

Reforming Capitalism¹

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ABSTRACT

This paper presents a model of the *stakeholder* corporation and analyzes the equilibrium of an economy with stakeholder firms. The analysis is based on a model of a production economy that differs from the standard approach based on states of nature. The property that differentiates it from the standard model (which justifies the *shareholder* approach to the corporation) is that firms' choices of investment influence the probability distributions of their outputs, and hence exert external effects on consumers and employees: as a result profit maximization and competitive behavior do not lead to Pareto optimality. Using a Coasian approach to resolve the problem of externalities, we show that if firms issue not only equity shares but also marketable property rights for employees and consumers, and if firms' managers maximize the total values of their firms (shareholder value plus consumer and employee values) then Pareto optimality of equilibrium is restored when agents are identical. In the more realistic case where agents are heterogeneous, reforming capitalism by giving some weight to employee and consumer surpluses in the objective of the firms always increases social welfare.

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1 Introduction

Everyone knows that corporations are not just cash machines for their shareholders, but that they also provide goods and services for their consumers, as well as jobs and incomes for their employees. Everyone, that is, except most economists. Indeed in the debate on the social responsibility of corporations, the majority of academic economists share the view expressed in unambiguous terms by Friedman (1970) “there is one and only one social responsibility of business—to use its resources and engage in activities designed to increase its profits”. By contrast, proponents of the ‘stakeholder’ view of corporations assert that managers should pay attention not only to the profits of the shareholders but also to the welfare of their employees and consumers. The orthodox view held by most economists is a tradition inherited from the Anglo-American view of corporations, while the so-called non-orthodox stakeholder view is that held in countries such as Japan and most continental European countries, in particular France and Germany.

The way in which a society views the role of a corporation can be traced to its legal system and to the social norms which shape the way individuals think about the role of institutions. Common Law countries such as the UK and the US view a corporation as a piece of private property and through their legal structure place exclusive emphasis on the shareholders as the owners of the firm. Civil Law countries such as France and Germany view corporations as ‘mini-societies’ and place emphasis on the responsibility of the firm to its employees as well as its shareholders. Social norms have pushed this view of the corporation to its extreme form in Japan where the responsibility to the interest of employees and other stakeholders such as suppliers outweighs that to the shareholders (see Yoshimori (1995)).

Viewed in historical perspective the stakeholder view of the corporation has been gaining momentum in all advanced economies over the last hundred years: the changing legal structures and the evolution of social norms have come to make most large corporations aware that they need to expand the focus of their responsibilities to a larger group than the shareholders, to include employees and consumers as well as other groups such as suppliers and subcontractors involved in their long-term productive relationship. Despite this century-long shift in the rest of society’s view of the corporate entity, the remarkable phenomenon is the hegemony of the Anglo-American view of the corporation among economists: to this day the idea that the corporation should serve the exclusive interest of its shareholders remains the dominant paradigm for corporate governance (Schleifer-Vishny (1997)). Although recently there are some signs of a willingness to change (Tirole (2001, 2006)) mainstream economics has not kept abreast of the evolution of society’s view of the role and responsibilities of a corporation, and continues to advocate shareholder value maximization as the primary responsibility of the management of a corporation.

As pointed out by Tirole (2001) one of the impediments to developing a stakeholder theory of the corporation acceptable to academic economists has been the lack of a formal model which can serve to articulate the basic ideas of the stakeholder approach. Underlying the faith of corporate finance in the virtue of profit maximization is the Arrow-Debreu model in which uncertainty is described by states of nature. In such a model, if there are contingent markets for all commodities, an Arrow-Debreu equilibrium leads to a Pareto optimum: in equilibrium firms maximize the present value of their profit and all agents, whether shareholders, workers

or consumers agree with this objective for the firm. There are no unaccounted for externalities which might prevent profit maximization from being the best criterion. This is what Friedman had in mind when he said that businessmen are wrong to look for a broader sense of ‘social responsibility’, since the profit criterion accurately takes into account the interests of all the agents in the economy and hence automatically acts in the social interest.

