Fall 2009, MATH 407, Final Exam

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Instructor — S. Lototsky (KAP 248D; x0–2389; lototsky@math.usc.edu)

Name: ______________________________

Instructions:

- No books, notes, or calculators.
- You have 120 minutes to complete the exam.
- **Show your work**.

Problem	Possible	Actual	Problem	Possible	Actual
1	20		6	20	
2	20		7	20	
3	20		8	20	
4	20		9	20	
5	20		10	20	
Total	100		Total	100	

Problem 1. A total of 10 identical gifts are to be distributed among 7 children. How many results are possible if every child is to receive at least one gift?

Problem 2. Suppose that A and B are independent events for which $P(A) = 0.3$ and $P(B) = 0.5$. What is the probability that either A or B occurs, but not both?

Problem 3. A population contains twice as many females as males. In this population, 5% of males and 0.25% of females are color-blind. A color-blind person is selected at random. Compute the probability that the person is male.

Problem 4. A fair coin is tossed 10 times. Let X be the difference between the number of heads and the number of tails. Find (a) the possible values of X (note: X can be both positive and negative) (b) $P(X = 0)$.

Problem 5. Let X be a random variable with uniform distribution on $[2, 5]$. Define the random variable Y by $Y = \ln X$. Compute the probability density function of the random variable Y.

Problem 6. A man and a woman decide to meet at a certain location. The arrival time of the man is uniformly distributed between 12:10pm and 12:40pm. The arrival time of the woman is uniformly distributed between 12pm and 1pm. The man and the woman arrive independently of each other. Compute the probability that the man arrives first.

Problem 7. For a randomly selected group of 100 people, compute the expected number of distinct birthdays (that is, the expected number of the days of the year that are a birthday of at least one person in the group).

Problem 8. A fair die is rolled until the total sum of all rolls exceeds 300. Compute approximately the probability that more than 80 rolls are necessary. Note that, for a single roll of the die, the expected value and variance of the outcome are $7/2$ and $35/12$, respectively. Use the continuity correction. Leave the answer in the form $P(\xi < r)$, where ξ is a standard normal random variable and r is a suitable real number.

Problem 9. Customers arrive at a bank at a Poisson rate λ. Suppose that two customers arrives during the first hour. Compute the probability that at least one arrived during the first 20 minutes.

Problem 10. Starting with a random number generator producing independent random variables that are uniform on $[0, 1]$, describe a method of generating a random variable with the distribution function $F(x) = x^{10},\ 0 < x < 1$.