MATH 125 Fall 2020 (S. Lototsky)

Computer Project 1 Due Friday, September 25

You are free to use any software package and any help you want. The software package you choose should support symbolic calculations.

Every page you turn in must have your name and last four digits of your student ID number **printed** on it.

The objective of this assignment is to learn basic features of the software package you are using.

Problem 1.

Plot the graph of the function

$$h_{10}(x) = \sum_{k=1}^{10} \frac{\sin\big((k!)^2 x\big)}{k!}$$

for $x \in [0,1]$, $x \in [0,0.1]$, $x \in [0,0.01]$, $x \in [0,0.001]$, and then the graph of the derivative of the function for $x \in [0,1]$. Recall that, for a positive integer k, $k! = 1 \cdot 2 \cdot 3 \cdot \dots \cdot k$, and $0! = 1$.

What you will turn in:

1. Five separate graphs.

2. Printout of the commands you used to generate the graphs.

Each graph must have a title, labelled axes, and the scale along each axis.

If you want to go further. What will happen to the above function as we add more and more terms? More precisely, consider the function

$$h_n(x) = \sum_{k=1}^{n} \frac{\sin\big((k!)^2 x\big)}{k!}.$$

What happens as $n \to \infty$? Will the limiting function be continuous? differentiable? Explain your conclusion the best you can.

Problem 2. Consider the function

$$f(x) = \sin^2\left(\cos\left(\frac{x}{\sqrt{x^2+1}}\right)\right).$$

1. Let the computer find $f'(x)$, $f''(x)$, and $f'(1)$.

2. On the same coordinate system, plot the curve $y = f(x), x \in [0,2]$ and the tangent line to the curve at the point $x = 1$.

3. Make two separate graphs of $f'(x)$ and $f''(x)$, $x \in [0,2]$.

What you will turn in:

1. The computer generated expressions for the derivatives, and the input used to produce the results.

2. Three pictures as described above with the corresponding inputs used to produce the pictures.

Each graph must have a title, labelled axes, and the scale along each axis.

If you want to do more. Compute $f'(x)$ and $f''(x)$ by hand and try to reconcile the results with the computer output.

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Computer Project 2 Due Friday, October 30

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Problem 1. The goal is to try Newton's method.

Use Newton's method to find the roots of the equation $x^5 - 4x + 1 = 0$ accurate up to six decimals. Evaluate the function $f(x) = x^5 - 4x + 1$ at each of the roots you found.

You will turn in the printout of your work. Please highlight the important numbers (for example, the roots you found).

Suggestion: First look at the graph of $f(x) = x^5 - 4x + 1$ to determine the number and approximate locations of the roots. Then, for each root, run the iteration $x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$ until the first six decimals of x_n and x_{n+1} are the same.

If you want to do more. For the root that is most distant from the origin, make a graphical illustration of the method. You might have to change the scale of the graph to make it look nice.

Problem 2. The goal is to try different methods of approximating definite integrals.

Consider the definite integral

$$I = \int_0^5 \frac{dx}{\sqrt{1+\sin^2 x}}.$$

1. Ask the software package you are using to evaluate the integral. Let I_0 be the number you got. **Make sure I_0 is indeed a real number in the form $a_0.a_1a_2...$ with at least 8 decimals**.

2. Compute approximate values of I using left endpoint rule, right endpoint rule, midpoint rule, and trapezoidal rule. Use $n = 50$ (that is, divide the interval $[0, 5]$ into 50 sub-intervals of equal size). Using I_0 as a benchmark, compute the error for each method. Which method is the most accurate?

You will turn in the printout of your work. Please highlight the important numbers (for example, the approximations of I you found). I am asking for three pages maximum.

If you want to go further. Evaluate the same integral using Simpson's rule. Take $n = 50$.