To the extent that an equilibrium model based on states of nature provides a realistic modeling of production under uncertainty, the Arrow-Debreu model provides a solid theoretical foundation for the shareholder model of the corporation, and serves to vindicate the view advocated by Friedman. The Arrow-Debreu model is however of far more limited applicability than the literature following Arrow (1953) and Debreu (1959) would have us believe, and this for two reasons. The first is that the Arrow-Debreu (AD) model is based on assumptions about the way productive uncertainty within firms can be described and the nature of the markets on which firms are supposed to operate which are difficult, if not impossible, to match to anything we observe, or could even imagine observing, in the real world. The conditions that would need to be satisfied are the following:

- (1) it must be possible to make a complete enumeration of *primitive causes (states of nature)* which, when combined with the actions (investment) of the firms, serve to explain the firms’ different possible outcomes;
- (2) these primitive causes must:
 - (i) have probabilities which are exogenous and *independent of the actions* of the firms;
 - (ii) be *known* and understood, not only by the manager of each firm, but *by all agents* in the economy;
 - (iii) be *sufficiently simple* to describe and verify to form the basis for contracts traded on markets.

In the standard Arrow-Debreu model these contracts are (promises to make) forward delivery of goods contingent on the primitive causes (states of nature). In a sequential interpretation of the AD model goods are exchanged on spot markets and financial contracts are used to distribute income across time and the states, all prices and contracts being contingent on the primitive causes.

Secondly, even if all these conditions could be satisfied, on which Arrow (1971) raised serious doubts, he also argued that the technical conditions required to obtain existence of an equilibrium may be violated by the unavoidable presence of indivisibilities or nonconvexities: to quote Arrow (1971)

“ . . . we have seen that it is possible to set up formal mechanisms which under certain conditions will achieve an optimal allocation of risk by competitive methods. However, the empirical validity of the conditions for the optimal character of competitive allocations is considerably less likely to be fulfilled in the case of uncertainty than in the case of certainty and, furthermore, many of the economic institutions which would be needed to carry out the competitive allocation in the case of uncertainty are in fact lacking.”

Perhaps if we were describing farming and used the dependence of a crop of wheat on the temperature, sunshine, and humidity in a certain locality over the growing season as the

primitive states, the approach might provide a useful first approximation, if the requisite contracts on such states were available. But these conditions, as we explain in Section 2, are far too demanding for the state-of-nature approach to provide a useful way of describing the complex production processes inherent in modern corporations.

This led two of us (Magill-Quinzii (2009)) to propose an alternative model of production under uncertainty in which the primitives are the possible outcomes of the firms, and investment influences the probability distribution of these outcomes: states of nature are left unspecified in the background and only the probability distribution of the possible outcomes of the production process is known. Such a model is more realistic since agents trade contracts based only on observable outcomes. However this alternative modeling of uncertainty brings with it an important consequence: since firms' actions affect the probability distribution of their outputs, they also affect the expected utilities of all agents in the economy.

In this paper we pursue the analysis of these externalities further, noting that by shifting the probability towards outcomes where it is more productive, and thus where the prices of the goods it produces are lower, and the wages of its employees are higher, the actions of each firm influence the expected utilities of its consumers and employees, and this external effect is not internalized by the market: as a result the profit criterion is no longer the correct 'social criterion' for the firm. There is thus a radical change in the view of the role and objective of a corporation when we move from an equilibrium model of production based on states of nature to an equilibrium model of production based on the probability approach advocated in this paper. From a logical perspective there is (not surprisingly) nothing wrong with Friedman's argument that the socially responsible objective of the firm is to maximize profit—it is just that he is assuming the presence of markets which permit every firm to fully internalize the consequences of its actions on all other agents—and we argue that in practice such markets do not exist.

The probability approach leads to a different criterion: since firms' actions have external effects on employees and consumers, the socially optimal criterion for the firm is to maximize the sum of the profit of the shareholders, the surplus of the employees and the surplus of the consumers—providing a precise formal criterion for a stakeholder theory of the corporation. Thus the apparent mystery in the conflict between the traditional shareholder-based profit-maximizing approach advocated by economists and the stakeholder approach to the corporation widely advocated by non-economists seems to be resolved: it reduces to whether we model markets based on highly complex and mostly unobservable states of nature or whether, as we propose, contracts and prices are based on firms' outcomes and firms can influence the probabilities of their outcomes.

Using the probability approach to modeling the production of firms, we introduce a concept of equilibrium in which firms maximize the sum of the surplus of shareholders, workers and consumers, which we call a *stakeholder equilibrium*, and show that such an equilibrium is Pareto optimal. This is an important first step: however it does not explain why corporate managers would be induced to apply the surplus criterion or, equally fundamentally, how they would acquire the requisite information on the surpluses of workers and consumers to apply the criterion.

