# MATH 125(S. Lototsky)

Practice Problems: Application of Differentiation, Integration

1. Find the minimal and maximal values of the function $f(x) = x^4 + 4x^3 + 4x^2$ on the following sets:
(a) $[-1/2, 1/2]$;
(b) $(-2, -0.5] \bigcup (0, 1]$ (The symbol $\bigcup$ means the union of the two sets, that is, you consider the values of $f$ both in $[-2, -0.5)$ and in $(0, 1]$.)
(c) $(-\infty, +\infty)$.

2. Suppose that $f'(x) = x(x-1)^2(x+1)^3(x-2)(x+3)$. Find the intervals where the function $f$ is increasing and decreasing, and the points of local min and max. How will your answer change if $f'(x) = x(x-1)^2(x+1)^3(2-x)(x+3)$?

3. Find all asymptotes of the following functions:
(a) $f(x) = \dfrac{\sqrt{x^2+1}}{x-1}$.
(b) $f(x) = \dfrac{x^2}{x^2-1}$.
(c) $f(x) = \dfrac{x^3+x^2+2x+1}{x^2+1}$.

4. Find the largest area of the rectangle that can be inscribed in a right triangle with sides 3, 4, and 5 cm, if
(a) the rectangle and the triangle share right angle.
(b) one of the sides of the rectangle is on the hypothenuse of the triangle.

5. Evaluate the following limit:

$$\lim_{n\to\infty} \frac{3}{n} \sum_{k=0}^{n} \sqrt{1 + \frac{3k}{n}}.$$

6. Compute $f'(x)$ if

$$f(x) = \int_{\sin x}^{x^2+1} \cos(\sqrt{t})dt.$$

7. Evaluate the following integrals:
(a) $\displaystyle\int_0^{\sqrt{\pi}} x\sin(x^2)dx$ (b) $\displaystyle\int x\sin(2x)dx$ (c) $\displaystyle\int_0^1 \frac{x^2-1}{\sqrt{x}}dx$ (d) $\displaystyle\int_0^1 \frac{xdx}{\sqrt{1+5x^2}}$ (e) $\displaystyle\int_0^{\pi/2} \frac{\cos(x)}{(5-\sin(x))^2}dx$

8. Show that

$$\frac{1+\sqrt{2}}{4} < \int_{\pi/4}^{\pi/2} \frac{\sin(x)}{x}dx < \frac{\sqrt{2}}{2}.$$

9. Evaluate the following limit:

$$\lim_{h\to 0} \frac{1}{h} \int_2^{2+h} \sqrt{1+t^2}dt.$$

10. Sketch the graph of the function $f(x) = \dfrac{x}{2+3x^2}$.

## Answers

Watch out for misprints...

1. $f(x) = x^2(x+2)^2$, $f'(x) = 4x(x+1)(x+2)$. (a) Max is $f(1/2)$, min is 0. (b) Max is $f(1)$, min does not exist (would be 0, but points $-2$ and 0 are excluded from the set. (c) Max does not exist ($f$ can be as large as you want), min is 0.

2. Increasing on $(-\infty, 3]\bigcup[-1,0]\bigcup[2,+\infty)$, decreasing on $[-3,-1]\bigcup[0,2]$; points of local max are $-3, 0$; points of local min are $-1, 2$. If you have $(2-x)$ instead of $(x-2)$, then min/max and increading/decreasing will switch places.

3. (a) $y = \pm 1$ as $x \to \pm\infty$; $x = 1$. (b) $x = \pm 1$; $y = 1$. (c) $y = x + 1$.

4. Denote by $x$ and $y$ the sides of the rectangle. Then, in (a), you maximize $xy$ given $4x+3y = 12$, and, in (b), you maximize $xy$ given $12x + 25y = 60$ (depending on your notations, you might have $x$ and $y$ switched. The answer is $\boxed{3 \text{ cm}^2}$ in both cases.

5. $3\int_0^1 \sqrt{1+3x}dx = 14/3$.

6. $f'(x) = 2x\cos(\sqrt{x^2+1}) - \cos(x) \cdot \cos\sqrt{\sin(x)}$.

7. (a) $-0.5\cos(x^2)|_0^{\sqrt{\pi}} = 1$ (b) $(\sin(2x) - 2x\cos(2x))/4 + c$ (by parts) (c) $-8/5$ (d) $(\sqrt{6}-1)/5$ (e) $1/(5-u)|_0^1 = 1/20$ with $u = \cos(x)$.

8. From above — by left point rule; from below — by trapezoidal rule with no extra partition of the interval.

9. $f'(2) = \sqrt{5}$, where $f(x) = \int_0^x \sqrt{1+t^2}dt$.

10. Function is odd; for $x > 0$, have local (in fact, global) max at $\sqrt{2/3}$, and inflection at $\sqrt{2}$; horizontal asymptote $y = 0$.