Spring 2018, MATH 408, Final Exam

Monday, May 7, 2018, 8–10am, THH 212

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Name: ____

Instructions:

- No books or notes of any kind.
- Turn off cell phones.
- All the necessary tables are provided, but you are also welcome to use the corresponding statistical functions on your calculator.
- Answer all questions and clearly indicate your answers.
- **Each problem is worth 20 points.**
- **Show your work!** Points might be taken off for a correct answer with no explanations.

Problem	Possible	Actual	Problem	Possible	Actual
1	20		6	20	
2	20		7	20	
3	20		8	20	
4	20		9	20	
5	20		10	20	
Total	100		Total	100	

Problem 1. The following results were obtained for about 1,000 families: average height of husband 68 inches, SD 2.5 inches; average height of wife 63 inches, SD 2.7 inches, correlation coefficient $r = 0.8$. Of the women who were married to men of height 70 inches, what percentage were over 67 inches?

Problem 2. To test whether a die is fair, 78 rolls were made, and the corresponding outcomes were as follows:

Face value	Observed frequency
1	10
2	11
3	18
4	17
5	13
6	9

Estimate the P-value if the χ^2 test is used and explain whether you would consider the die fair.

Problem 3. Let $X_1, \dots, X_n$ be an independent random sample from a population with pdf

$$f(x;\theta) = \frac{\theta}{x^{\theta+1}}, \ x \geq 1,$$

where $\theta > 1$.

Compute the method of moments estimator $\widehat{\theta}_n$ of θ.

Problem 4. Consider an independent random sample $X_1, \ldots, X_{26}$ of size 26 from a normal population, and assume that

$$\sum_{k=1}^{26} X_k = 260 \text{ and } \sum_{k=1}^{26} X_k^2 = 2700.$$

Construct a 99% confidence intervals for the mean of the population.

Problem 5. A study reports that freshmen at four-year public universities work 10.2 hours a week for pay, on average, and the SD is 8.5 hours; at two-year community colleges, the average is 11.5 hours and the SD is 8.6 hours. Assume these data are based on two independent simple random samples, each of size 196. Is the **difference** between the weekly work hours statistically significant?

Problem 6. In 1970, 55% of freshmen at a certain college studied 40 hours or more per week. In 2005, the percentage decreased to 50%. The percentages are based on two independent simple random samples, each of size 225. Would you consider this **decrease** statistically significant? Why or why not?

Problem 7. Assume that

$$X_1 = 10, X_2 = 30, X_3 = 50, X_4 = 20, X_5 = 40, X_6 = 60$$

is an independent random sample from a population with a continuous cdf $F_X = F(x)$, and assume that

$$Y_1 = 22, Y_2 = 45, Y_3 = 64, Y_4 = 12, Y_5 = 53, Y_6 = 31$$

is an independent random sample from a population with cdf $F_Y = F(x - \theta)$. Compute the P-value of the sign test for the null hypothesis $\theta = 0$ against the alternative $\theta > 0$. The binomial coefficients for $n = 6$ are $1, 6, 15, 20, 15, 6, 1$.

Problem 8. Fill in the rest of the following two-way ANOVA table.

Source	SS	df	MS	F	Prob $> F$
Columns		4			
Rows	87				
Error	348	20			
Total	592	29			

Problems 9. Let $X_1, \ldots, X_n$ be an independent random sample from a population with pdf

$$f(x;\theta) = \frac{1}{\sqrt{2\pi}\,\theta}\, e^{-\frac{(x-\theta)^2}{2\theta^2}},$$

that is, normal with mean $\theta > 0$ and variance θ^2. Compute the maximum likelihood estimator $\hat{\theta}_n$ of θ. [You will end up with a quadratic equation; make sure to choose the correct root. To simplify computations, write $\bar{X}_n = \frac{1}{n}\sum_{i=1}^n X_i$, $V_n = \frac{1}{n}\sum_{i=1}^n X_i^2$].

Problem 10. Let $X_1, \ldots, X_n$ be an independent random sample from a population with pdf

$$f(x;\theta) = \frac{1}{\sqrt{2\pi\theta}}\, e^{-\frac{(x-\theta)^2}{2\theta}},$$

that is, normal with both mean and variance equal to $\theta > 0$. Define the random variable

$$V_n = \frac{1}{n}\sum_{k=1}^{n} X_k^2$$

and consider the hypothesis testing problem $H_0 : \theta = 1$ against the alternative $H_1 : \theta = 200$.

Select the correct statement from the two listed below and explain your reasoning:

(a) The test constructed using the Neyman-Pearson lemma should reject H_0 if V_n is large.

(b) The test constructed using the Neyman-Pearson lemma should reject H_0 if V_n is small.