Publications results for "Author=(friz) AND Publication Type=(Books)"

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Citations
From References: 236
From Reviews: 4

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A course on rough paths.
With an introduction to regularity structures. Universitext. *Springer, Cham,* 2014. xiv+251 pp. ISBN: 978-3-319-08331-5; 978-3-319-08332-2
60-02 (34F05 35R60 60H07 60H10 60H15)

The theory of rough paths was developed by Terry Lyons at the end of the 1990's (see his foundational paper [Rev. Mat. Iberoamericana 14 (1998), no. 2, 215–310; MR1654527]). The purpose of the theory is to develop an integral and differential calculus with respect to paths of low regularity in the Hölder or p-variation sense.

In particular, under suitable conditions, the theory makes it possible to give a sense to integrals of the type

$$\int z_t dx_t$$

and solutions of differential equations like

$$y_t = y_0 + \int_0^t V(y_s)dx_s$$

where $x, y\colon [0, +\infty) \to \mathbb{R}^d$ and $z\colon [0, +\infty) \to \mathbb{R}^d \otimes \mathbb{R}^d$ typically have low α-Hölder regularity and $V = (V_1, \dots, V_d)$ is a collection of vector fields. If $\alpha > 1/2$, the theory is consistent with Young's integration theory (see [L. C. Young, Acta Math. 67 (1936), no. 1, 251–282; MR1555421] and [T. J. Lyons, Math. Res. Lett. 1 (1994), no. 4, 451–464; MR1302388]). When $\alpha \leq 1/2$, the key insight is to add to the path x an extra information $\mathbb{X}$ that satisfies some algebraic compatibility and regularity conditions. The maps $(x, \mathbb{X}) \to \int z_t dx_t$ and $(x, \mathbb{X}) \to y$ are then continuous in a convenient topology.

The rough paths theory has quickly gained popularity, since it provides a fresh if not revolutionary robust perspective on Itô's theory of stochastic integrals and stochastic differential equations. More recently, ideas from rough paths theory have been used in M. Hairer's theory of regularity structures [see Invent. Math. 198 (2014), no. 2, 269–504; MR3274562], which gives a sense to solutions of ill-posed stochastic partial differential equations, like the KPZ equation.

There are already several books and monographs which are devoted to the theory of rough paths [see for instance T. J. Lyons and Z. M. Qian, System control and rough paths, Oxford Math. Monogr., Oxford Univ. Press, Oxford, 2002; MR2036784; T. J. Lyons, M. Caruana and T. Lévy, Differential equations driven by rough paths, Lecture Notes in Math., 1908, Springer, Berlin, 2007; MR2314753; P. K. Friz and N. B. Victoir, Multidimensional stochastic processes as rough paths, Cambridge Stud. Adv. Math., 120, Cambridge Univ. Press, Cambridge, 2010; MR2604669]. The one under review is complementary to those existing references in several ways. First, the theory has now reached a sufficient degree of maturity that allows a presentation in an unprecedented simple manner. In particular, for pedagogical reasons, the authors focus on the construction of the needed extra information $\mathbb{X}$ and the corresponding theory in the case where $\alpha > 1/3$. This choice allows them to introduce the relevant definitions and topologies in a straightforward way, without the algebraic difficulties that appear for $\alpha \leq 1/3$. Secondly, unlike the other references, the presentation is more oriented toward the theory of controlled rough paths that was developed by M. Gubinelli [J. Funct. Anal. 216 (2004), no. 1, 86–140; MR2091358]. This approach stresses and makes precise the fact that the object $\int z_t dx_t$ may be constructed as soon as z is controlled in a suitable sense by $(x, \mathbb{X})$. Finally, the book presents the most recent developments of the theory, like the applications of the rough paths ideas in the context of stochastic partial differential equations. In particular, a gentle introduction to Hairer's regularity structure theory is given in the last three chapters of the book.

The book can easily be used as a support for a graduate course on rough paths theory. There are several exercises throughout the text intended to test the understanding of the reader. It is also useful to experts in the area, since it presents in an accessible way the unique point of view of two experts who themselves have largely contributed to the theory which is exposed here. Some of the most recent developments are explained and several (sometimes unexpected) connections to other areas of mathematics are given.

Reviewed by Fabrice Baudoin