Spring 2016, MATH 408, Final Exam

May 6, 2016, 11am–1pm, THH 210

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Name: ____

Instructions:

- No books or notes of any kind.
- Turn off cell phones.
- All the necessary tables are provided, but you are also welcome to use the corresponding statistical functions on your calculator.
- Answer all questions and clearly indicate your answers.
- **Each problem is worth 20 points.**
- **Show your work!** Points might be taken off for a correct answer with no explanations. Wrong answer with no explanations is worth zero points.

Problem	Possible	Actual	Problem	Possible	Actual
1	20		6	20	
2	20		7	20	
3	20		8	20	
4	20		9	20	
5	20		10	20	
Total	100		Total	100	

Problem 1. The following results were obtained for about 1,000 families: average height of husband 69 inches, SD 2.4 inches; average height of wife 64 inches, SD 2.6 inches, correlation coefficient $r = 0.66$. Of the women who were married to men of height 72 inches, what percentage were under 64 inches?

Problem 2. To test whether a die is fair, 72 rolls were made, and the corresponding outcomes were as follows:

Face value	Observed frequency
1	9
2	10
3	17
4	16
5	11
6	9

Estimate the P-value if the χ^2 test is used.

Problem 3. Fill in the rest of the following one-way ANOVA table.

Source	SS	df	MS	F	Prob $> F$
Rows	157				
Error		20			
Total	592	23			

Problem 4. Consider an independent random sample $X_1, \ldots, X_{26}$ of size 26 from a normal population, and assume that

$$\sum_{k=1}^{26} X_k = 260 \text{ and } \sum_{k=1}^{26} X_k^2 = 2700.$$

Construct 95% confidence intervals for the mean and standard deviation of the population.

Problem 5. A study reports that freshmen at four-year public universities work 10.2 hours a week for pay, on average, and the SD is 8.5 hours; at two-year community colleges, the average is 11.5 hours and the SD is 8.6 hours. Assume these data are based on two independent simple random samples, each of size 225. Is the **difference** between the weekly work hours statistically significant?

Problem 6. In 1970, 55% of freshmen at a certain college studied 40 hours or more per week. In 2005, the percentage changed to 50%. Is this **change** statistically significant? You may assume that the percentages are based on two independent simple random samples, each of size 196.

Problem 7. Assume that

$$X_1 = 1, X_2 = 3, X_3 = 5, X_4 = 2, X_5 = 4, X_6 = 6$$

is an independent random sample from a population with a continuous cdf $F_X = F(x)$, and assume that

$$Y_1 = 2, Y_2 = 4, Y_3 = 6, Y_4 = 1, Y_5 = 5, Y_6 = 3$$

is an independent random sample from a population with cdf $F_Y = F(x - \theta)$. Compute the p-value of the sign test for the null hypothesis $\theta = 0$ against the alternative $\theta > 0$.

Problem 8. Fill in the rest of the following two-way ANOVA table.

Source	SS	df	MS	F	Prob $> F$
Columns		4			
Rows	87				
Error	348	20			
Total	592	29			

Problems 9. A coin-making machine produces pennies in such a way that, for each coin, the probability to turn up heads is uniform on $[0,1]$. A coin pops out of the machine, flipped **three** times and **lands heads twice**. Compute the Bayesian point estimate and a 90% confidence, or credible, interval for the (posterior) probability p that the coin turns up heads.

Problem 10. Let $X_1, \ldots, X_n$ be an independent random sample from a population with uniform distribution on the interval $(\theta - 1, \theta + 1)$. Compute **the method of moments** estimator $\widehat{\theta}_n$ of θ, and the bias and the MSE of $\widehat{\theta}_n$.