SETS, SUBSETS, AND INTERSECTIONS

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Abstract. This 12-week session for Kindergarten and 1st grade kids focuses on sets. We begin with the concept of inclusion (subsets), then move on to disjoint and overlapping sets, and the ever useful principle of Inclusion-Exclusion. The mathematical content includes polygons and reinforcement of small number addition (and very small number subtraction). Logical reasoning focuses on depth-first search and working problems from both ends.

Contents

1. Odd One Out

For our opening meeting, we aim to break the stereotype of math as having absolute answers obtained from regimented systems. We also dissociate it from numbers, to the extent possible, by focusing entirely on concepts in a context where all answers can be correct, if they have the correct reasoning behind them.

1.1. **Puzzle: *The odd one out* by Britta Teckentrup.** This is a spotting puzzle written in verse, where the kids have to find the one odd animal on each page based on the rhyming clue. This warms up their observational skills to get us started.
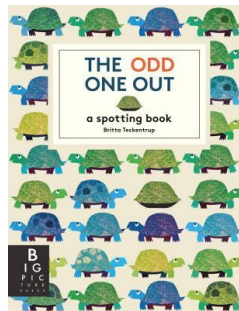
1.2. **Concept: All answers can be right.** Lay out picture cards in groups of four, and ask the kids what pictures they see. Agree with any reasonable description (e.g. the wagon looks like a cart), and then read the word on the card while pointing to it. Explain that one of these cards is the odd one out, and ask them which.

Each card has a good answer, and see if they can find one. As an example, I used a blue car, a red bike, a red wagon and a red drum. The answers given by the kids were: the *drum* is odd because you can ride on the others but not on the drum; the *car* is odd because the others are red and the car is not; the *wagon* is odd because the others have twos (two headlights, two wheels, two drumsticks) and the wagon does not (I didn't mention about axles); the *bike* is odd because the others have solid wheels but the bike does not. Go slowly, letting everyone who wants to chime in do so. Do 4-6 rounds, depending on interest.

1.3. **Game: *SET* by SET Enterprises.** Reduce the game by using only the solid shapes (27 of 81), and explain the rules by

- there are three attributes: shape, color, number
- they can be all the same or all different for each attribute

Lay out a 3x3 grid of cards and ask the kids to point to "sets". Check each one attribute at a time, but don't remove them yet. Clear the cards and do a 4x3 grid, now with remove and replace going in turns to find sets. If a child is stuck, point to two cards and ask him/her to finish the set by figuring out what shape, what color, and what number the third card must be.

1.4. **Story: *Which one doesn't belong?* by Christopher Danielson.** We come together as a large group for the story. Read slowly through the book allowing the kids to make their reasons for which doesn't belong. This story gives the explanations page by page for a few example, then leaves the questions open for the rest. Go through slowly, listening to ideas as the kids get engaged.
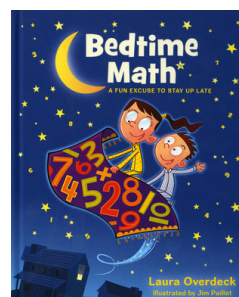
2. Quadrilaterals

We present definitions for quadrilaterals, rectangles, and squares to begin to introduce the concepts of sets and subsets. Manipulatives use paper shapes (suitable for folding) and peg boards for making polygons.

2.1. **Puzzle: *Bedtime Math* by Laura Overdeck.** Read one *Bedtime Math* story and ask the "Wee-Ones" puzzle afterward. These are the super easy counting ones, and we want to start off with everyone feeling confident. Try to wait until most of your group figures out the answer, then ask each child to share his/her answer if he/she has one! Its OK if they didn't see how to or didn't want to figure it out. Whether they all agree or not, say "lets check it" and demonstrate with counting on your figures. Go very slowly for the check. Ask if they want another puzzle, and go with consensus. Do at most 2 puzzles.

2.2. **Concept: Quadrilaterals and subsets.** Begin with the first of our three basic definitions for the day.

Definition 2.1. A *quadrilateral* is a flat, closed shape with four straight sides.

Show examples with cards with circles, ovals, triangles, rectangles, squares, and diamonds. Ask the kids to sort them into a quadrilateral pile and a non-quadrilateral pile. Check each shape by counting the sides (how many sides on a circle? – leave that question up in the air).

