Spring 2017, MATH 407, Mid-Term Exam 2

Monday, April 17, 2017

Instructor S. Lototsky (KAP 248D; x0–2389; lototsky@math.usc.edu)

Name: ____

Circle the time of your discussion section: **10am** **11am**

Instructions:

- No books, notes, or calculators.
- You have 50 minutes to complete the exam.
- **Show your work**.

Problem	Possible	Actual
1	10	
2	10	
3	10	
4	10	
5	10	
Total	50	

Problem 1. A fair die is rolled until the total sum of all rolls exceeds 350. Compute approximately the probability that at most 100 rolls are necessary. Note that, for a single roll of the die, the expected value and variance of the outcome are $7/2$ and $35/12$, respectively. Use the continuity correction. Leave the answer in the form $P(Z < r)$ or $P(Z > r)$ (whichever applies), where Z is a standard normal random variable and r is a suitable real number.

Problem 2. Customers arrive at a bank according to a Poisson process. Suppose that three customers arrive during the first hour. Compute the probability that at least one arrived during the first 20 minutes. [**To help you out**: the arrival times are order statistics $U_{(1)}, U_{(2)}, U_{(3)}$ of the uniform distribution on $[0, 1]$, with time measured in hours. Your objective is to compute the probability that the smallest of the three is less than $1/3$, so it makes sense to go with the complement: compute the probability that the smallest is bigger than $1/3$.]

Problem 3. Let X, Y be independent random variables, both exponential with mean 1.
(a) Find the joint density of $U = X + Y$ and $V = X/(X + Y)$.
(b) Are the random variables U and V independent? Explain your conclusion.

Problem 4. For a randomly selected group of 50 people, compute the expected number of distinct birthdays (that is, the expected number of the days of the year that are a birthday of at least one person in the group).

Problem 5. The joint probability density function of two random variables X and Y

$$f_{X,Y}(x,y) = \begin{cases} Cx, & \text{if } x^2+y^2 \le 1,\ x>0, y\ge 0, \\ 0, & \text{otherwise.} \end{cases}$$

(a) Are X and Y independent? Justify your answer.

(b) Compute $E(X|Y)$. Note: there is no need to know C.