Spring 2015, MATH 408, Final Exam

May 8, 2015, 11am–1pm, SLH 102

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Name: ____

Instructions:

- No books or notes of any kind.
- Turn off cell phones.
- Answer all questions and clearly indicate your answers.
- **Each problem is worth 20 points.**
- **Show your work!** Points might be taken off for a correct answer with no explanations. Wrong answer with no explanations is worth zero points.

Problem	Possible	Actual	Problem	Possible	Actual
1	20		6	20	
2	20		7	20	
3	20		8	20	
4	20		9	20	
5	20		10	20	
Total	100		Total	100	

The data table:

$$\begin{bmatrix} 15 & 8 & 6 & 4 & 1 \\ 7 & 5 & 8 & 9 & 4 \\ 14 & 4 & 13 & 11 & 12 \\ 10 & 8 & 7 & 4 & 0 \\ 16 & 4 & 2 & 8 & 1 \end{bmatrix}$$

Problem 1. The following results were obtained for about 1,000 families: average height of husband 68 inches, SD 2.5 inches; average height of wife 63 inches, SD 2.5 inches, correlation coefficient $r = 0.6$. Of the men who were married to women of height 60 inches, what percentage were under 64 inches?

Problem 2. In a certain town, there are exactly 10,000 residents. The table below summarizes the relationship between sex and participation in the most recent election.

	Men	Women
Voted	2,825	3,575
Didn't vote	1,475	2,125

Are sex and voting participation independent? [Note: this problem is NOT about chi-square test.]

Problem 3. In the data table on the first page, assume that each column is an independent random sample from a normal population. Assume that the five populations are independent, with the same standard deviation. Estimate the p-value of the F test for the null hypothesis that the population means are all the same.

Problem 4. In the data table on the first page, assume that the first column is an independent sample from a normal population. Construct 95% confidence intervals for the mean and the standard deviation.

Problem 5. A study reports that freshmen at four-year public universities work 10.2 hours a week for pay, on average, and the SD is 8.5 hours; at two-year community colleges, the average is 11.5 hours and the SD is 8.6 hours. Assume these data are based on two independent simple random samples, each of size 144. Is the **difference** between the weekly work hours statistically significant?

Problem 6. In 1970, 59% of freshmen at a certain college studied 40 hours or more per week. In 2005, the percentage had dropped to 50%. Is this **drop** statistically significant? You may assume that the percentages are based on two independent simple random samples, each of size 169.

Problem 7. Consider the data table on the first page. Assume that row number three is an independent sample from a population with a continuous cdf $F_X = F(x)$, and assume that row number four is an independent sample from a population with cdf $F_Y = F(x - \theta)$. Compute the p-value of the sign test for the null hypothesis $\theta = 0$ against the alternative $\theta > 0$. [In other words, the third row is "clearly" bigger than the fourth row, and we want to quantify this observation using the sign test].

Problem 8. In the data table on the first page, assume that the entries are independent normal random variables with the same standard deviation. Below is part of the corresponding two-way ANOVA table (aka randomized block design). Fill out the rest of the table. Please provide some justifying computations.

Source	SS	df	MS	F	Prob $> F$
Columns	210				
Rows	83				
Error					
Total	483				

Problems 9. A coin-making machine produces pennies in such a way that, for each coin, the probability to turn up heads is uniform on $[0, 1]$. A coin pops out of the machine, flipped twice and lands heads twice. Compute the Bayesian point estimate and the 95% confidence, or credible, interval for the (posterior) probability p that the coin turns up heads.

Problem 10. Let $X_1, \ldots, X_n$ be an independent random sample from a population with uniform distribution on the interval $(\theta, 1)$. Compute the MLE $\widehat{\theta}_n$ of θ, and the bias and the MSE of $\widehat{\theta}_n$.