Fall 2018, MATH 407, Mid-Term Exam 2

Wednesday, November 14, 2018

Instructor S. Lototsky (KAP 248D; x0–2389; lototsky@usc.edu)

Name: _____

Circle the time of your discussion section: **2pm** **3pm**

Instructions:

- No books, notes, or calculators.
- Turn off cell phones.
- Show your work/explain your answers.
- You have 50 minutes to complete the exam.

Problem	Possible	Actual
1	10	
2	10	
3	10	
4	10	
5	10	
Total	50	

Problem 1. A fair die is rolled until the total sum exceeds 320. Using the continuity correction, compute approximately the probability that **more that** 100 rolls are necessary. Note that, for a single roll of the die, the expected value and variance of the outcome are $7/2 = 3.5$ and $35/12 \approx (1.7)^2$, respectively. **Leave your answer in the form** $P(Z < r)$ **or** $P(Z > r)$**, where** Z **is the standard normal random variable and** r **is a suitable real number, and indicate, with some explanation, whether your answer is bigger than** 0.2 **or less that** 0.2**.**

Problem 2. Customers arrive at a bank according to a Poisson process. Suppose that three customers arrive during the first hour. Compute the probability that at least one arrived during the first 20 minutes.

Problem 3. Let X and Y be independent exponential random variables, each with mean value 1. Compute the probability density function of the random variable $U = \dfrac{X}{X+Y}$.

Problem 4. For a group of 100 people, compute the expected number of days that are not a birthday of anybody in the group and, in the line below, circle the interval to which you think your answer belongs. Assume 365 days in the year and that everybody in the group is equally likely to be born on any particular day, independent of other people.

$$[0, 100] \qquad [101, 200] \qquad [201, 300] \qquad [301, 365]$$

Problem 5. The joint probability density of the random variables X and Y is

$$f_{X,Y}(x, y) = \begin{cases} Cx^2, & \text{if } x^2 + y^2 \leq 1,\ x > 0, \\ 0, & \text{otherwise.} \end{cases}$$

(a) Compute $\mathbb{E}(Y^2|X)$. [Note: there is no need to know C.]

(b) Explain why X and Y are not independent.