Calculus I: Background Information

From Algebra

Straight line:

point (x_0, y_0) and a slope m: $y = y_0 + m(x - x_0)$,

two points (x_1, y_1) and (x_2, y_2): $\dfrac{y - y_1}{y_2 - y_1} = \dfrac{x - x_1}{x_2 - x_1}$.

Vertical line: $x = x_0$; horizontal line: $y = y_0$.

$$(x+y)^2 = x^2 + 2xy + y^2,\ (x+y)^3 = x^3 + 3x^2y + 3xy^2 + y^3,$$
$$(x-y)^2 = x^2 - 2xy + y^2,\ (x-y)^3 = x^3 - 3x^2y + 3xy^2 - y^3;$$
$$(x+y)^n = \sum_{k=0}^{n} \frac{n!}{k!(n-k)!} x^k y^{n-k},\ n! = 1 \cdot 2 \cdot \ldots \cdot n, 0! = 1;$$
$$x^2 - y^2 = (x-y)(x+y),\ x^3 - y^3 = (x-y)(x^2 + xy + y^2),\ x^3 + y^3 = (x+y)(x^2 - xy + y^2);$$
$$\sum_{k=0}^{n} x^k = 1 + x + x^2 + \ldots + x^n = \frac{1 - x^{n+1}}{1 - x};$$
$$\sqrt{x^2} = |x|;$$
$$a > 0 \ \Rightarrow\ a^x = e^{x \ln a},\ a^{x+y} = a^x a^y;$$
$$\ln = \log_e;\quad x, y > 0 \ \Rightarrow\ \ln(xy) = \ln x + \ln y,\ \ln x^r = r \ln x,\ \ln(1/x) = -\ln x \quad [r = -1];$$
$$\log_a b = \frac{\log_c b}{\log_c a}$$

From Trigonometry

$$\tan x = \frac{\sin x}{\cos x}, \cot x = \frac{\cos x}{\sin x},\ \sec x = \frac{1}{\cos x},\ \csc x = \frac{1}{\sin x}$$
$$\cos^2 x + \sin^2 x = 1,\ \tan^2 x + 1 = \sec^2 x,\ \cot^2 x + 1 = \csc^2 x;$$
$$\sin(x \pm y) = \sin x \cos y \pm \cos x \sin y,\ \cos(x \pm y) = \cos x \cos y \mp \sin x \sin y,$$
$$\sin 2x = 2 \sin x \cos x,\quad \cos 2x = \cos^2 x - \sin^2 x = 2\cos^2 x - 1 = 1 - 2\sin^2 x,$$
$$\cos^2 x = \frac{1 + \cos 2x}{2},\quad \sin^2 x = \frac{1 - \cos 2x}{2}.$$

From Pre-Calculus

$y = f(x)$ is a function $\Leftrightarrow$ vertical line test;
$y = f(x)$ is one-to-one ($a \neq b \ \Rightarrow\ f(a) \neq f(b)$) $\Leftrightarrow$ horizonal line test.
$f(x)$ (strictly) increasing $\Leftrightarrow a > b \ \Rightarrow f(a) > f(b)$
Domain of f: where f is defined; range of f: where f takes its values.
Inverse function: if $y = f(x)$ is one-to-one, then $x = f^{-1}(y)$:

$$y = f\big(f^{-1}(y)\big),\ x = f^{-1}\big(f(x)\big)$$

Domain of f^{-1} is the range of f; Range of f^{-1} is the domain of f.
The graphs $y = f(x)$ and $y = f^{-1}(x)$ are symmetric about the line $x = y$.

Even function: $f(x) = f(-x)$; odd function: $f(x) = -f(-x)$.

From geometry

Object	Volume	Surface area
Cylinder	$\pi R^2 H$	$2\pi R(R+H)$
Cone	$\frac{1}{3}\pi R^2 H$	$\pi R(R+\sqrt{R^2+H^2})$
Ball	$\frac{4}{3}\pi R^3$	$4\pi R^2$