# Fall 2013, MATH 407, Mid-Term Exam 1 

Wednesday, October 9, 2013
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Name: $\qquad$

Circle the time of your discussion section: 8am 9am 10am

## Instructions:

- No books, notes, or calculators.
- You have 50 minutes to complete the exam.
- Show your work.

| Problem | Possible | Actual |
| :---: | :---: | :---: |
| 1 | 10 |  |
| 2 | 10 |  |
| 3 | 10 |  |
| 4 | 10 |  |
| 5 | 10 |  |
| Total | 50 |  |

Problem 1. Consider two events $A$ and $B$ such that $P(A)=P(B)=0.6$.
(a) Explain why the events cannot be mutually exclusive.
(b) Suppose that the events are independent. Compute $P(A \bigcup B)$.

Problem 2. In a certain community, $42 \%$ of the families own a dog and $30 \%$ of the families own a cat. $20 \%$ of the families that own a dog also own a cat. A randomly selected family owns a cat. What is the probability that this family also owns a dog?

Problem 3. Two teams play a series of games until one of the teams wins two games. In every game, both teams have equal chances of winning and there are no draws. Compute the expected number of the games played.

Problem 4. Consider the function

$$
f(x)= \begin{cases}C\left(2 x-x^{2}\right) & 0<x<2 \\ 0 & \text { otherwise }\end{cases}
$$

(a) Could $f$ be a cumulative distribution function? If yes, explain why and determine $C$; if not, explain why.
(b) Could $f$ be a probability density function? If yes, explain why and determine $C$; if not, explain why.

Problem 5. Let $U$ be uniform on the interval $(0,1)$. Identify the distribution of $\ln U$.

