} \mathbf{E}_\zeta[P_t D_y F(\zeta)]\,(\mathbf{M} - \mathbf{L})(\mathrm{d}y)\,\mathrm{d}t$$
$$\le \int_{\mathbb{Y}} |\mathbf{M} - \mathbf{L}|(\mathrm{d}y)$$

Generic Theorem

Theorem

$$\mathrm{d_R}(\, PPP(\mathbf{M})|_{[0,a]}\,,\, \xi(\eta)|_{[0,a]}) \\ \leq \mathrm{d_{TV}}(\mathbf{L},\mathbf{M}) + 3\cdot 2^{k+1}\,(\mathbf{L})(\mathbb{Y})\, r(\mathrm{dom} f)$$

where

$$r(\mathrm{dom} f) := \sup_{\substack{1\le\ell\le k-1,\\ (x_1,\dots,x_\ell)\in\mathbb{X}^\ell}} \mathbf{K}^{k-\ell}(\{(y_1,\dots,y_{k-\ell})\in\mathbb{X}^{k-\ell} : \\ (x_1,\dots,x_\ell,y_1,\dots,y_{k-\ell})\in \mathrm{dom} f\})$$

Example (cont'd)

Theorem

$$\mathrm{d}_{\mathrm{R}}\big(\, PPP(\mathbf{M})|_{[0,a]}\,,\, \xi(\eta)|_{[0,a]}\big) \le C_a t^{-1}$$

Compound Poisson approximation

The process

$$L_t^{(b)}(\mu) = \frac{1}{2} \sum_{(x,y)\in\mu^2_{t,\neq}} \|x-y\|^b \mathbf{1}_{\|x-y\|\leq\epsilon_t}$$

Theorem (Reitzner-Schulte-Thäle (2013) w.o. conv. rate)

Assume

- $t^2\epsilon_t^b \xrightarrow{t\to\infty} \lambda$

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- $N \sim Poisson(\kappa_d\lambda/2)$
- $(X_i,\, i\ge 1)$ *iid, uniform in* $B_d(\lambda^{1/d})$
- *Then*

$$d_{TV}(t^{2b/d} L_t^{(b)}, \sum_{j=1}^N \|X_j\|^b) \le c(|t^2\epsilon_t^b - \lambda| + t^{-\min(2/d,1)})$$

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