MATH 705, Seminar in Probability (39806R), Fall 2011.
Class meetings: W noon-1:50 pm, in WPH 204.

Information on this and linked pages changes frequently.

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Office Hours: M noon-1pm, W 2-3pm, F 10-11am. Walk-ins and appointments at other time are welcome.

The objective of this semester: To discuss the concept of the local time beyond the standard Brownian motion.

The plan: Every week, each participant will make a short presentation on a specific topic related to the general theme of the seminar.

The action

August 31: Attempts to compute the local time of a continuously differentiable process suggest that the ``natural'' measure of time for this computation should be the quadratic variation of the process rather then the physical time (for the standard Brownian motion, they are the same).

September 7. A more detailed discussion with some concrete conclusions:
1) All formulas for local time for a semimartingale (a fortiori, continuous semimartingales) involve the quadratic variation of the continuous component of the semimartingale.
Consulting Protter's book p. 216, Corollary 2, or Revuz and Yor's book p. 224 Corollary 1.6 \int L^a_t da= [X,X]^c_t, where [X,X]^c_t is the continuous component of the quadratic variation of a semimartingale X.

For specific semimartingales such as Levy processes - say characterized by (S, b, m), where S is the associated matrix - the continuous component of the quadratic variation is Sdt.
In Applebaum's book the measure used to compute the time spent on a set is the Lebesgue measure dt - it will result "proportional" to the measure Sdt he "should" have used. Note that they are "consistent" in the B. motion case, where S is the identity. A puzzle here is that Applebaum states the Tanaka formula in p. 244. Theorem 4.4.29 and he remarks that the local time appearing in this Tanaka formula coincides with the local time introduced before. To us they are equal up to a multiplication by a constant matrix. In particular, all semimartingales for which [X,X]^c_t=0 have null local time.
2) Using the dt in the "occupation measure" has its own interest, in general not related to local time. We recall the notion of empirical measure studied in Large Deviations.
3) We also discussed briefly how the notion of local time can be extended to fBM, by using distributions (delta function), so that a generalized Tanaka formula holds true: it turns out a weighted local time is needed, where the weight depends on the Hurst parameter (s^(2H-1)).

September 14. Discussed some possible applications of the local time, with emphasis on the Ito formula for convex functions. One discovery: a function of a Brownian motion is a semi-martingale if and only if the function is a difference of two convex functions.

September 21. Started the discussion of the three ways to look at the local time, following Chapter 19 of Kallenberg's ``Foundations of modern probability''.

September 28. Continued the discussion of the occupation time and excursions.

October 5. Discussed various ways to simulate the local time process using MatLab. Eventually realized that our conclusion (following one of Kallenberg's constructions) that L_t does not exceed t, was wrong.

October 12. After another attempt to understand Kallenberg's discussion, decided to look at the local times of other specific processes, starting with the Orstein-Uhlenbeck.

October 19. The main key-word search hit on local time for the Orstein-Uhlenbeck process was an article ``Statistics of local time and excursions for the Ornstein-Uhlenbeck process'' by J. Hawkes and A. Truman. Because of paper's heavy use of quantum-mechanical ideas, discussed foundations of the Hilbert-space model of quantum mechanics.

October 26. Discussed the quantum harmonic oscillator and its connection with the Ornstein-Uhlenbeck process.

November 2. Worked out the main ideas behind the connection between the Ornstein-Uhlenbeck process and the stochastic quantization of the gournd state of the quantum harmonic oscillator.

November 9. Discussed the stochastic quantization of the second energy state of the quantum harmonic oscillator and of the ground sate of the hydrogen atom. Noticed some connections with the Bessel processes and the CIR model.

November 16. Discussed the connection of the stochastic quantization of the ground sate of the hydrogen atom with the Bessel processes and the CIR model.

November 23. No meeting (student holiday - extra day for Thanksgiving break)

November 30. Summary and course evaluation.