Go around the group, starting and ending with yourself, asking for examples of quadrilaterals you see in the room. On your first turn pick a generic rectangle (not a square). On your second turn, pick a non-rectangle.

Definition 2.2. A *rectangle* is quadrilateral in which every angle is a right angle.

Discuss right angles. Show that the *Bedtime Math* book has right angles, so it is a rectangle. Ask the students to make a right angle with their elbows. Have them make a right angle with their bodies and the floor, then with their bodies and the wall. Bring everyone back to order for another sorting game.

Give a visual explanation that a shape cut into paper is a rectangle if (and only if) when you fold it in half either way, the two sides touch and are the same length. Demonstrate with an example and non-example. Pass out the paper shapes for them to test it on. Have them categorize the shapes by: not a quadrilateral (5), quadrilateral but not a rectangle (4), rectangle (9). Count the totals for each pile.

Problem A. Which are there more of: quadrilaterals or rectangles?

Don't give away any solutions; just listen to the answers and encourage discussion. Always ask "why" and probe for their *reasons.*

Definition 2.3. A *square* is rectangle in which every side is the same length.

Pick up the pile of rectangle paper shapes and move the others away (maintaining the piles). Have the kids sort out which are squares and which are not. Remind them of the fold test for rectangles (demonstrate with an example and

non-example). Ask them to come up with a fold test for squares. Let them play with it for a while until someone figures it out. If they don't get it after a few minutes, show that the rectangle fold test works on both and ask if there is another direction they can fold. Again, count the totals for each pile.

Problem B. Which are there more of: rectangles or squares?

Tally the totals for each set a piece of paper; they count and you write. Write Squares and draw one, Rectangle and draw a generic one (not a square), Quadrilateral and draw a generic one (not a rhombus or trapezoid). Now write the counts that they come up with. Count each pile first. Then ask leading questions about the squares to get them to figure out that squares are rectangles, so we need to add them to the count! Remember to count up on your fingers. Repeat for quadrilaterals. See if they can figure out which number to add to the quadrilateral total.

Close the concept with the peg boards. Ask each child to make a red square, a blue rectangle, and a green quadrilateral on his/her board. Then have them see what funny quadrilaterals they can make! (Bonus for non-convex!)

2.3. **Game:** *Three Little Pigs* **by SmartGames.** Introduce the game with the pigs only (no wolf). Show them a puzzle board and explain how to set up the game. Let them help to set up the puzzle. Take suggestions for how to orient the board to match the puzzle in the book. Let the kids place the pigs.

Show the houses and explain the pigs cannot be in houses. Demonstrate a good way to place a piece and an incorrect way. Then let the kids have a go at the puzzle. The main concepts to emphasize here are:

- Just try something!
- If you get stuck, back it out until you are not stuck.
- If a choice got you stuck, then make a different choice.

You may give them a hint, but dont phrase it as such: what if we start with this piece here (solutions are on the back if you need it, but dont show them the solution page, even after they solve it). Keep going through the easy level puzzles until 1 minute before story time. Have the kids help you pack up the game and tidy up the shapes (good lesson in that, too!).

2.4. **Story:** *If you were a Quadrilateral* **by Molly Blaisdell.** Come together as a group for the story. Take the time to let the kids look at the richly detailed pictures and comment on the story or illustrations. Ask where they see quadrilaterals, rectangles, squares, etc. Enunciate the new vocabulary (polygon, parallelogram, etc) and talk a little about each. End by asking them to think about what kind of quadrilateral they are!
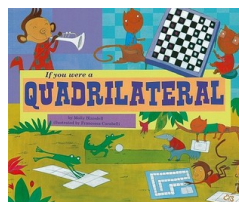
3. This is like that

This week we build on the concepts of similarities, similar to the first lesson. To keep things grounded, we use objects and beetles before getting to more abstract attributes like shape and number. We'll spend less time on the concept than last week and more time on the game. These activities are also easier to do in pairs or individually, to allow the kids to get more engagement time on their own.
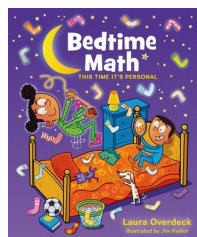

3.1. **Puzzle: *Bedtime Math* by Laura Overdeck.** Read a *Bedtime Math* story and ask the "Wee-Ones" puzzle afterward. Again, try to wait until most of your group figures out the answer, then ask each child to share his/her answer if he/she has one! Its OK if they didn't see how to or didn't want to figure it out. Whether they all agree or not, say "lets check it" and demonstrate with counting on your figures. Do up to 3 puzzles, based on interest of the group. Move faster by letting only one kid answer and one other kid check.

