The orchestra in the eyes of the wave equation

Sound : $u = u(t, x)$ $\quad t > 0,$
$x \in G \subset \mathbb{R}^n$
$n = 1, 2, 3$

$$\frac{\partial u}{\partial t} = c^2 \Delta u + f(t, x)$$ – the wave equation

Also need
Initial conditions (IC): $u(0, x)$, $\frac{\partial u}{\partial t}(0, x)$
(initial displacement and initial speed)

Boundary conditions (BC): Values of $u(t, x)$ for $x \in \partial G$
[for x on the boundary of G]

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1) $G = [0, L]$, zero BC: $u(t,0) = u(t,L) = 0$.

strings

$c = \sqrt{\frac{\text{tension}}{\text{density}}}$

$f(t, x)$ – "the bow technique"

flute

c = "speed of sound in air" ($\sim 343 \frac{\text{meters}}{\text{second}}$)

2) $n = 1$ (one-dimensional object); c – speed of sound in air.

$$\underbrace{u(t, \text{"open end"}) = 0, \quad \frac{\partial u}{\partial x}(t, \text{"the other end"}) = 0}$$

$G = [0, L]$: clarinet

More complicated shapes:

sax, bassoon, all brass

3) $n = 1$, c – speed of sound in air.

$u(t, \text{"open end"}) = 0$, $u(t, \text{"the other end"})$ – input from the reed.

↓

oboe, english horn

4) $n = 2$, $G = \{(x, y) : x^2 + y^2 \leq R^2\}$

$c = \sqrt{\frac{\text{tension}}{\text{density}}}$ $\qquad u\big|_{\partial G} = 0$

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all drums

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sound is described by Bessel function
(as opposed to trig. functions)

5) $n = 3$ – "Bells".
