Spring 2013, MATH 245, Exam 2

Wednesday, March 13, 2013; 9–9:50am

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Name: _____

Circle the time of your discussion section: **2pm 3pm 4pm**

Instructions:

- No notes, books, calculators, etc.
- Answer all questions and clearly indicate your answers.
- **Show your work!** Points might be taken off for correct answer with no explanations. Wrong answer with no explanations is worth zero points.

Problem	Possible	Actual
1	10	
2	15	
3	10	
4	15	
Total	50	

Properties of the Laplace transform

Function	Laplace transform	Function	Laplace transform
$f(t)$	$F(s) = \int_0^{+\infty} e^{-st} f(t) dt$	$1 = u_0(t)$	$\dfrac{1}{s}$
$af(t) + bg(t)$	$aF(s) + bG(s)$	$\delta(t)$	1
$e^{-ct} f(t)$	$F(s + c)$	e^{at}	$\dfrac{1}{s-a}$
$f(t - c) = f(t - c)u_c(t)$	$e^{-cs} F(s), \ (c > 0)$	$\sin(at)$	$\dfrac{a}{s^2 + a^2}$
$f'(t)$	$sF(s) - f(0)$	$\cos(at)$	$\dfrac{s}{s^2 + a^2}$
$f''(t)$	$s^2 F(s) - sf(0) - f'(0)$	$\dfrac{t}{2a} \sin(at)$	$\dfrac{s}{(s^2 + a^2)^2}$
$\int_0^t f(\tau) d\tau$	$\dfrac{F(s)}{s}$	$\dfrac{1}{2a^3} \big(\sin(at) - at \cos(at) \big)$	$\dfrac{1}{(s^2 + a^2)^2}$
$tf(t)$	$-F'(s)$	$t^r, \ r > -1$	$\dfrac{\Gamma(r+1)}{s^{r+1}}$
$t^n f(t)$	$(-1)^n F^{(n)}(s)$	$t^n, \ n = 1, 2, 3, \ldots$	$\dfrac{n!}{s^{n+1}}$
$\dfrac{f(t)}{t}$	$\int_s^{+\infty} F(z) dz$	$f(t + T) = f(t), \ T > 0$	$\dfrac{\int_0^T e^{-st} f(t) dt}{1 - e^{-sT}}$
$f(ct), \ c > 0$	$\dfrac{1}{c} F(s/c)$	$\int_0^t f(t - \tau) g(\tau) d\tau$	$F(s)G(s)$

Problem 1. Compute the Laplace transform of $t^2 e^{-t} \cos 5t$.

Problem 2. Compute the inverse Laplace transform of $\dfrac{5s}{s^2 - s - 6}$.

Problem 3. Solve the initial value problem $y'' - 4y' + 4y = 0$, $y(0) = y'(0) = 1$ using the Laplace transform.

Problem 4. Compute trace, determinant, eigenvalues, and eigenvectors of the matrix

$$\begin{pmatrix} 3 & 1 \\ 5 & -1 \end{pmatrix}$$