Fall 2022, MATH 407, Mid-Term Exam 1

Wednesday, October 5, 2022

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Instructions:

- No books, notes, or help from other people. Think twice (or more) before using a calculator.
- Turn off cell phones.
- Show your work/explain your answers.
- You have 50 minutes to complete the exam.
- There are five problems; 10 points per problem.
- Upload the solutions to GradeScope.

Problem 1. A box contains 5 white balls, 10 blue balls, 15 green balls and 20 red balls, [50 total, well mixed]. Fourteen (14) balls are taken out of the box, all at once.

Compute the probability that two (2) of those balls are white, three (3) are blue, four (4) are green, and five (5) are red. `For your final answer, do not expand/evaluate any of the binomial coefficients.`

Problem 2. Consider two events A and B such that $P(A) = 0.5, P(B) = 0.6$.

(a) Explain why the events cannot be mutually exclusive.

(b) Suppose that the events are independent. Compute $P(A \bigcup B^c)$, where B^c denotes the complement of the event B. In other words, you need the probability that either A happens or B does not happen.

Problem 3. In a certain town, there are twice as many cars as trucks, and 10% of trucks and 5% of cars have manual transmission. A vehicle is selected at random, and it has manual transmission. Compute the probability that the vehicle is a truck. `Simplify your answer as much as possible.`

Problem 4. Consider the function

$$f(x) = \begin{cases} 0 & x \le 0 \\ C(2x - x^2) & 0 < x < 1 \\ C & x \ge 1, \end{cases}$$

where $C > 0$.

(a) Could f be a cumulative distribution function? If yes, determine the value of C; if not, explain why.

(b) Could f be a probability density function? If yes, determine the value of C; if not, explain why.

`Make sure to draw the graph of` f.

Problem 5. Let X be a standard exponential random variable. In particular, $X \ge 0$, the pdf of X is e^{-x}, $x \ge 0$, and $\mathbb{P}(X > x) = e^{-x}$, $x \ge 0$. Define the random variable Y by $Y = e^{-2X}$. Compute the probability density function of the random variable Y. `Make sure to indicate the range of possible values for` Y.