3.2. **Concept: Connections and Patterns.** To approach the concept of intersections of sets, we build on the "odd one out" theme with two activites taken from the Natural Math book *Socks are Like Pants, Cats are Like Dogs* by Malke Rosenfeld and Gordon Hamilton.


The first activity is "This is Like That". The kids will each get a board with a graph (mostly cycles) and a set of pictures. Talk about the pictures as a group so that each one is clear to each child. The rules here are that two objects can be placed next to each other (with an edge connecting them) when they have something in common.
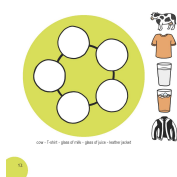
As an example: a cow, an orange shirt, a glass of milk, a glass of orange juice, a leather jacket are to be placed on a 5-cycle. One solution is cow to milk to orange juice to orange shirt to leather jacket to cow. We'll do four to six of these, depending on interest. Start with the first round as a group, so that everyone gets the idea of what to do, and then break into pairs or individuals for the subsequent rounds. Have the kids explain their solution, and sticker them with encouragement when they have a good explanation (doesn't have to be perfect, but don't let just anything go... challenge them to do better).

Next up we'll do "Beetle Sorting". The kids will each be given a page with images of beetles. On the left is a member of the beetle family they are looking for; on the right is a member of the beetle family they should try to avoid; in the middle are the candidates from which to choose. The purpose here is to work on their pattern recognition. It's amazing how well they can do with no instruction, so just let them have a go at it. There are 8 beetle challenges here, so again start with the first as a group so that everyone is on the same page (pun intended). Then let them work in pairs or individually.
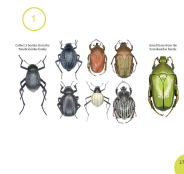
I recommend pairs if two kids are well matched in temperament and ability, and individual otherwise. Feel free to give the kids who work faster more puzzles to do.

The final activity for this lesson combines both of the prior ones and uses bug counters that come in 4 colors, 3 species, and 3 sizes. I might filter this down for each group to limit the number of possibilities. Let the kids have a moment to get familiar with the bugs before explaining the activity. With the bugs in the middle, explain that you will take turns choosing one bug to place in front of you, going *clockwise* (explain that!); start so that the child on your left goes first. S/he can place any bug in front of her/him. The next child must place a bug that has one attribute the same (color, species, or size), and so on, until the turn is back to you. You have to match both the bugs to your left and to your right. Talk about the dilema with the kids, and make a wrong choice to test before you find a correct one. Let a different person start each round, and talk about who has the easiest, hardest, and mediumest plays.

3.3. **Game: SmartGames Trio.** Each group will get a different game this week, so that each group has 2 copies of the same game. Pair the kids off as you see fit based on temperament. The three games will be "Three Little Pigs", "Little Red Riding Hood", and "Squirrels Go Nuts!", all by Smart Games. They are similar in their puzzle set up and the manipulation aspect, but each has it's own twist.

"Little Red Riding Hood" is a path game; encourage the kids the work the problem from both ends (i.e. from Red and from the House). "Squirrels go Nuts" is a sliding game; encourage the kids to solve one sub-puzzle (squirrel) at a time.

3.4. **Story: *Pattern Fish* by Trudy Harris.** This is a rhyming puzzle book with beautiful illustrations (by Anne Canevari Green). Every other page has a big animal with a pattern and the text describes the pattern leaving one missing piece for the reader to guess before the page turns to reveal the answer. The patterns begin simple (yellow–black–yellow–black–yellow–???) and increase in complexity (wiggle–jiggle–jiggle–float) by the end. There is not a running story, so it's easy to end this book early without it feeling unfinished.
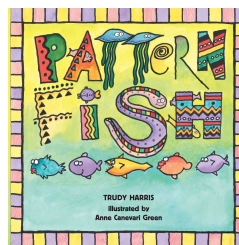
4. Two plus two can be three

Switching up our concept slightly, we're going to try having "stations" this week, and we'll rotate the kids through them in groups. The four components of the circle will be the four stations, so the kids will work through them at different times. Eventually we should let the kids have more autonomy in selecting their activities, so this is the beginning of that.

