Fall 2018, MATH 407, Mid-Term Exam 1

Wednesday, October 3, 2018

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Name: _____

Circle the time of your discussion section: 2pm   3pm

Instructions:

- No books, notes,or calculators.
- Turn off cell phones.
- Show your work/explain your answers.
- You have 50 minutes to complete the exam.

Problem	Possible	Actual
1	10	
2	10	
3	10	
4	10	
5	10	
Total	50	

Problem 1. Consider two events A and B such that $P(A) = P(B) = 0.6$.
(a) Explain why the events cannot be mutually exclusive.

(b) Suppose that the events are independent. Compute $P(A \bigcup B)$.

Problem 2. Five balls are placed at random in five boxes. Compute the probability that there are no empty boxes.

Problem 3. A charitable lottery has 10,000 tickets, of which 200 win prizes and the rest win nothing. You buy 50 tickets.

(a) Compute, approximately, the number of the prize-winning tickets you expect to find.

(b) In the line below, circle the number you think is the closest to the probability that, out of 50 tickets, none are prize-winning, and explain your reasoning.

$$\frac{1}{100}, \quad \frac{1}{10}, \quad \frac{1}{5}, \quad \frac{1}{3}, \quad \frac{2}{5}, \quad \frac{1}{2}, \quad \frac{3}{5}, \quad \frac{2}{3}, \quad \frac{4}{5}$$

Problem 4. Let C be a positive real number and consider the function

$$h(x) = \begin{cases} C(2x + x^2) & 0 < x < 1, \\ 0 & x < 0, \\ 3C & x > 1. \end{cases}$$

(a) Could h be a cumulative distribution function? If yes, explain why and determine C; if not, explain why.

(b) Could h be a probability density function? If yes, explain why and determine C; if not, explain why.

Problem 5. Let U be uniform on the interval $(-1, 1)$. Compute the probability density function of $\ln(U + 1)$.