MATH 407: The Computer Project

Problem 1. Find a procedure for sampling uniformly on the surface of the sphere.

(a) Use computer to generate a thousand points that are random, independent, and uniform on the unit sphere, and print the resulting picture.

(b) By putting sufficiently many independent uniform points on the surface of the Earth (not literally but using a computer model, of course), estimate the areas of Antarctica and Africa, compare your results with the actual values [about 5.5 million square miles/15 million square kilometers for Antarctica and about 12 million square miles/30 million square kilometers for Africa], and make a few comments (e.g. are the relative errors similar? would you expect them to be similar? if not, which one should be bigger? how does accuracy improve if you use more points? etc.)

Problem 2. Get a computer program for distinguishing a randomly generated sequence of zeroes and ones from a cooked-up one. You are welcome to write the program yourself or use what can you find on the web or in some book. Test your program on the following two sequences: the sequence consisting of the concatenation of all numbers in binary form[1]

$$0\,1\,10\,11\,100\,101\,110\,111\,1000\ldots$$

and a similar sequence consisting of the concatenation of all prime numbers in binary form

$$0\,1\,10\,11\,101\,111\,1011\ldots$$

The first sequence (the fractional part of the Champernowne number) is known to be random when considered in base 10; the second sequence (the fractional part of the Copeland-Erdös constant in binary form) is known to be random. In both cases, randomness is understood in a very specific way,[2] and you are welcome to discuss this point too.

Please note:

- You are welcome to use any software package and any help, including any on-line resource you can find.
- If you are not comfortable with computers, start early and ask for help often.
- Do not miss classes: throughout the semester, we will discuss various questions related to the project, both at the lectures and in discussions.

[1] spacing is introduced only for convenience: to indicate how the numbers are appearing

[2] The official name is *normal*, as in *normal number*; while it is known that most numbers are normal in every base, it is very hard to show that a particular number is normal, even in one particular base.