4.1. **Puzzle: *Bedtime Math* by Laura Overdeck.** Begin with a few *Bedtime Math* stories and ask the "Wee-Ones" puzzle afterward, then the "Little Kids" puzzle after that. Try to get them to find efficient ways to solve the problems, for instance by counting by 2s or 5s or 10s. Do several of these, and then move on to riddles.
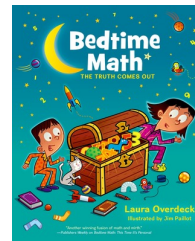
Problem A. Two fathers and two sons take a walk and find three apples. Which are there more of, people or apples?

Ask them to count how many people and compare that with the number of apples. Ask if they are certain.

Problem B. Two fathers and two sons take a walk and find three apples. Each person gets one apple. How does that work?

Explain that this is a riddle: it seems wrong now but there is a good explanation. The goal is to puzzle it out. Kids who have done the concept already will be in a better position to figure this out. Don't give away the answer!

4.2. **Concept: Intersections of sets.** Once again using our bug counters, we're going to explore the concept of intersections of sets. To begin, you will have two circles places with a small gap between on the table. Pour out all of the bugs (i've filtered down to a reasonable number) onto the middle of the table, not in either circle. Ask the kids to tell you how many different colors there are (4) and how many different species/kind of bug (3). Which are there more of, colors or kinds?

Next do quick sorting games to get them warmed up. Encourage them to wait for you to complete the instructions, then do the sorting as quickly as possible. Use the colored index cards for colors and the labeled white cards for kinds.

Problem C. Put all blue bugs in one circle and all red bugs in the other.

Make sure they leave the other colors alone. Count to determine which circle has more bugs, then count up to determine how many bugs total.

Problem D. Put all spiders in one circle and all butterflies in the other.

Count and compare to find the winner, then count up to get the total number of bugs. Warn them that the next one is a little tricky.

Problem E. Put all lady bugs in one circle and all red bugs in the other.

Watch to see what they do with the red lady bugs. If they just pick and no one objects, ask why that choice is better than the other. Ask if it is ok to pick one

circle and not the other. Ask if no circle is ok for the red lady bugs. Let them think about it and play with it. If they don't get it on their own after a while and no more energy is coming, then ask how they could put it in both circles. Eventually, you'll have a nice Venn diagram with the red lady bugs in the overlapping middle.

Now ask how many red and count. Then count how many lady bugs. Then compare. Then have them add the totals. Then have the count. Why are the numbers different? Wait for explanations.

4.3. **Game: SmartGames Trio.** The game station this week will be set up with two of each game. Each child should get his own game (though possibly not his first choice of game). The three games are "Three Little Pigs", "Little Red Riding Hood", and "Squirrels Go Nuts!", all by Smart Games. They are similar in their puzzle set up and the manipulation aspect, but each has it's own twist.


By now the games should be familiar, so to have the kids set up their own puzzles and solve them. Ask them to show you their solutions. Give small hints as needed.

4.4. **Story: *Shape* by Pluckrose and *Triangle* by Barnett and Klassen.** We'll have two books for the story station. One is a story and the other is interactive. Ask the group which they would like to do first, and remind them that they can get credit for the Read-a-thon!

Triangle tells the story of Triangle who goes to visit Square and plays a trick on him. I'm not a fan of the "play a mean trick to scare someone" theme, so I downplay that aspect. For the readers, you can let one of them read to the group if they want, or read by her/himself. For all, move your finger along with the text and emphasize the two words "triangle" and "square" when they appear. Pause to talk about why Square gets stuck, and of course debate the ending.

Shape is an interactive book that emphasizes the shapes we see in real life and how they relate to their mathematical ideals. The book is full of questions that you should let them answer. Emphasize the names of the shapes as they arise in the book and add some context (e.g. when you read "hexagon" for the first time, emphasize that it means a flat, closed figure with 6 sides).
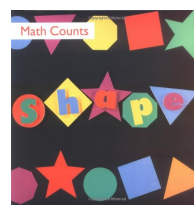
I'll also bring along the book "Which One Doesn't Belong" by Christopher Danielson that I read with them during the first circle. It might be nice to revisit this book now, if there's time.

5. Symmetry

This lesson has only three instructors, so the story aspect is combined into the puzzle for this week. Each section of the class should run roughly 15 minutes, to allow a couple of minutes of transition time between sections.

