

# Fall 2021, MATH 407, Mid-Term Exam 1

Wednesday, October 6, 2021

Instructor S. Lototsky (KAP 248D; x0-2389; lototsky@usc.edu)

## Instructions:

- No books, notes, calculators, or help from other people.
- Turn off cell phones.
- Show your work/explain your answers.
- You have 50 minutes to complete the exam.
- There are five problems; 10 points per problem.
- Upload the solutions to GradeScope.

**Problem 1.** A box contains 10 blue balls, 15 green balls and 20 red balls, [45 total, well mixed]. Nine balls are taken out of the box, all at once.

Compute the probability that three of those balls are blue, four are green, and two are red. Do not simplify/evaluate binomial coefficients.

**Problem 2.** The probability that a student passes the first midterm is 0.6, the probability that the student passes the second midterm is 0.7, and the probability that the student passes at least one of the two midterms is 0.8. Compute the probability that the student passes both midterms.

**Problem 3.** In a certain town, there are twice as many cars as trucks, and 5% of trucks and 0.5% of cars have manual transmission. A vehicle is selected at random, and it has manual transmission. Compute the probability that the vehicle is a car.

**Problem 4.** A charitable lottery has 10,000 tickets, of which 200 win prizes and the rest win nothing. You buy 50 tickets.

(a) Compute the number of the prize-winning tickets you expect to find.

(b) Which of the following numbers is closest to the probability that, out of 50 tickets, none are prize-winning:

$$\frac{1}{100}, \quad \frac{1}{10}, \quad \frac{1}{5}, \quad \frac{1}{3}, \quad \frac{1}{2}, \quad \frac{2}{3}, \quad \frac{3}{4},$$

Explain your conclusion.

**Problem 5.** Let  $X$  be a random variable with uniform distribution on  $[-2, 2]$ . Define the random variable  $Y$  by  $Y = -\ln(X + 2)$ . Compute the probability density function of the random variable  $Y$ . Make sure to indicate the range of possible values for  $Y$ .