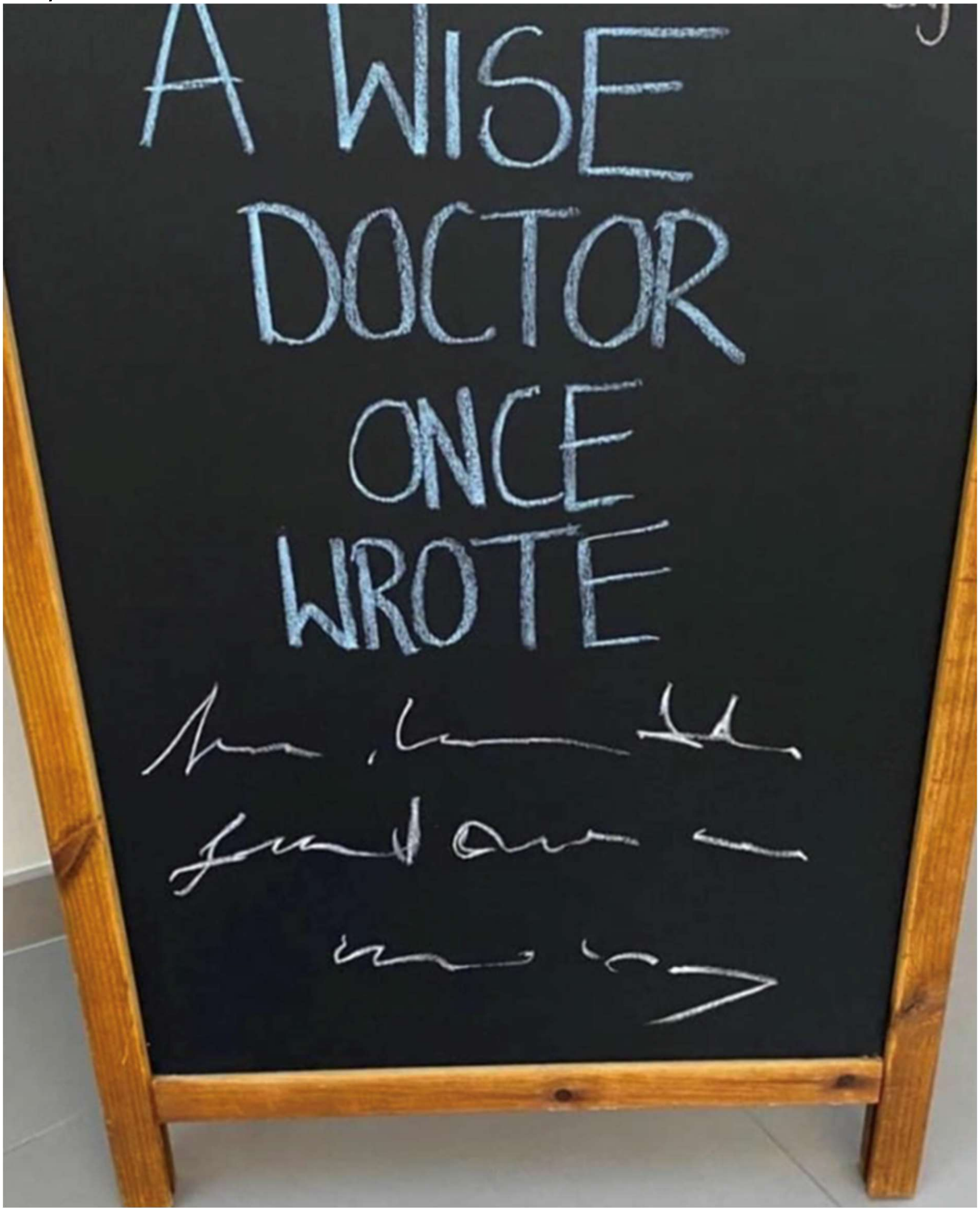
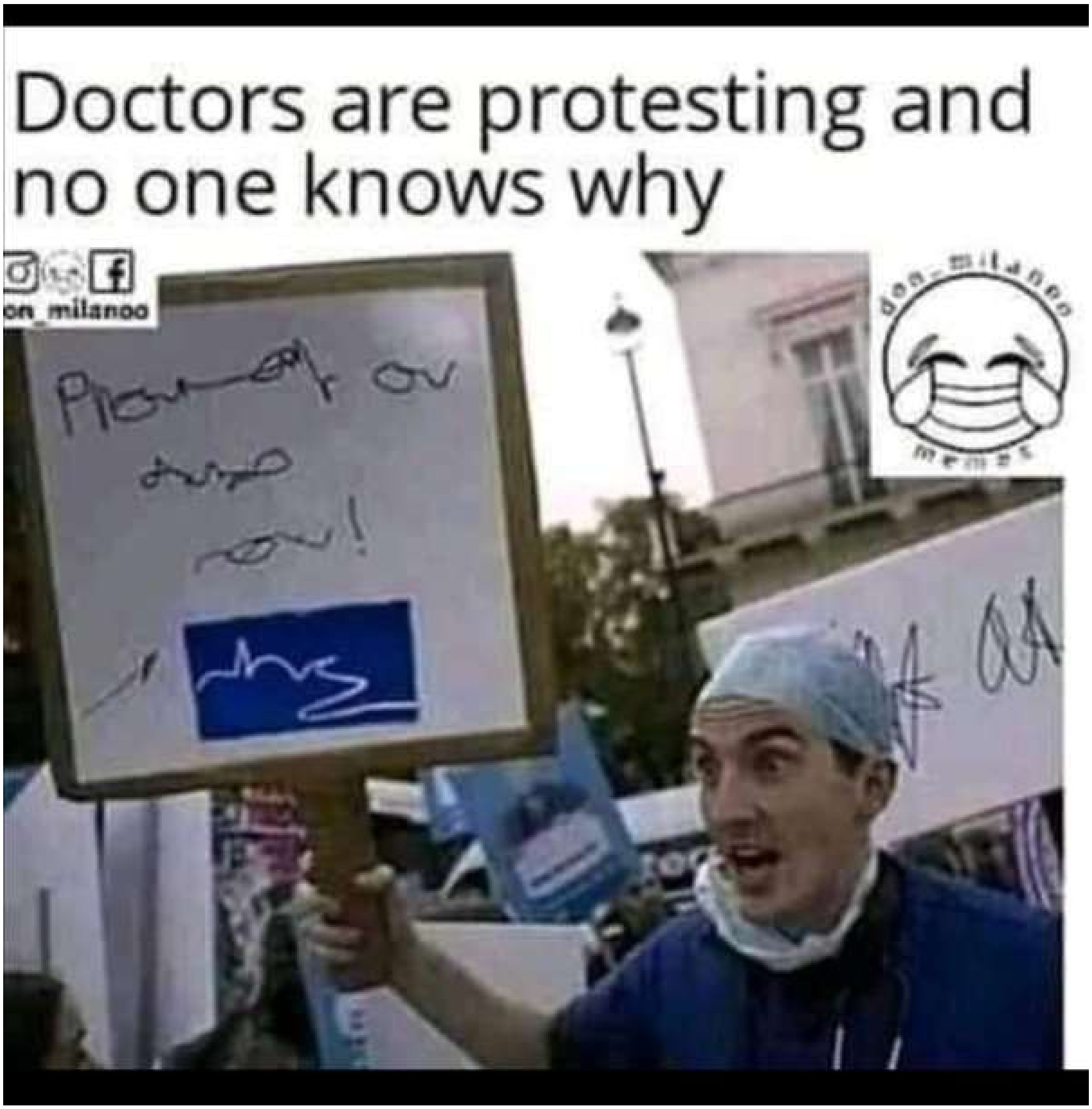
On Handwriting