We explore a market approach to the internalization of externalities based on Coase's (1960) fundamental insight that properly defining property rights, and organizing efficient markets

for trading them, allows externalities to be internalized and restores the efficiency of a market equilibrium. To this end we introduce two collections of tradeable rights associated with a firm: employee rights, which give to the bearer the right to work for the firm, and consumer rights which give to the bearer the right to purchase the firm's output on the spot market. The firm's charter specifies that the manager should maximize the total market value of the firm's rights—the market value of equity plus the market value of employee and consumer rights. Just as an equity holder who has bought the right to get a share of the firm's profit gets voting power in the corporation to ensure that his interests are duly taken into account, so the holders of employee and consumer rights must get voting power on the firm's decisions to ensure that the total value criterion is applied.

Section 2 introduces the simplest model which permits us to formalize these ideas, consisting of a single firm and identical investors, employees and consumers. The firm can make an investment at date 0 which increases the probability of having an improved technology at date 1—we refer to this as the “probability model” since investment influences the probability of the outcome. In this model if the firm uses the “capitalist” criterion of maximizing the present value of profit then there is always underinvestment relative to the social optimum. Since such an inefficiency does not arise in an Arrow-Debreu equilibrium, we examine what would be involved in specifying the same underlying economy in an Arrow-Debreu fashion with explicit states of nature, and conclude that the detailed data required to even specify the model are not in practice available. Furthermore for this economy, even if such data were available, no Arrow-Debreu equilibrium exists.

In Section 3 we show that if the firm adopts the stakeholder criterion of maximizing the profit plus the employee and consumer surplus, then it is led to the socially optimal level of investment. Adopting the stakeholder rather than the profit criterion augments the firm's perception of the benefits from the “good” outcome, leading it to invest more, thereby solving the externality problem. When all agents are identical there is a device which ensures that the firm's manager will adopt the stakeholder criterion: this consists in issuing tradeable employee and consumer rights. For when agents are homogeneous the prices of the rights correctly reflect the employee and consumer surpluses, so that maximizing the total value of the firm leads to the socially optimal investment.

When agents are heterogeneous this device no longer works with the same magic. For when employees have different disutilities of work and consumers have different utilities for the good, the prices of the rights reflect the surplus of the marginal buyer of the rights, namely the one with the lowest valuation among the buyers of the rights: thus the total value of the rights no longer correctly reflects the total surplus. In this case there is a trade-off between excluding some agents from the markets (by restricting the issue of rights) and obtaining a more accurate evaluation of the total surplus. For if many rights are issued, few if any employees and consumers are excluded from trading with the firm, but the prices of the rights may be very low since they reflect the surplus of the least interested employee or consumer. While if fewer rights are issued their prices may more accurately reflect the average surpluses, but more agents are excluded from the “club” of “authorized” employees and consumers.

In Section 4 we show however that there is always a way of ensuring that rights retain a part of their magic, for the trade-off can be resolved so that the equilibrium with rights always improves on the capitalist equilibrium. If the employee with the highest disutility of work has

a *positive* surplus, and/or the consumer with the least interest in the produced good gets a *positive* surplus from buying it, then even without exclusion, when as many rights are issued as there are potential employees and consumers, maximizing total value leads to an improvement over the capitalist equilibrium. When the least interested employees and consumers get *zero* surplus from participating in the market, and enough rights are issued to permit all employees and consumers to participate, then the prices of the rights are zero and the rights equilibrium coincides with the capitalist equilibrium. We show however that marginally restricting the the supply of rights and forcing payment of small fixed fee to belong to the firm’s “club” of employees and consumers, always increases welfare relative to the capitalist equilibrium. The welfare loss associated with the exclusion of some potential employees and consumers is more than compensated by the gain in welfare from the increased investment induced by a more accurate evaluation of the total surplus. Thus reforming capitalism by issuing rights can always lead to an increase in welfare, even if it only marginally shifts the focus of the firm’s responsibility from shareholders to the combined interests of its employees, consumers and shareholders.

The paper is organized as follows: Section 2 presents the benchmark model with homogeneous agents and Section 3 studies the concept of a stakeholder equilibrium for such an economy. Section 4 extends the analysis to an economy with heterogeneous agents.

