Math 129 **Exam 2** **Spring 2023**

Name:_____

Printed (as registered on Blackboard)

USC ID #

Instructions:

- Do not open this exam until you are told to begin. You will have 50 minutes for the exam.
- **Show all your work**. Unless explicitly stated otherwise in a particular question, if there is no work supporting your answer, you will not receive credit for the problem.
- If you need more space for a problem, there is a blank page at the end of the exam.
- You are allowed to have one page of notes, 8.5" × 11", hand written on both sides. No collaboration is allowed. No calculators or electronic devices are allowed. *Turn off your cell phone.*
- Cheating will result in a zero on this exam and the student will be reported to the Office of Academic Integrity.

Question	Points
1	20
2	20
3	20
4	20
5	20
Total:	100

1. (20 points)

   (a) Find the value of the series

$$\sum_{n=2}^{\infty} \frac{3^{2n-2}}{5^{n+3}}.$$

   (b) Determine if the follow series converges or diverges. Clearly state any test(s) you use and show that they are applicable.

$$\sum_{n=129}^{\infty} \frac{n^2 + 1 - \cos(n)}{\sqrt{n^5 - n^2 + n - 8}}.$$

2. (20 points) Determine whether the following series are absolutely convergent (ac), conditionally convergent (cc) or divergent (div). Circle your answer for each part. Clearly state any test(s) you use and show that they are applicable.

(a) $\displaystyle\sum_{n=0}^{\infty} \frac{\sin(n)}{n^2 - n - 1}$. ac cc div

(b) $\displaystyle\sum_{k=2}^{\infty} (-1)^k \frac{\ln(k)}{k}$. ac cc div

3. (20 points) Consider the following power series:

$$\sum_{n=0}^{\infty} \frac{x^n}{5^n(n+1)^2}$$

(a) Show that the *radius* of convergence of the power series is 5. That is, $R = 5$.

(b) Find the *interval* of convergence of the power series.

4. (20 points) Consider the following Taylor series:

$$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!}$$

with $R = \infty$.

(a) Find a Taylor series for $f(x) = x^5 e^{-x^2}$.

(b) Find a series for $\displaystyle\int_0^{\frac{1}{10}} f(x)\,dx$.

(c) Find the best upper bound for the error that you can, if we approximate the series in Part (b) by its first 3 non-zero terms. *Briefly justify the technique you use.*

5. (20 points) Consider the function f defined by a power series:

$$f(x) = \sum_{n=0}^{\infty} \frac{x^n}{4^n(n+3)}$$

with $R = 4$.

(a) Explicitly write out the first 3 non-zero terms of the series.

(b) Find a power series for $f'(x)$ and state its radius of convergence.

(c) Find a series for $f'(-3)$.

(d) Find the minimal number of terms we can use to approximate the series in Part (c) so that the error is no more than $\frac{1}{10}$.

blank page.