Spring 2022, MATH 408, Exam 2

Monday, April 18; 10–10:50am

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Instructions:

- You should have access to a calculator or some other computing device, and to the distribution tables (normal, t, χ^2, and F). Instead of the tables, you are welcome to use the statistical functions available on your computing device.
- Answer all questions and clearly indicate your answers; upload the solutions to GradeScope.
- **There are five problems; each problem is worth 10 points.**

Problem 1. Fill in the rest of the following two-way ANOVA table.

Source	SS	df	MS	F	Prob $> F$
Columns	157	4	39.25	2.25	0.1
Rows	87	5	17.4	1	>0.1
Error	348	20	17.4		
Total	592	29			

(or 0.44 if on calculator)

Problem 2. To test whether a die is fair, 72 rolls were made, and the corresponding outcomes were as follows:

Face value	Observed frequency	Expected $= \frac{72}{6}$	$(\text{obs}-\text{exp})^2$
1	9	12	9
2	10	12	4
3	17	12	25
4	16	12	16
5	11	12	1
6	9	12	9

Estimate the p-value if the χ^2 test is used.

$$\sum \frac{(\text{obs}-\text{exp})^2}{\text{exp}} = \frac{9+4+25+16+1+9}{12} = \frac{30+34}{12} = \frac{64}{12}$$

$$\text{p-value} = \mathbb{P}\left(\chi^2_5 > \frac{64}{12}\right) = \mathbb{P}(\chi^2_5 > 5.33)\ > 0.1$$

(or 0.38)

Problem 3. Compute the Spearman rank correlation coefficient for the data set

$$X_1 = 2, X_2 = 4, X_3 = 6, X_4 = 1, X_5 = 5, X_6 = 3;$$

$$Y_1 = 3,\ Y_2 = 1,\ Y_3 = 5,\ Y_4 = 2,\ Y_5 = 4,\ Y_6 = 6.$$

$(\mathrm{Rank}(X) - \mathrm{Rank}(Y))^2$ $\quad 1 + \underbrace{9 + 1} + 1 + \underbrace{1 + 9} = 22$

Indicate the formula you are using and show your work. Keep in mind that your final answer should be in the interval $[-1, 1]$.

$$r_s = 1 - \frac{6 \sum (\mathrm{Rank}(X) - \mathrm{Rank}(Y))^2}{n(n^2-1)} = 1 - \frac{6 \cdot 22}{6 \cdot 35} = 1 - \frac{22}{35} = \frac{13}{35}$$

$n = 6$

Problems 4. Assume the following is an independent sample from a population with a continuous cdf $F_X = F(x)$:

$$X_1 = 14 \quad X_2 = 6 \quad X_3 = 10.5 \quad X_4 = 11 \quad X_5 = 12,$$

and assume that the following is an independent sample from a population with cdf $F_Y = F(x - \theta)$

$$Y_1 = 14.5 \quad Y_2 = 8 \quad Y_3 = 7 \quad Y_4 = 9 \quad Y_5 = 13.$$

$1(Y > X)$ $\quad 1 \quad 1 \quad 0 \quad 0 \quad 1 \quad \to 3.$

Compute the p-value of the sign test for the null hypothesis $\theta = 0$ against the alternative $\theta > 0$.

You will need the binomial coefficients $1, 5, 10, \underbrace{10, 5, 1}$.

$P(Y > X) > \frac{1}{2}$

$$\text{p-value} = \mathbb{P}\left(B(5, \tfrac{1}{2}) \geq 3 \right) = \frac{10 + 5 + 1}{2^5} = \frac{16}{32} = \frac{1}{2}$$

Problem 5. A coin-making machine produces pennies with unknown probability p to turn up heads; this probability is equally likely to be any number between 0 and 1.

A coin pops out of the machine, flipped 23 times and lands heads 9 times. Compute the Bayesian estimate $\hat{p}$ of p.

$$\hat{p} = \int_0^1 p f^*(p)\, dp = \mathbb{E}(\mathrm{Beta}(10, 15)) = \frac{10}{25} = \frac{2}{5}$$

$$f^* \sim \mathrm{Beta}(9+1,\ 23 - 9 + 1)$$