Why we need a note taker



A variation



A real thing

Another real thing

En Theorema Analyticum continens Criterium, utrum formula differentialis differentialia cujusvis gradus involvens, sit integrabilis nec ne!

Cujuscunque gradus fuerit formula differentialis proposita, ea ponendo $\partial y = p\partial x$; $\partial p = q\partial x$; $\partial q = r\partial x$; etc. semper ad hanc formam revocatur $V\partial x$, in qua erit V quantitas ex litteris x, y, p, q, r etc. utcunque conflata, quae ergo more solito differentiata praebebit talem formam:

$$\partial V = M\partial x + N\partial y + P\partial p + Q\partial q + R\partial r \text{ etc.}$$

Jam dico quoties formula $V\partial x$ integrationem admittit, toties fore $N - \frac{\partial P}{\partial x} + \frac{\partial\partial Q}{\partial x^2} - \frac{\partial^3 R}{\partial x^3} + \text{etc.} = 0$ ac vicissim.

Exemplum. Sit proposita haec formula $\frac{y\partial x\partial y - x\partial y^2 + xy\partial\partial y}{yy\partial x}$ sumto ∂x constante quae per positiones factas abit in $\frac{yp - xpp + xyq}{yy}\partial x$, ut sit $V = \frac{p}{y} - \frac{xpp}{yy} + \frac{xq}{y}$ haec quantitas differentietur, fietque

$M = -\frac{pp}{yy} + \frac{q}{y}$; $N = -\frac{p}{yy} + \frac{2xpp}{y^3}$; $P = \frac{1}{y} - \frac{2xp}{yy}$; $Q = \frac{x}{y}$; $R = 0$. $S = 0$ [illegible]

Hinc fit $\partial P = -\frac{\partial y}{yy} - \frac{2p\partial x}{yy} - \frac{2x\partial p}{yy} + \frac{4xp\partial y}{y^3} = -\frac{p\partial x}{yy} - \frac{2p\partial x}{yy} - \frac{2xq\partial x}{yy} + \frac{4xpp\partial x}{y^3}$

et $\frac{\partial P}{\partial x} = -\frac{3p}{yy} - \frac{2xq}{yy} + \frac{4xpp}{y^3}$.

Porro $\partial Q = \frac{\partial x}{y} - \frac{x\partial y}{yy} = \frac{\partial x}{y} - \frac{xp\partial x}{yy}$ unde $\frac{\partial Q}{\partial x} = \frac{1}{y} - \frac{xp}{yy}$, et denuo differentiando

$\frac{\partial\partial Q}{\partial x} = -\frac{\partial y}{yy} - \frac{p\partial x}{yy} - \frac{x\partial p}{yy} + \frac{2xp\partial y}{y^3} = -\frac{p\partial x}{yy} - \frac{p\partial x}{yy} - \frac{xq\partial x}{yy} + \frac{2xpp\partial x}{y^3}$

ideoque $\frac{\partial\partial Q}{\partial x^2} = -\frac{2p}{yy} - \frac{xq}{yy} + \frac{2xpp}{y^3}$. Quare habebitur

$$N - \frac{\partial P}{\partial x} + \frac{\partial\partial Q}{\partial x^2} - \text{etc.} = \begin{array}{l} -\frac{p}{yy} + \frac{2xpp}{y^3} - \frac{xq}{yy} \\ +\frac{3p}{yy} - \frac{4xpp}{y^3} + \frac{2xq}{yy} \\ -\frac{2p}{yy} + \frac{2xpp}{y^3} - \frac{xq}{yy} \end{array} = 0$$

Ergo formula $V\partial x$ est integrabilis. Manifesto autem integrale est $\frac{x\partial y}{y\partial x}$