Summary of Normal Distribution

1. If $X \sim \mathcal{N}(\mu, \sigma^2)$, then

$$\frac{X-\mu}{\sigma} \sim \mathcal{N}(0,1)$$

2. If $Z \sim \mathcal{N}(0,1)$, and $a > 0$, then
   - the values of $P(0 < Z < a)$ are obtained from the table;
   - $P(-a < Z < 0) = P(0 < Z < a)$;
   - $P(Z > a) = P(Z < -a) = 0.5 - P(0 < Z < a)$;
   - $P(|Z| > a) = 2P(Z > a)$;
   - $P(Z < a) = P(Z > -a) = 0.5 + P(0 < Z < a)$.

Note: $P(Z < a) > 0.5$ if and only if $a > 0$; $P(Z > b) > 0.5$ if an only if $b < 0$. For example,
   - $P(Z < 1.1) = 0.5 + P(0 < Z < 1.1) = 0.8643$;
   - If you know that $P(Z > c) = 0.6179$, then $c < 0$ and $P(0 < Z < |c|) = 0.1179$, which means that $|c| = 0.3$ and $c = -0.3$.

Drawing a picture of the “Bell Curve” is very helpful

3. If $Y_1, \ldots, Y_n$ are independent so that $Y_k \sim \mathcal{N}(\mu_k, \sigma_k^2)$ and $a_1, \ldots, a_n$ are real numbers, then $a_1Y_1 + \ldots + a_nY_n \sim \mathcal{N}(\mu, \sigma^2)$, where

$$\mu = a_1\mu_1 + \ldots + a_n\mu_n, \quad \sigma^2 = a_1^2\sigma_1^2 + \ldots + a_n^2\sigma_n^2.$$

In particular,
   - If $Y_k,\ k = 1, \ldots, n$, are iid $\mathcal{N}(\mu, \sigma^2)$, then $Y_1 + \ldots + Y_n \sim \mathcal{N}(n\mu, n\sigma^2)$,
   - If $Y_1 \sim \mathcal{N}(\mu_1, \sigma_1^2)$ and $Y_2 \sim \mathcal{N}(\mu_2, \sigma_2^2)$ are independent, then $Y_1 - Y_2 \sim \mathcal{N}(\mu_1 - \mu_2, \sigma_1^2 + \sigma_2^2)$.

Summary of Central Limit Theorem (CLT)

1. **Basic result:** if $X_1, \ldots, X_n$ are iid with mean μ and standard deviation σ, and $n > 30$, then $X_1 + \ldots + X_n$ is approximately normal with mean $n\mu$ and standard deviation $\sqrt{n}\sigma$, while the sample mean $\bar{X}_n = (X_1 + \ldots + X_n)/n$ is approximately normal with mean μ and standard deviation $\sigma/\sqrt{n}$. In the end, though, everything reduces to

$$\frac{X_1 + \ldots + X_n - n\mu}{\sigma\sqrt{n}} \approx \mathcal{N}(0,1).$$

2. **CLT for Binomial distribution:** if $np(1-p) > 5$, then $\mathcal{B}(n,p) \approx \mathcal{N}(np, np(1-p))$.
In the problems:
   (1) Identify the “success” event.
   (2) Compute the probability of success p.
   (3) Check that $np(1-p) > 5$.
   (4) Use continuity correction. For example, $P(X > m) = P(X \geq m+1) = P(X > m + 0.5)$.
   (5) Normalize: $\dfrac{X - np}{\sqrt{np(1-p)}} \approx \mathcal{N}(0,1)$.
   (6) Use the table of the normal distribution keeping in mind the properties listed above.
3. **CLT for general distributions:**
   (1) Compute the expected value and standard deviation for the distribution. If the distribution is continuous, you might need integration.
   (2) Check whether the question is asking for the sum or for the sample mean.
   (3) If the distribution is discrete, use continuity correction.
   (4) Remember to normalize properly.