Fall 2013, MATH 407, Final Exam

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Name: ______________________________

Instructions:

- No books, notes, or calculators.
- You have 120 minutes to complete the exam.
- **Show your work**.

Problem	Possible	Actual	Problem	Possible	Actual
1	20		6	20	
2	20		7	20	
3	20		8	20	
4	20		9	20	
5	20		10	20	
Total	100		Total	100	

Problem 1. A total of 10 identical gifts are distributed at random among 7 children. Compute the probability that every child receives at least one gift.

Problem 2. Suppose that A and B are independent events for which $P(A) = 0.3$ and $P(B) = 0.4$. What is the probability that either A or B occurs, but not both?

Problem 3. A population contains twice as many females as males. In this population, 5% of males and 0.25% of females are color-blind. A color-blind person is selected at random. Compute the probability that the person is male.

Problem 4. Compute the proportion of all the four-children families with more girls than boys. Assume that that boys and girls are equally likely.

Problem 5. Let X be a standard normal random variable. Define the random variable Y by $Y = e^X$. Compute the probability density function of the random variable Y.

Problem 6. For a randomly selected group of 100 people, denote by X the number of days in a 365-day year that are not a birthday of any person in the group. Compute the expected value of X.

Problem 7. Let X and Y be independent standard random variables. Explain why the random variables $X + Y$ and $X - Y$ are independent.

Problem 8. Let X, Y be independent random variables, both exponentially distributed with mean 1.

(a) Compute the joint density of $U = X + Y$ and $V = X/(X + Y)$.

(b) Are random variables U and V independent? Explain your answer.

Problem 9. A fair die is rolled until the total sum of all rolls exceeds 290. Compute approximately the probability that at most 80 rolls are necessary. Note that, for a single roll of the die, the expected value and variance of the outcome are $7/2$ and $35/12$, respectively. Use the continuity correction. Leave the answer in the form $P(Z > r)$, where Z is a standard normal random variable and r is a suitable real number.

Problem 10. Customers arrive at a bank according to a Poisson process. Suppose that three customers arrived during the first hour. Compute the probability that nobody arrived during the first 15 minutes.