Fall 2021, MATH 407, Mid-Term Exam 2

Wednesday, November 17, 2021, 9:00-9:50am

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Instructions:

- No books, notes, calculators, or help from other people.
- Turn off cell phones.
- Show your work/explain your answers.
- You have 50 minutes to complete the exam.
- There are five problems; 10 points per problem.
- Upload the solutions to GradeScope.

standard normal pdf: $(2\pi)^{-1/2}e^{-x^2/2}$; Gamma(a,b) pdf: $b^a\big(\Gamma(a)\big)^{-1}x^{a-1}e^{-bx}$; Exponential with mean θ is Gamma$(1,1/\theta)$, Beta(a,b) pdf: $\big(B(a,b)\big)^{-1}x^{a-1}(1-x)^{b-1}$; Poisson, mean μ, pmf: $e^{-\mu}k^{\mu}/k!$.

Problem 1. For a randomly selected group of 50 people, compute the expected number of distinct birthdays (that is, the expected number of the days of the year that are a birthday of at least one person in the group). Assume 365 days in a year.

Problem 2. The joint probability density function of two random variables X and Y

$$f_{X,Y}(x,y) = \begin{cases} Cx, & \text{if } \ x^2+y^2 \le 1, \ \ x \ge 0, \ y \ge 0, \\ 0, & \text{otherwise.} \end{cases}$$

Compute $\mathbb{E}(X|Y)$. Note: there is no need to know C.

Problem 3. At a particular location, there is, on average, one earthquake every 4 days. Assuming that the earthquakes follow Poisson process, compute, approximately, the probability that there are more than 100 earthquakes in 360 days. Leave your answer in the form $\mathbb{P}(\mathcal{N} < r)$ or $\mathbb{P}(\mathcal{N} > r)$, where $\mathcal{N}$ is a standard normal random variable and r is a real number. Then circle the interval that contains your answer:

$$(0, 0.1) \qquad [0.1, 0.3) \qquad [0.3, 0.5) \qquad [0.5, 1)$$

Problem 4. Let X, Y be independent exponential random variables with $\mathbb{E}(X) = \mathbb{E}(Y) = 1/2$. Compute the probability density functions of the random variables $X + Y$ and $X/(X+Y)$.

Problem 5. Customers arrive at a bank at a Poisson rate λ. Suppose that two customers arrive during the first hour. Compute the probability that at least one of the customers arrived during the first 15 minutes.