Spring 2023, MATH 408, Exam 2

Monday, April 17; 11–11:50am

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Instructions:

- You should have access to a calculator or some other computing device, and to the χ^2 and F distribution tables. Instead of the tables, you are welcome to use the statistical functions available on your computing device.
- Answer all questions and clearly indicate your answers; upload the solutions to GradeScope.
- **Each problem is worth 10 points.**

Problem 1.
Below is part of a two-way ANOVA table for $b = 5$ blocks and $k = 6$ treatments. Fill out the rest of the table.

Source	SS	df	MS	F	Prob $> F$
Blocks	83	4	20.75	2.18	$F_{4,20} > 2.18$
Treatments	210	5	42	4.42	$F_{5,20} > 4.42$
Error	190	20	9.5		
Total	483	29			

> 0.1 0.108250

$\in (0.005, 0.01)$ 0.00712432

Problem 2.
To test whether a die is fair, 62 rolls were made, and the corresponding outcomes were as follows:

expected $= 62/6 = 10\frac{1}{3} = \frac{31}{3}$

Face value	Observed frequency
1	7
2	10
3	14
4	14
5	10
6	7

$(\text{expected} - \text{obs.})^2$

$2 \times (3\,^1/_3)^2$

$2 \times (^1/_3)^2$

$2 \times (3\,^2/_3)^2$

$\frac{100}{9} + \frac{1}{9} + \frac{1}{9}$

$2 \cdot \frac{222}{9}$

$31/3$

$= \frac{444}{93} = 4.77$

$= \frac{148}{31}$

Estimate the P-value if the χ^2 test is used.

$P(\chi^2_5 \geq 4.77) > 0.1$

$= 0.44458623$

Problem 3. Assume the following is independent sample from a population with a continuous cdf $F_X = F(x)$:

R 4 5 3 1 2

14 15 13 11 12,

and assume that the following is an independent sample from a population with cdf $F_Y = F(x+\theta)$

8 10 7 4 0. $M = \sum 1(X>Y) = 5.$

R 4 5 3 2 1

$\text{p-value} = P(B(5,\tfrac{1}{2}) \geq 5) = \frac{1}{32}.$

Compute the P-value of the sign test for the null hypothesis $\theta = 0$ against the alternative $\theta > 0$.

Problem 4. For the two samples in Problem 3, compute the Spearman rank correlation coefficient.

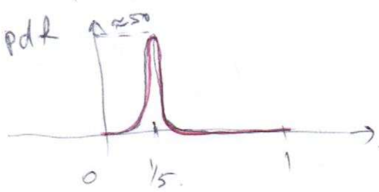
$\sum (R(x) - R(y))^2 = 0+0+0+1+1 = 2$

$r_s = 1 - \frac{6 \cdot 2}{5 \cdot 24} = \frac{9}{10} = 0.9.$

Problems 5. A coin-making machine produces pennies in such a way that, for each coin, the probability U to turn up heads is uniform on $[0,1]$. A coin pops out of the machine, flipped 2000 times and lands heads 400 times. Sketch the graph of the pdf and cdf of the posterior distribution of U.

$f^* = \text{Beta}(401, 1601) \approx \delta(\tfrac{4}{20}) = \delta(\tfrac{1}{5})$ — Point mass at $\frac{1}{5}$

pdf



cdf