# MATH 125 (S. Lototsky)

Practice Problems: Limits and Derivatives

1. Compute the following limits.

(a) $\lim_{x\to 2} \frac{2x^2-x-6}{x-2}$ (b) $\lim_{x\to 1} \frac{x-1}{\sqrt{x}-1}$

(c) $\lim_{h\to 0} \frac{\tan(\pi/4+h)-1}{h}$ (d) $\lim_{x\to 1} \frac{x-1}{|x^2-1|}$

(e) $\lim_{x\to 1^+} \frac{\sin\left(\frac{\pi}{2x}\right)}{\sqrt{x^5+3}}$.

2. Use the definition of derivative to compute $f'(2)$, where $f(x)=\sqrt{x}$.

3. Compute the derivative of the following function:

$$f(x) = \frac{\sqrt{x^2+1}+x}{\cos(\sin^3(x))}.$$

4. Compute the third derivative of $f(x) = \frac{x-1}{x+1}$.

5. Compute $y'(x)$ if $x^2y + xy^5 = 2$.

6. Let $f(x) = x^2+2x-1$ and $\varepsilon = 0.001$. Find $\delta$ so that $|f(x)-7| < \varepsilon$ for all $x$ satisfying $0 < |x-2| < \delta$.

7. For what $x$ is the function $f(x) = \tan\sqrt{x^2-1}$ continuous?

8. For what values of $A$ will the function

$$f(x) = \begin{cases} x^3, & x \le 0 \\ Ax+1, & x > 0 \end{cases}$$

be differentiable at $x=0$?

9. For what values of $A$ and $B$ will the function

$$f(x) = \begin{cases} x^3, & x \le 2 \\ Ax+B, & x > 2 \end{cases}$$

be differentiable at $x=2$?

10. Find an approximate value of $(16.00008)^{1/4}$.

11. Suppose that $f=f(x)$ is a differentiable function; $f(1)=2$, $f'(1)=-1$. Define $g(x) = x+\sqrt{2+f(x^2)}$. Write the equation of the tangent line to the curve $y=g(x)$ at the point $(1,3)$.

12. True of false: equation $x^3+2x=4$ has a solution on $(1,2)$? Explain your answer.

13. Sketch the graph of the function with the following properties: $f$ has a limit at $x=0$ but is not continuous at $x=0$; $f$ is continuous from the left at $x=1$, but $\lim_{x\to 1} f(x)$ does not exist; $f$ has a vertical asymptote at $x=2$, $\lim_{x\to 3} f(x) = -\infty$; $\lim_{x\to 4^-} f(x) = +\infty$; $\lim_{x\to 4^+} f(x) = -\infty$.

14. Boat A is 90 miles west of boat B. At noon, boat A starts going east at 15 miles per hour and boat B starts going north at 10 miles per hour. Assume that each boat moves with constant velocity all the time.

(a) At 4 pm, is the distance between the boats increasing or decreasing?

(b) Is there a moment before 5 pm when the distance between the boats is not changing?

**Answers**

Watch out for misprints...

1. (a) **7** $(2x^2 - x - 6 = (x-2)(2x+3))$
(b) **2** $(x - 1 = (\sqrt{x} - 1)(\sqrt{x} + 1)$
(c) **2** (it is the derivative of $\tan x$ at $x = \pi/4$
(d) **does not exist** (you get $1/2$ from the right and $-1/2$ from the left.)
(e) **1/2** (just put $x = 1$: the function is continuous there).

2. $\mathbf{\frac{1}{2\sqrt{2}}}$ (you compute $\lim_{x\to 2} \frac{\sqrt{x}-\sqrt{2}}{x-2}$; similar to 1(b).)

3.

$$f'(x) = \frac{x(1+x^2)^{-1/2} + 1}{\cos(\sin^3(x))} + \frac{(\sqrt{1+x^2} + x) \cdot 3\sin^2(x)\cos(x)\sin(\sin^3(x))}{\cos^2(\sin^3(x))}$$

(no comments...)

4. $\mathbf{\frac{12}{(x+1)^4}}$ $(f(x) = 1 - \frac{2}{x+1}.)$

5. $2xy + x^2y' + y^5 + 5xy^4y' = 0; \mathbf{y}'(x) = -\frac{2xy+y^5}{x^2+5xy^4}$ (implicit differentiation).

6. For example, $\delta = \varepsilon/7$ $(|f(x) - 7| = |(x-2)(x+4)|$ and $|x+4| < 7$ if $x$ is near 2.)

7. $|x| \geq 1$ and $x \neq \pm\sqrt{1 + (\pi/2 + \pi n)^2}, n = 0, \pm 1, \pm 2, \ldots$ (that's the domain of the function.)

8. **For no** $A$ (the function is not continuous at 0).

9. $\mathbf{A = 12, B = -16}$ (you need $8 = 2A + B$ for continuity and $12 = A$ for the derivative to exist).

10. $\mathbf{2 + 2.5 \cdot 10^{-6} = 2.0000025}$ (linear approximation with $f(x) = x^{1/4}$, $a = 16, \Delta x = 8 \cdot 10^{-5}; f'(a) = 1/32$.)

11. $\mathbf{y = 3 - (x-1)/2}$ $(g'(x) = 1 + xf'(x^2)(2 + f(x^2))^{-1/2}; g'(1) = -1/2)$.

12. **True.** (By intermediate value theorem)

13. There are many correct solutions.

14. (a) **Decreasing** (b) **Yes: some time between 4:09 and 4:10 pm**.

(Denote by $d$ the distance between the boats, and by $x$ and $y$, the distances from the boats A and B to the initial position of the boat B. Then $d^2 = x^2 + y^2$; $\dot{d} = (x\dot{x} + y\dot{y})/d$. You know that $\dot{x} = -15$, because A is approaching the initial position of B, $\dot{y} = 10$, and at 4pm, $x = 30, y = 40$, so that $\dot{d} = -1$. Since $x(t) = 90 - 15t, y(t) = 10t$, you can find $t$ when $\dot{d} = 0$; it is about 4.153 hours. )