Fall 2017, MATH 408, Final Exam

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Instructor — S. Lototsky (KAP 248D; x0–2389; lototsky@math.usc.edu)

Name: ____

Instructions:

- No books or notes of any kind.
- Turn off cell phones.
- All the necessary tables are provided, but you are also welcome to use the corresponding statistical functions on your calculator.
- Answer all questions and clearly indicate your answers.
- **Each problem is worth 20 points.**
- **Show your work!** Points might be taken off for a correct answer with no explanations.

Problem	Possible	Actual	Problem	Possible	Actual
1	20		6	20	
2	20		7	20	
3	20		8	20	
4	20		9	20	
5	20		10	20	
Total	100		Total	100	

Problem 1. The following results were obtained for 1,000 families: average height of husband 69 inches, SD 2.4 inches; average height of wife 64 inches, SD 2.6 inches, correlation coefficient $r = 0.6$. Of the women who were married to men of height 72 inches, what percentage were under 64 inches?

Problem 2. To test whether a die is fair, 72 rolls were made, and the corresponding outcomes were as follows:

Face value	Observed frequency
1	9
2	10
3	17
4	16
5	11
6	9

Estimate the p-value if the χ^2 test is used.

Problem 3. For the first-year students at a certain university, the correlation between SAT scores and the first-year GPA was 0.66. Assume the distribution of the scores is jointly normal. Predict the percentile rank on the first-year GPA for a student whose percentile rank on the SAT was 30%.

Problem 4. Consider an independent random sample $X_1, \dots, X_{36}$ of size $n = 36$ from a normal population, and assume that

$$\sum_{k=1}^{36} X_k = 432 \quad \text{and} \quad \sum_{k=1}^{36} X_k^2 = 8684.$$

Construct 95% confidence intervals for the mean and standard deviation of the population. [In the corresponding tables, use the average of the values for 30 and 40 degrees of freedom.]

Problem 5. A study reports that freshmen at four-year public universities work 10.2 hours a week for pay, on average, and the SD is 8.5 hours; at two-year community colleges, the average is 11.5 hours and the SD is 8.6 hours. Assume these data are based on two independent simple random samples, each of size 225. Is the **difference** between the weekly work hours statistically significant?

Problem 6. In 1970, 55% of freshmen at a certain college studied 40 hours or more per week. In 2005, the percentage decreased to 50%. Is this **decrease** statistically significant? You may assume that the percentages are based on two independent simple random samples, each of size 400.

Problem 7. Assume that

$$X_1 = 2.1, X_2 = 4.3, X_3 = 6.5, X_4 = 1.2, X_5 = 5.6, X_6 = 3.7$$

is an independent random sample from a population with a continuous cdf $F_X = F(x)$, and assume that

$$Y_1 = 1.1, Y_2 = 3.3, Y_3 = 5.7, Y_4 = 2.2, Y_5 = 4.8, Y_6 = 6.9$$

is an independent random sample from a population with cdf $F_Y = F(x - \theta)$. Compute the p-value of the sign test for the null hypothesis $\theta = 0$ against the alternative $\theta < 0$. [The binomial coefficients for $n = 6$ are $1, 6, 15, 20, 15, 6, 1$.]

Problem 8. Fill in the rest of the following two-way ANOVA table.

Source	SS	df	MS	F	Prob $> F$
Columns	571	4			
Rows					
Error	759	16			
Total	2065	24			

Problems 9. A coin-making machine produces pennies in such a way that, for each coin, the probability to turn up heads is uniform on $[0, 1]$. A coin pops out of the machine, flipped **three** times and **lands heads once**. Compute the Bayesian point estimate and a 90% confidence, or credible, interval for the (posterior) probability p that the coin turns up heads.

Problem 10. Let $X_1, \ldots, X_n$ be an independent random sample from a population with binomial distribution $\mathcal{B}(k,p)$ in which both k and p are unknown. Assume that

$$\bar{X}_n = \frac{1}{n}\sum_{i=1}^{n} X_i = 100, \quad \bar{\sigma}_n^2 = \frac{1}{n}\sum_{i=1}^{n}(X_i - \bar{X}_n)^2 = 20.$$

Compute the method of moments estimates $\hat{k}$ and $\hat{p}$ of k and p. [Recall that if $X \sim \mathcal{B}(k,p)$, then $\mathbb{E}X = kp$ and $\mathrm{Var}X = kp(1-p)$. Now replace $\mathbb{E}X$ and $\mathrm{Var}X$ with the provided sample quantities and solve for $\hat{k}$ and $\hat{p}$].