Fall 2008, MATH 407, Final Exam

Monday, December 15, 2008, 11am-1pm

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Name: ______________________________

Discussion section time: ______________________________

Instructions:

- No books, notes, or calculators.
- You have 120 minutes to complete the exam.
- **Show your work**.

Problem	Possible	Actual	Problem	Possible	Actual
1	20		6	20	
2	20		7	20	
3	20		8	20	
4	20		9	20	
5	20		10	20	
Total	100		Total	100	

Problem 1. (cf. 17/17) A total of 7 different gifts are to be distributed among 10 children. How many results are possible if no child is to receive more than one gift?

Problem 2. (cf. 56/8) Suppose that A and B are mutually exclusive events for which $P(A) = 0.3$ and $P(B) = 0.5$. What is the probability that either A or B occurs?

Problem 3. (cf. 114/26) A population contains twice as many females as males. In this population, 5% of males and 0.25% of females are color-blind. A color-blind person is selected at random. What is the probability that the person is male?

Problem 4. (cf. 187/6) A fair coin is tossed three times. Let X be the difference between the number of heads and the number of tails. Find the distribution of X.

Problem 5. (cf. 251/39) Let X be an exponential random variable with parameter $\lambda = 1$. Define the random variable Y by $Y = \ln X$. Find the probability density function of the random variable Y.

Problem 6. (cf. 314/13) A man and a woman decide to meet at a certain location. The arrival time of the man is uniformly distributed between 12:15pm and 12:45pm. The arrival time of the woman is uniformly distributed between 12pm and 1pm. The man and the woman arrive independently of each other. What is the probability that the first to arrive waits no more than five minutes?

Problem 7. (cf. 411/21) For a randomly selected group of 100 people, compute the expected number of distinct birthdays (that is, the expected number of the days of the year that are a birthday of at least one person in the group).

Problem 8. (cf. 457/6) A fair die is rolled until the total sum of all rolls exceeds 300. Find approximately the probability that more than 80 rolls are necessary. Note that, for a single roll of the die, the expected value and variance of the outcome are $7/2$ and $35/12$, respectively. Use the continuity correction. Leave the answer in the form $P(\xi < r)$, where ξ is a standard normal random variable and r is a suitable real number.

Problem 9. (cf. 484/1) Customers arrive at a bank according to a Poisson process. Suppose that two customers arrived during the first hour. What is the probability that at least one arrived during the first 20 minutes?

Problem 10. (cf. 505/7) Starting with a random number generator producing independent random variables that are uniform on $[0, 1]$, describe a method of generating a random variable with the cumulative distribution function

$$F(x) = \begin{cases} 0, & x \leq 0, \\ x^{10}, & 0 < x < 1, \\ 1, & x \geq 1. \end{cases}$$