2 Benchmark Model with Homogeneous Agents

Consider a stochastic production economy with one firm, two dates ($t = 0, 1$) and three goods: a produced good, a composite good called “money” (used as the numeraire) and labor. At date 0 there is only money, a part of which can be used to finance investment by the firm. The investment is risky in that there are two possible outcomes: the production function at date 1 can be either $y = f_g(l)$ or $y = f_b(l)$, where f_g and $f_b : \mathbb{R}_+ \rightarrow \mathbb{R}$ are differentiable, increasing, concave and satisfy $f_s(0) = 0$, $s = g, b$. The marginal product of f_g is uniformly higher than that of f_b : $f'_g(l) > f'_b(l), \forall l > 0$, which implies that $f_g(l) > f_b(l), \forall l > 0$. Thus ‘g’ is the good outcome. The probability $\pi(a)$ of this outcome is determined by the amount of investment a made by the firm at date 0: π is increasing and concave, $0 < \pi(a) < 1, \forall a > 0$ (no amount of investment removes uncertainty), $\pi'(a) \rightarrow \infty$ as $a \rightarrow 0$, and $\pi'(a) \rightarrow 0$ as $a \rightarrow \infty$. To retain the symmetry of notation, we let $\pi_s(a)$ denote the probability of outcome $s, s = g, b$, with $\pi_g(a) = \pi(a)$ and $\pi_b(a) = 1 - \pi(a)$.

There are three “classes” of agents: workers, consumers and capitalists. Each consists of a continuum of identical agents of mass 1. The representative worker, who is endowed with 1 unit of labor at date 1, consumes only money and has the utility function

$$U^w(m, \ell) = m_0 + \delta \sum_{s=g,b} \pi_s(a) \left(m_s - v(\ell_s) \right),$$

where $m = (m_0, m_g, m_b)$ is a worker’s consumption of money and ℓ_s is the quantity of labor sold to the firm in outcome $s, s = g, b$. The discount factor satisfies $0 < \delta \leq 1$ and the disutility of labor, $v(\ell) : \mathbb{R}_+ \rightarrow \mathbb{R}$, is differentiable, convex and increasing, with $v(0) = 0$, $v'(\ell) \rightarrow \infty$ if $\ell \rightarrow 1$ and $v'(\ell) \rightarrow 0$ if $\ell \rightarrow 0$.

The representative consumer, who consumes both money and the produced good, has the utility function

$$U^c(m, c) = m_0 + \delta \sum_{s=g,b} \pi_s(a) \left(m_s + u(c_s) \right),$$

where $c = (c_g, c_b)$ is the consumption of the produced good in the two outcomes, and u is differentiable, strictly concave and increasing, with $u(0) = 0$ and $u'(c) \rightarrow \infty$ if $c \rightarrow 0$.

Finally the representative capitalist, who is the owner of the firm, consumes only money and has the utility function

$$U^k(m) = m_0 + \delta \sum_{s=g,b} \pi_s(a) m_s.$$

The money endowments $e^i = (e_0^i, e_1^i)$, $i = w, c, k$ are assumed to be sufficiently large so that non-negativity constraints on consumption never bind. We let $e_0 = e_0^w + e_0^c + e_0^k$, $e_1 = e_1^w + e_1^c + e_1^k$ denote the aggregate endowment of money at date 0 and 1. We let $\mathcal{E} = (U, e, f, \pi)$ denote the economy with preferences, endowments and technology $(U^i, e^i)_{i=w,c,k}$ and technology characterized by $f = (f_g, f_b)$ and π .

The general model outlined above is the reference model for our analysis: it focuses on uncertainty regarding the efficiency of the production process at date 1. It can be readily modified to cover uncertainty regarding the quality (good or bad) of the commodity produced at date 1, where a commodity of good quality is one that is free of defects, and the probability of obtaining a product without defects is an increasing function of the expenditure a invested at date 0. To adapt the model to this case, it is sufficient to make the utility of the representative consumer for the produced good dependent on the outcome: let $u_s(x)$ denote the utility of consuming x units of quality s . Whatever the quality outcome, investing l units of labor produces $f(l)$ units, so that the quantity produced is no longer outcome dependent.² If the production function f and the utility functions u_s satisfy the standard assumptions made above and if $u_g(x) > u_b(x)$, $u'_g(x) > u'_b(x)$ for all $x \geq 0$, then all the results derived for the reference model carry over to the model with uncertain quality.

