Exam 2 is cumulative, but will emphasize material covered since Exam 1: §8.0, §8.2–8.7, but only the parts of §8.5–7 covered in class (equivalently Homework #5a–10a).

Exercise 1. Evaluate the following series, or show that they are divergent.

(a) $\displaystyle\sum_{n=0}^{\infty} \frac{2^{3n+1}+4}{\pi^{2n-1}}$.

(b) $\displaystyle\sum_{n=1}^{\infty} \frac{\sin\left(\frac{1}{n}\right)}{\tan\left(\frac{1}{n}\right)}$.

Exercise 2. Determine if the following series are convergent or divergent. Be sure to clearly state any test(s) you use.

(a) $\displaystyle\sum_{n=0}^{\infty} \frac{\sqrt{n}+1}{\sqrt{n^3}-\sqrt[3]{n^2}-1}$.

(b) $\displaystyle\sum_{n=0}^{\infty} \frac{n^2+n-1}{n^4-n^2+1}$.

(c) $\displaystyle\sum_{n=0}^{\infty} \frac{n!}{\sqrt{(2n)!}}$.

(d) $\displaystyle\sum_{n=1}^{\infty} \frac{2^n-n^2+\log_2(n)}{\log_3(n)+n^3-3^n}$.

(e) $\displaystyle\sum_{k=3}^{\infty} \frac{1}{(\ln k)^3}$.

Exercise 3. Determine if the following series are absolutely convergent, conditionally convergent, or divergent. Be sure to clearly state any test(s) you use.

(a) $\displaystyle\sum_{n=3}^{\infty} (-1)^n \frac{3n^{2e}}{n^{2\pi}-4}$.

(b) $\displaystyle\sum_{n=1}^{\infty} (-1)^n\left(1+\tfrac{1}{n}\right)$.

(c) $\displaystyle\sum_{m=2}^{\infty} \frac{(-1)^{m+1}}{m(\ln m)^{1.1}}$.

(d) $\displaystyle\sum_{k=129}^{\infty} \sin\left(\frac{1}{k}\right)$.

Exercise 4. For which values of p does $\sum_{k=3}^{\infty} \frac{1}{k(\ln k)^p}$ converge. Prove your answer.

Exercise 5. Find the interval of convergence of the power series $\sum_{n=2}^{\infty} \frac{\ln(n)}{\sqrt{n}} x^n$.

Exercise 6. Find the interval of convergence of the power series $\sum_{k=0}^{\infty} \frac{x^{2k}}{7^{3k}}$.

Exercise 7. Consider the function given by the power series $f(x) = \sum_{n=0}^{\infty} (-1)^n \frac{x^n}{2n+1}$ with radius of convergence 1.

(a) Find the power series of $f'(x)$ and state its radius of convergence.

(b) Find a series for $f'(\frac{1}{3})$.

(c) Find the minimal number of terms that we need to use to approximate the above series so that the error is less than $\frac{1}{100}$.

Exercise 8. Harry is lost in the Forbidden Forrest. Hermione is trying to find him.

(a) Hermione determines that the amount of magical energy required for a *finding spell* is $f(x) = \frac{M}{1+x^2}$ where $M > 0$ is Merlin's constant. Find the Taylor series for $f(x)$ and state its radius of convergence.

(b) Because Harry is wearing his cloak of invisibility, she needs a more powerful spell. The amount of energy for the *powerful spell* is $p(x) = \frac{Mx^{13}}{1+x^2}$. Find the Taylor series for $p(x)$ and state its radius of convergence.

(c) The amount of energy required to search the entire forest is $\int_0^{1/2} p(x)\,dx$. Find a series for this integral. Write the general term in any valid form. Explicitly write out the first 3 non-zero terms as fractions.

(d) Estimate the error if Hermione approximates the series in (c) by its first 3 non-zero terms.