

Vector calculus: Kepler's laws, Euler equations for the solid body and the tumbling box, the three main integral theorems in one form $\int_{\partial G} \omega = \int_G d\omega$, integration without integration (gradient over a curve by the fundamental theorem of calculus, a linear form over a region by the centroid argument), continuity equation, material derivative and the Euler equations for fluids, Maxwell's equations, two formulas of Green.

Complex analysis: Cauchy-Riemann equations, conjugate harmonic functions, integral theorem of Cauchy, integral formula of Cauchy, Liouville's theorem, fundamental theorem of algebra, conformal mappings (by Möbius and Zhukovsky), Taylor series, Laurent series, analytic functions, classification of singular points, residues, residue integration, power series for ODEs and the Fuchs-Frobenius theorem.

Fourier analysis: Fourier series and approximation by trigonometric polynomials, convergence (point-wise, uniform, and mean-square), Parseval's identity, Fourier transform and Fourier integral, discretization of a signal and the sampling theorem, Plancherel's identity, Poisson's summation formula, connections (between Fourier series, Fourier transform, Fourier sine and cosine transforms, Laplace transform, z -transform, and the discrete Fourier transform), convolution in discrete and continuous time, approximation of a continuous function by polynomials (The Weierstrass Approximation Theorem), Chebyshev polynomials, Hilbert space and the inequalities of Bessel and Cauchy-Schwarz.

Partial differential equations: classification (order, elliptic/hyperbolic/parabolic, linear/semi-linear/quasi-linear), method of characteristics for first-order linear equations and for second-order linear equations in two independent variables; TRANSPORT EQUATION on the line; WAVE EQUATION on the line (general solution and the initial-value problem); WAVE EQUATION in two and three dimensional space (the method of descent and the Huygens principle); HEAT EQUATION (on the line with non-uniqueness, on the interval with uniqueness, and on the half-line in the homework); general ideas about separation of variables, eigenfunction expansion, and the Sturm-Liouville theory; the HELMHOLTZ EQUATION (general properties of the eigenvalues and eigenfunctions of the Laplacian in a bounded region, closed-form solutions for the Dirichlet Laplacian in a rectangle, disk, and ball); LAPLACE'S AND POISSON'S equations (uniqueness in a general domain, closed-form solution in a rectangle, limitations of the closed-form solutions); numerical methods (general ideas about discretization, specific examples for the heat and wave equations on the interval.)

Extracurricular activities

- (1) The Riemann zeta function and connections with number theory
- (2) Cake and Lazy Caterer's numbers
- (3) Continuous nowhere differentiable function
- (4) Infinitely differentiable function that is not analytic
- (5) Schrödinger equation and two important models from quantum mechanics (harmonic oscillator and hydrogen atom)
- (6) The telegraph equation and the story of the first transatlantic cable
- (7) Navier-Stokes equations
- (8) Burgers equation and Hopf-Cole transformation