5.1. **Puzzle: Tiny Polka Dots.** Open the puzzle station with one or two stories from the *Bedtime Math* series, asking the wee-ones and little kids puzzles afterward.

We begin to build a stronger number sense using the *Tiny Polka Dots* cards. There are six stacks of cards, each stack containing a card representing each number between 0 and 10. We'll run through three games with the deck: number sorting, adding to 5, addition memory.

Problem A. Sort the cards by the same number.

We start with an easy sorting exercise to get the kids used to the cards. For this, use the numeral cards (yellow on purple) and the three stacks "dice" (red on teal), "parity" (blue/green on red), and "random" (orange on green). Start by laying out the numeral cards in a row. Have the other cards shuffled and divided into roughly even piles, one pile per kid plus one short stack for you. Begin with your stack and demonstrate how to match up the cards with the correct number. Have the kids help, then distribute the piles and let them have a go at sorting their own. Randomly check a few cards verbally as a group after the chaos subsides. Ask which cards were the hardest to sort and which were the easiest?

Problem B. Match the cards if the number of dots add to 5.

Pair the kids off; for an odd number, you get to play as well. Each kid gets one stack representing the numbers 0 to 5. S/he should lay out his cards across from his partner, and they should try to match up their cards based on value. Then, they should try to match them up when the dots add to 5.

Problem C. Memory where a match is two cards that add to 5.

Divide your group into two subgroups, each with two or three kids. You'll preside over both groups to keep the order. You can begin the memory game with looking for number matches, or jump right into matching cards that add to 5, depending on how the group did with the previous exercise. Use 12 cards, two stacks of the number 0 to 5, placed face down in a 3x4 grid.

5.2. **Concept: Pegboard symmetry.** For the concept this week we're back to the peg boards to consider symmetry in its different forms. This week we'll focus on two types of symmetry: mirror and radial. These activities will work best if you have a few boards already set up with examples, and pre-set boards for the first exercise as well. Also, limit the number of rubber bands by passing them out yourself and hoarding the pile; this will help to keep order.

Copy the pattern. Using one peg board as the model, make a pattern with one rectangle, one square and one triangle; don't get too complicated to begin. Take a moment to remind them the definitions of *quadrilateral* (a flat, closed shape with four sides), *rectangle* (quadrilateral with all right angles), and *square* (rectangle

with all sides equal length). Then pass out the right number and colors of rubber bands to have each kid create the same pattern as yours. If a kid finishes much earlier than the others, have her/him make a second copy on the back of the board. Check if the kids copied it or did a mirror image. Talk about the difference.

Mirror your partner's creation. Pair the kids off. This is a great teaching moment if you want to ask the kids if they can all be paired *even*ly or if there will be an *odd* person out. If there is an odd number, ask the kids for ideas how to resolve the problem (either you join in or let one group have three).

Take turns creating a pattern, then have the partner copy the pattern as a mirror image. Show them how to check the solution by holding the boards face to face to see if they match. Switch so that each kid gets a turn to create and copy.

Radial symmetry. Divide each board into quadrants using two rubber bands along the center. Demonstrate making a pattern with at most 2 rubber bands in one quadrant. Have the kids make their own patterns, again in only one quadrant. Then each person (including you!) should copy her/his own patter around to the next quadrant, then the next, then the next. Also take a moment to discuss *quad*rants, how many there are, and the semantic similarity with *quad*rilateral.

If time remains, then let them repeat any of those activities or just play with the peg boards. You can suggest challenges for them to create: a house, a person, a robot, etc, or simply let them challenge themselves. Once it's open ended like this, make sure that you're playing, too!

5.3. **Game: SmartGames Trio.** The game station this week will be set up with two of each game. Each child should get his own game (though possibly not his first choice of game). The three games are "Three Little Pigs", "Little Red Riding Hood", and "Squirrels Go Nuts!", all by Smart Games.

5.4. **Story:** *Penguin Place Value* **by Kathleen Stone.**
Penguin Place Value by Kathleen Stone is great for rein- forcing the grouping numbers concept, and it's an easy 2-3 minute read. This will start to teach the kids about group- ing by tens, and you can let them count the fish and trays and talk about counting by 10's versus counting by 1's. For more of a challenge, you can do *The Grapes of Math* by Greg Tang, which is a rhyming number riddle book.
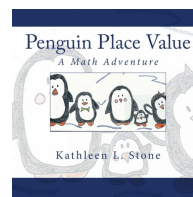
6. Intersections

This lesson again has only three instructors, so the story aspect is combined into the puzzle for this week. Each section of the class should run roughly 15 minutes, to allow a couple of minutes of transition time between sections.

