Fall 2013, MATH 407, Mid-Term Exam 2

Wednesday, November 20, 2013

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Name: ____

Circle the time of your discussion section: **8am** **9am** **10am**

Instructions:

- No books, notes, or calculators.
- You have 50 minutes to complete the exam.
- **Show your work**.

Problem	Possible	Actual
1	10	
2	10	
3	10	
4	10	
5	10	
Total	50	

Problem 1. A fair die is rolled until the total sum of all rolls exceeds 300. Compute approximately the probability that at least 80 rolls are necessary. Note that, for a single roll of the die, the expected value and variance of the outcome are $7/2$ and $35/12$, respectively. Use the continuity correction. Leave the answer in the form $P(Z < r)$, where Z is a standard normal random variable and r is a suitable real number.

Problem 2. Customers arrive at a bank according to a Poisson process. Suppose that two customers arrives during the first hour. Compute the probability that at least one arrived during the first 20 minutes.

Problem 3. Let X, Y be independent random variables, both uniform on $(0, 1)$. Find the joint density of $X + Y$ and $X/(X + Y)$.

Problem 4. For a randomly selected group of 100 people, compute the expected number of distinct birthdays (that is, the expected number of the days of the year that are a birthday of at least one person in the group).

Problem 5. The joint probability density function of two random variables X and Y

$$f_{XY}(x, y) = \begin{cases} Cy, & \text{if } \ x^2 + y^2 \le 1, \ |x| \le 1, y \ge 0, \\ 0 & \quad \text{otherwise.} \end{cases}$$

(a) Are X and Y independent? Justify your answer.

(b) Compute $E(X|Y)$. Suggestion: keep your computations to a minimum. In particular, there is no need to know C.