

Fall 2022, MATH 407, Mid-Term Exam 2

Wednesday, November 16, 2022

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Instructions:

- No books, notes, or help from other people. Think twice (or more) before using a calculator.
- Turn off cell phones.
- Show your work/explain your answers.
- You have 50 minutes to complete the exam.
- There are five problems; 10 points per problem.
- Upload the solutions to GradeScope.

Problem 1. 10 balls are dropped at random into five boxes so that the balls are dropped independently of one another and each ball is equally likely to land in any of the boxes. Denote by X the number of non-empty boxes. Compute the expected number $E(X)$ of non-empty boxes.

Problem 2. A fair die is rolled 200 times. Compute, approximately, the probability that the total sum S of the rolls is exactly 700. For a single roll, the expected value is 3.5 and the standard deviation is (approximately) 1.7. You can leave the answer in the form $P(|Z| < r)$, where Z is standard normal random variable and r is a suitable number.

Problem 3. Let U and V be independent random variables, such that U is uniform on $(0, 1)$ and V is uniform on $(0, 2\pi)$. Define the random variables X and Y by $X = \sqrt{U} \cos(V)$, $Y = \sqrt{U} \sin(V)$. Compute the joint pdf of the random vector (X, Y) .

Problem 4. Let X and Y be iid standard normal random variables. Determine the value of the number a so that the random variables $X + 2Y$ and $aX + Y$ are independent.

Problem 5. Customers arrive at a bank at a Poisson rate λ . Suppose that four customers arrived during the first hour. Compute the probability that at least one of the four customers arrived during the last 15 minutes.