Fall 2020, MATH 408, Final Exam

Monday, November 23; 11am–1pm

Instructor — S. Lototsky (KAP 248D; x0–2389; lototsky@usc.edu)

Instructions:

- If you have a question, please write to the instructor using the private chat function of the zoom meeting. Other than that, do not communicate with anybody during the exam.
- You should have (and use!) a calculator or other computing device and three distribution tables: normal, t, and χ^2. Instead of the tables, you are welcome to use the corresponding statistical functions on your computing device.
- Answer all questions and clearly indicate your answers.
- **Each problem is worth 10 points.**

Problem 1. Given the set of numbers 39, 51, 63, 70, 55, and assuming that this is a sample from a normal population, construct the 95% confidence interval for the standard deviation.
To get full credit, indicate the values of the sample mean, sample standard deviation, and the quantiles of the corresponding distribution you need to construct the confidence interval.

Problem 2. Let $X_1, \ldots, X_n$ be an independent random sample from exponential distribution with unknown mean θ. Construct the maximum likelihood estimator of θ.

Problem 3. A study reports that freshmen at public universities work 11.2 hours a week for pay, on average, and the s_n is 8.8 hours; at private universities, the average is 9.5 hours and the s_n is 7.3 hours. Assume these data are based on two independent simple random samples, each of size 1,000. Is the difference between the averages due to chance? Explain your conclusion by stating the corresponding null and alternative hypotheses and computing the p-value.

Problems 4. Let $X_1, \ldots, X_n$ be a random sample from exponential distribution with mean θ. Construct the most powerful test with Type-I error equal to 0.05 for testing $H_0 : \theta = 1$ against $H_1 : \theta = 2$.

Problem 5. Fill in the rest of the following two-way ANOVA table.

Source	SS	df	MS	F	Prob $> F$
Columns	8201				
Rows		4			
Error	8290	20			
Total	23157	29			

Problem 6. To test whether a die is fair, 66 rolls were made, and the corresponding outcomes were as follows:

Face value	Observed frequency
1	8
2	9
3	15
4	16
5	8
6	10

Estimate the p-value if the χ^2 test is used.

Would you consider the die fair? Explain your conclusion.

Problem 7. Assume that

$$X_1 = 2, X_2 = 4, X_3 = 6, X_4 = 1, X_5 = 5, X_6 = 3$$

is an independent random sample from a population with a continuous cdf $F_X = F(x)$, and assume that

$$Y_1 = 3, Y_2 = 5, Y_3 = 4, Y_4 = 2, Y_5 = 6, Y_6 = 1$$

is an independent random sample from a population with cdf $F_Y = F(x - \theta)$. Compute the p-value of the sign test for the null hypothesis $\theta = 0$ against the alternative $\theta > 0$.

You will need the binomial coefficients $1, 6, 15, 20, 15, 6, 1$.

Problems 8. Compute the Spearman rank correlation coefficient for the data set

$$X_1 = 2, X_2 = 4, X_3 = 6, X_4 = 1, X_5 = 5, X_6 = 3;$$
$$Y_1 = 6,\ Y_2 = 5,\ Y_3 = 3,\ Y_4 = 2,\ Y_5 = 4,\ Y_6 = 1.$$

Indicate the formula you are using and show your work. Keep in mind that your final answer should be in the interval $[-1, 1]$.

Problem 9. For the first-year students at a certain university, the correlation between SAT scores and first-year GPA was 0.66. Assume the distribution of the scores is jointly normal. Predict the percentile rank on the first-year GPA for a student whose percentile rank on the SAT was 35%.

Problems 10. A coin-making machine produces pennies with unknown probability p to turn up heads; this probability is equally likely to be any number between 0 and 1.

A coin pops out of the machine, flipped 21 times and lands heads 6 times. Compute the Bayesian estimate $\hat{p}$ of p.