Fundamental Theorems

Arithmetics $N = p_1^{n_1} p_2^{n_2} \cdots p_K^{n_K}$ (prime factorization)
=> the prime number theorem => the Riemann Hypothesis

Algebra $P_N(z) = a(z - z_1)^{n_1} (z - z_2)^{n_2} \cdots (z - z_K)^{n_K}$,
$n_1 + \ldots + n_K = N$
=> Galois theory.

Calculus $\frac{d}{dt} \int_0^t f(s)\, ds = f(t)$ (=> the Index theorem)

Linear Algebra Rank–Nullity [?]

Statistics Glivenko-Cantelli [!]
(empirical cdf → true cdf)

Signal processing Sampling Theorem [?]
(Nyquist-Kotel'nikov-Raabe-Someya-Shannon)

Numerical analysis consistency + stability => convergence [!]

How about

Weierstrass approximation theorem

Karl Theodor Wilhelm Weierstrass
(1815–1897)

Statement a continuous function on a bounded interval can be approximated by a polynomial.

Note The Theorem covers the Weierstrass function (continuous, not differentiable)

Proofs

a) using Heat equation - original by Weierstrass himself

b) using Fourier Series - in a typical analysis class

c) using Law of Large Numbers (for Binomial distribution) - well-suited for a probability class

To remember

a) Chebyshev polynomials.

b) (Non) Analytic functions.