A summary of ANOVA

One-way layout

$$Y_{ij} = \theta_i + \varepsilon_{ij}, \quad \varepsilon_{ij} \text{ are iid } \mathcal{N}(0, \sigma^2),\ i = 1, \ldots, k,\ j = 1, \ldots, n.$$

To test

$$H_0 : \theta_1 = \theta_2 = \cdots = \theta_k$$

against the alternative that $\theta_p \neq \theta_q$ for at least one pair (p, q), $p \neq q$, define

$$N = nk;\ \bar{Y}_{i\bullet} = \frac{1}{n}\sum_{j=1}^{n} Y_{ij},\ \bar{Y}_N = \frac{1}{N}\sum_{j=1}^{n}\sum_{i=1}^{k} Y_{ij};$$

$$\text{APV} = n\sum_{i=1}^{k}(\bar{Y}_{i\bullet} - \bar{Y}_N)^2,\ \text{IPV} = \sum_{j=1}^{n}\sum_{i=1}^{k}(Y_{ij} - \bar{Y}_{i\bullet})^2,$$

and reject H_0 at level α if

$$\frac{\text{APV}/(k-1)}{\text{IPV}/(N-k)} > F_{k-1,N-k,\alpha}.$$

Remember that in this setting

$$\text{TV} = \text{APV} + \text{IPV}.$$

Randomized block design

$$Y_{ij} = \theta_i + \mu_j + \varepsilon_{ij}, \quad \varepsilon_{ij} \text{ are iid } \mathcal{N}(0, \sigma^2),\ i = 1, \ldots, k,\ j = 1, \ldots, n.$$

We define

$$N = nk,\ M = (k-1)(n-1);$$

$$\bar{Y}_{i\bullet} = \frac{1}{n}\sum_{j=1}^{n} Y_{ij},\ \bar{Y}_{\bullet j} = \frac{1}{k}\sum_{i=1}^{k} Y_{ij},\ \bar{Y}_N = \frac{1}{N}\sum_{j=1}^{n}\sum_{i=1}^{k} Y_{ij};$$

$$\text{APV} = n\sum_{i=1}^{k}(\bar{Y}_{i\bullet} - \bar{Y}_N)^2,\ \text{ABV} = k\sum_{j=1}^{n}(\bar{Y}_{\bullet j} - \bar{Y}_N)^2,\ \text{TV} = \sum_{j=1}^{n}\sum_{i=1}^{k}(Y_{ij} - \bar{Y}_N)^2,$$

$$\text{IPV} = \text{TV} - \text{APV} - \text{ABV}.$$

(a) When testing

$$H_0 : \theta_1 = \theta_2 = \cdots = \theta_k$$

against the alternative that $\theta_p \neq \theta_q$ for at least one pair (p, q), $p \neq q$, reject H_0 at level α if

$$\frac{\text{APV}/(k-1)}{\text{IPV}/M} > F_{k-1,M,\alpha}$$

(b) When testing the null hypothesis

$$H_0 : \mu_1 = \mu_2 = \cdots = \mu_n$$

against the alternative that $\mu_p \neq \mu_q$ for at least one pair (p, q), $p \neq q$, reject H_0 at level α if

$$\frac{\text{ABV}/(n-1)}{\text{IPV}/M} > F_{n-1,M,\alpha}.$$