About the Buffon needle

The problem A needle of length L is tossed in a random way on the floor with infinitely many parallel lines d units apart, $d > L$. What is the probability p that the needle crosses a line?

The origin Georges-Louis Leclerc, compte de Buffon (1707–1788), French naturalist and intellectual; 1733: statement; 1777: solution.

The answer

$$p = \frac{2L}{\pi d}.$$

Informal argument By common sense, $p = cL/d$: the longer the needle and closer the lines, the more likely the needle to cross a line. All we need is the number c.

Note that, in the case of a needle, p is also the *average number* of intersections. An extra stretch of imagination suggests that the *average number* of intersections should not depend on the shape of the needle but only on its length, that is, should be the same for a needle or a *noodle*, as long as both have the same length L, that is, the average number of intersections should be cL/d for either a needle or a noodle.

Now take a noodle in the form of the circle of diameter d so that $L = \pi d$. Then the number of intersections is always 2, that is, $2 = c(\pi d)/d$, that is, $c = 2/\pi$.

Rigorous derivation Introduce the distance X from the mid-point of the needle to the nearest line and the angle θ between the needle and a line. The needle intersects a line if

$$X \leq \frac{L}{2} \sin \theta.$$

A random toss means that X is uniform on $(-d/2, d/2)$, θ is uniform on $(0, \pi)$, and X and θ are independent. The answer follows after a simple integration.

Statistical computation of π From

$$p = \frac{2L}{\pi d},$$

we conclude that

$$\pi \approx \frac{2L}{d} \times \frac{\text{total number of tosses}}{\text{number of times the needle crossed a line}}$$

Recall

$$\pi = 3.141592654...$$

A physical experiment Rudolf Wolf (1816–1893), Swiss; conducted the experiment between 1849 and 1953 with $d = 45\,mm$, $L = 36\,mm$. He made 5000 tosses, got 2532 intersections, resulting in

$$\pi \approx \frac{360,000}{2532 \cdot 45} \approx 3.159558$$

Very reasonable...

A "mathematical" experiment Mario Lazzarini, Italian; conducted the experiment in 1901 with $d = 30\,mm$, $L = 25\,mm$. He made 3408 tosses, got 1808 intersections, resulting in

$$\pi \approx \frac{3408 \cdot 5}{1808 \cdot 3} \approx 3.14159292$$

Amazing...

Making sense of Lazzarini's experiment

Fact 1: $3408 = 213 \cdot 16 = 71 \cdot 3 \cdot 16$, $1808 = 113 \cdot 16$, that is,

$$\frac{3408 \cdot 5}{1808 \cdot 3} = \frac{71 \cdot 3 \cdot 16 \cdot 5}{113 \cdot 16 \cdot 3} = \frac{355}{113}.$$

Fact 2:

$$\pi \approx \frac{355}{113} \approx 3.14159292$$

is a famous rational approximation of π: the best there is if you limit the number of digits on top and bottom to three (even to four or five...).

So Lazzarini knew what he wanted ($113n/213$ crossings in n tosses) and ran a "sequential" experiment until he got it (this is the most probable explanation; we will never know for sure).

Note also that, to claim accuracy to 6th decimal in the outcome, one would need to claim similar accuracy in the measurements of d and L, which is not easily achievable, especially in 1901.

Further reading

- Lee Badger. Lazzarini's Lucky Approximation of π. *Mathematics Magazine*, Vol. 67 (1994), No. 2, pp. 83–91.
- Ivars Peterson. Buffon's Needling Ants, `http://www.sciencenews.org/view/generic/id/459/description/Buffons_Needling_Ants`