MATH 445

Computer Project 1   Due Friday, October 25, 2024

Please note:

- You are welcome to use any software package and any help, including any on-line resource you can find.
- If you are not comfortable with computers, start early and ask for help often.
- Submit your work as a single pdf file, by uploading to the corresponding portal on Brightspace.
- Do not miss classes: throughout the semester, we will discuss various questions related to the project, both at the lectures and in discussions.

Problem 1. The main objective is to practice plotting graphs; thinking about the results is somewhat optional, but is highly recommended.

Part 1. Plot the graph of the function

$$h_{10}(x) = \sum_{k=1}^{10} \frac{\sin\Big((k!)^2 x\Big)}{k!}$$

for $x \in [0,1]$, $x \in [0,0.1]$, $x \in [0,0.01]$, $x \in [0,0.001]$. **Make sure to use the right step size for plotting: to get meaningful pictures, the step size must be small enough and must depend on the length of the interval.**

What you will turn in:

(1) Four separate graphs.
(2) Printout of the program you used to generate the graphs.

Each page you turn in must have your name and date *printed* on it. Each graph must have a title, labeled axes, and the scale along each axis.

Part 2. Consider the function

$$h(x) = \sum_{k=1}^{\infty} \frac{\sin\Big((k!)^2 x\Big)}{k!}.$$

For what $x \in [0,1]$ will this function be

(1) defined?
(2) continuous?
(3) differentiable?

Provide a one-sentence explanation for each of your answers.

Problem 2. The objective is to compute Fourier coefficients numerically and to analyze the Gibbs phenomenon.

Let $f = f(x)$ be a 2π-periodic function defined for $x \in (-\pi, \pi]$ by $f(x) = x$. Let $S_n(x) = \sum_{k=1}^{n} b_k \sin(kx)$, where $b_k = (1/\pi)\int_{-\pi}^{\pi} f(x)\sin(kx)dx$.

Do the following:

(1) Plot the graphs of $S_n(x)$ for $n = 10, 50, 100$ and $x \in [-\pi, \pi]$. The choice of the procedure to compute b_k is up to you. Keep in mind that if you divide the interval $[-\pi, \pi]$ to approximate the integral, your step size must be small enough to "see" the oscillations of the sines.
(2) Estimate $\max_{x\in[-\pi,\pi]} S_n(x)$ for $n = 10, 50, 100$.
(3) Compute $\lim_{n\to\infty} \frac{S_n(\pi-\pi/n)}{\pi}$ (either compute analytically or guess from the graphs). This is a quantitative measure of the Gibbs phenomenon.

What you will turn in:

(1) Three separate graphs with the corresponding values of $\max_{x\in[-\pi,\pi]} S_n(x)$.
(2) Printout of the program you used to generate the graphs. Please indicate the procedure you used to compute the Fourier coefficients b_k.
(3) The numerical value of $\lim_{n\to\infty} \frac{S_n(\pi-\pi/n)}{\pi}$ and the corresponding explanations.

Each page you turn in must have your name and date *printed* on it. Each graph must have a title, labeled axes, and the scale along each axis.

Instructor: Sergey Lototsky, KAP 248D.

Computer Project 2 Due Friday, December 6, 2024

Please note:

- You are welcome to use any software package and any help, including any on-line resource you can find. Be careful when using an "off-the-shelf" PDE solver: figuring out all the details about the way it works can take longer than writing your own code.
- If you are not comfortable with computers, start early and ask for help often.
- Submit your work as a single pdf file, by uploading to the corresponding portal on Brightspace.
- Do not miss classes: throughout the semester, we will discuss various questions related to the project, both at the lectures and in discussions.

The objective of this assignment is to see how implicit numerical schemes work for parabolic and hyperbolic equations.

Problem 1. Consider the heat equation

$$u_t(x,t) = 0.25u_{xx}(x,t),\ 0 < t \le 2,\ 0 < x < 1,$$

with $u(0,t) = u(1,t) = 0$ and

$$u(x,0) = \begin{cases} 20x, & 0 \le x \le 1/2 \\ 20(1-x), & 1/2 \le x \le 1. \end{cases}$$

Solve it numerically by the Crank-Nicholson method taking $h = k = 0.1$. Plot a 3-D graph of the result. Then compare the result with the Fourier series solution (use your judgement as to how many terms to keep in the Fourier series: this is your only chance to get to the exact solution as close as possible).

What you will turn in:

(1) The graph $(x, t, \bar{u}(x,t))$, where $\bar{u}$ is the numerical solution you got.
(2) The graph $(x, t, |\bar{u}(x,t) - u(t,x)|)$, where u is the Fourier series solution.
(3) Printout of the program you used to generate the graphs.

Each page you turn in must have your name and date *printed* on it. The graph must have a title, labeled axes, and the scale along each axis.

Problem 2. Consider the wave equation

$$u_{tt}(x,t) = u_{xx}(x,t),\ 0 < t \le 2,\ 0 < x < 1,$$

with $u(0,t) = u(1,t) = u_t(x,0) = 0$, $u(x,0) = x(1-x)$.

Solve it numerically by the implicit method taking $h = k = 0.1$. Plot a 3-D graph of the result. Then compare the result with the exact solution (unlike the heat equation, you can get the exact solution without the Fourier series.)

What you will turn in:

(1) The graph $(x, t, \bar{u}(x,t))$, where $\bar{u}$ is the numerical solution you got.
(2) The graph $(x, t, |\bar{u}(x,t) - u(t,x)|)$, where u is the exact solution. This time, you have at least two choices for the exact solution: you can either truncate the Fourier series of the solution or you can use d'Alembert's formula and get the truly exact solution in the form

$$u(t,x) = \frac{S_{f,s}(x+ct) + S_{f,s}(x-ct)}{2}$$

where $S_{f,s}$ is the Fourier sine series of the function $f(x) = x(1-x)$.
(3) Printout of the program you used to generate the graphs.

Each page you turn in must have your name and date *printed* on it. The graph must have a title, labeled axes, and the scale along each axis.