6.1. **Puzzle: Pairity.** We use Tiny Polka Dots to explore the concept of *even*.

Problem A. What does it mean to call a number even?

Some kids might not be familiar with the *concept*, others might have a memorized answer. Try to dig deeper on the latter and engage the curiosity of the former. If there are an even number of cookies, then two kids can each have the same number of them. Check understanding on small numbers, going verbally through 1 to 5.

The next task will be to pass out paper and ask the kids to draw what *even* means. If someone write the number 2, you can comment along the lines of "that's the symbol we might use, but what do you think of when you think of the *concept*?". Apply stickers generously. Then out come the dots.

Using the parity cards (blue and green on red), have the kids count the total number of dots and then write that number of a white board. Then ask if the number is even or odd. Demonstrate how they can check by matching up (creating a *bijection*), the blue and the green dots. Have them sort the numbers into two lines, odd and even, and then in increasing order within the lines. Write the numbers down on the board, using different colors for even and odd.

Problem B. Is 0 even or odd?

Ask the question, listen to answers, don't pass judgement. Let them think about it, and validate any reasonable explanation (even if they argue for odd).

Next use the dice cards (red on teal) and ask if there is a good way to check *parity* for these cards. See if they notice the symmetry and the "one in the middle" property. If this is going well, then have them do the circle cards (blue on orange) by a similar symmetry argument. You can use the numeral cards (yellow on purple) for reference above the dotted ones.

Problem C. How can we tell is a super big number is even or odd?

To help with this, list numbers in columns that hint to the previous problem:

$$\begin{array}{cccccccccc} & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 10 & 11 & 12 & 13 & 14 & 15 & 16 & 17 & 18 & 19 \end{array}$$

Have them (or you do it if there are too many kids to keep order) circle the even numbers. Ask what they notice. Write down a few larger two-digit numbers and ask if they are even or odd. Ask which column the numbers will go in. Play with the idea until they are all comfortable with parity.

Problem D. Consider parity of even+even, odd+odd, and even+odd.

Talk about *remainders*, the left over bit of an odd number. I like to use the parity cards for this one to show how those two left over dots combine. Again, don't give away any answers, but have them do experiments and discover the patterns.

6.2. **Concept: Creating intersections.** We begin with the bug counters once again, but we'll reverse the concept from the previous one. This time the kids will be tasked to find 3 bugs so that exactly 2 are the same color and exactly 2 are the same species. Each kid should have two circles to help check his/her answer.

Problem E. How many red bugs? How many spiders? How many all together? How much is $2 + 2$? What's going on?

Now we'll make this more contextual with picture of everyday things. The goal is to create two sets with exactly one object in the intersection. We'll always take an odd number of pictures where there is some answer that splits the groups in half. For example, a spider, an octopus, and a clown fish. The spider and octopus have 8 legs, and the octopus and clown fish live in the ocean, so the octopus is in the middle. Divide a large group into smaller ones and have them do these together in pairs or triples. Make sure they all agree on the answers.

After this, we'll go back to the abstract with the game of SET. Reduce the game by using only the solid shapes (27 of 81), and explain the rules by

- there are three attributes: shape, color, number
- they must be all the same or all different

Lay out a 3x3 grid of cards and ask the kids to point to "sets". Check each one attribute at a time, but don't remove them yet. Clear the cards and do a 4x3 grid, now with remove and replace going in turns to find sets. If a child is stuck, point to two cards and ask him to finish the set by figuring out what shape, color, and number the third card is.

6.3. **Game: SmartGames Trio.** The game station once again has the three games "Three Little Pigs", "Little Red Riding Hood", and "Squirrels Go Nuts!"

6.4. **Story:** ***Sheep won't sleep* by Judy Cox.** This is a comical counting book where herds of animals are counted by 1's, 2's, 5's, and 10's. The pictures are vibrant and the story is cute, so it should hold their interest and get them counting by different amounts. After the book, have the kids do the count by 1's until 10, then by 2's until 30, then by 5's until 50, then by 10's until 100. Write the benchmark switch points on a white board and write the numbers (perhaps in a different color) as they count.