Fundamental Theorems

Arithmetics $N = p_1^{n_1} p_2^{n_2} \cdots p_k^{n_k}$ (prime factorization)
$\Rightarrow$ the prime number theorem $\Rightarrow$ the Riemann Hypothesis

Algebra $P_N(z) = a(z - z_1)^{n_1} (z - z_2)^{n_2} \cdots (z - z_k)^{n_k}$,
$n_1 + \ldots + n_k = N$
$\Rightarrow$ Galois theory.

Calculus $\frac{d}{dt} \int_0^t f(s)\, ds = f(t)$ $\Rightarrow$ the Index theorem

Numerical analysis consistency + stability $\Rightarrow$ convergence [!]

How about

Weierstrass approximation theorem

Karl Theodor Wilhelm Weierstrass

(1815 – 1897)

Statement a continuous function on a bounded interval can be approximated by a polynomial.

Note The Theorem covers the Weierstrass function (continuous, not differentiable)

Proofs

a) using Heat equation - original by Weierstrss himself

b) using Fourier Series - in a typical analysis class

c) using Law of Large Numbers (for Binomial distribution) - well-suited for a probability class

To remember

a) Chebyshev polynomials.

b) (Non) Analytic functions.