2.1 Socially Optimal Investment

Given the quasi-linearity of the agents' preferences, a Pareto optimum is an allocation (a, m, c, ℓ) which maximizes the sum of the agents' utilities

$$\max_{(a, m, c, \ell) \geq 0} \sum_{i=w,c,k} \left(m_0^i + \delta \sum_{s=g,b} \pi_s(a) m_s^i \right) + \delta \sum_{s=g,b} \pi_s(a) (u(c_s) - v(\ell_s))$$

subject to the resource constraints

$$\sum_{i=w,c,k} m_0^i + a = e_0, \quad \sum_{i=w,c,k} m_s^i = e_1, \quad c_s = f_s(\ell_s), \quad s = g, b.$$

²At the cost of making the proofs more cumbersome, both the utility and the production functions could be taken to be dependent of the outcome.

This is equivalent to finding (a^*, ℓ^*) which solves

$$\max_{(a, \ell) \geq 0} e_0 - a + \delta \sum_{s=g, b} \pi_s(a)(e_1 + u(f(\ell_s)) - v(\ell_s)).$$

(a^*, ℓ^*) is characterized by the first-order conditions

$$u'(f_s(\ell_s^*))f'(\ell_s^*) = v'(\ell_s), \quad s = g, b \quad (1)$$

$$\delta \sum_{s=g, b} \pi'_s(a^*)(u(f_s(\ell_s^*)) - v(\ell_s^*)) \leq 1, \quad = \text{ if } a^* > 0 \quad (2)$$

Since the presence of e_1 in the social utility function does not influence the production choice, we may define the social welfare in state s by

$$W_s = u(c_s) - v(\ell_s), \quad s = g, b$$

factoring out the utility of the exogenous amount of money e_1 . Consider the pair of equations (1): since $u(f_s(\ell_s))$ is strictly concave, the left side of (1) is decreasing while the right side is increasing. Since $u'(c) \rightarrow \infty$ when $c \rightarrow 0$ and $v'(\ell) \rightarrow \infty$ when $\ell \rightarrow 1$ there is always a solution to (1), and the solution $\ell^* = (\ell_g^*, \ell_b^*)$ is unique. Let $W_s^* = (u(f_s(\ell_s^*)) - v(\ell_s^*))$, $s = g, b$. The first-order condition (2) can then be expressed as

$$\delta \pi'(a^*)(W_g^* - W_b^*) = 1 \quad \text{if } W_g^* > W_b^*, \quad a^* = 0 \text{ otherwise} \quad (3)$$

and the equation in (3) has a unique solution since $\pi'(a)$ decreases from ∞ to 0.

2.2 Capitalist Equilibrium

We now analyze a market equilibrium of the above economy in which the firm chooses its investment and labor in the best interests of its shareholders (the capitalists) maximizing the present value of its profit, and consumers and workers buy the firm's production and sell their labor services on spot markets. Agents trade in addition on a pair of asset markets to redistribute their income. We will show that the real side of such a market equilibrium can be summarized by a pair (\bar{a}, \bar{l}) consisting of the firm's investment and labor choice, and this pair can be compared with the Pareto optimal choice (a^*, l^*) derived above.

At each date there is a spot market for the composite commodity (money) with price normalized to 1. At date 0 agents trade a riskless bond promising one unit of money in each outcome $s = g, b$ at date 1, and given the agents' preferences its price is $\frac{1}{1+r}$ where r is the interest rate. There is also an equity market at date 0 on which the capitalists sell their ownership of the firm, the price of equity being q_e . At date 1 for each outcome $s = g, b$ there are spot markets for labor and the produced good with prices (w_s, p_s) , $s = g, b$.

The firm makes two choices, the amount a to invest at date 0 and the amount of labor $l = (l_g, l_b)$ to hire in each of the possible outcomes at date 1. The labor is chosen in each outcome to maximize

$$R_s(w_s, p_s) = p_s f(l_s) - w_s l_s, \quad s = g, b$$

taking the spot prices (w_s, p_s) as given. Assuming that the firm correctly anticipates the spot prices and its future labor decision, it chooses its investment a at date 0 to maximize the (net) present value of profit, which in this case is just the discounted expected profit since agents are risk neutral. The firm's combined choice problem amounts to choosing (a, l) to maximize its value for the shareholders

$$SV(a, l; w, p) = \sum_{s=g,b} \frac{\pi_s(a)}{1+r} (p_s f(l_s) - w_s l_s) - a. \quad (4)$$

given the spot prices (w, p) . The three groups of agents trade on the spot and financial markets and have sequential budget equations at date 0 and in each outcome at date 1 of the form

$$\begin{aligned} m_0^i &= e_0^i - \frac{1}{1+r} z^i - q_e \theta^i + \xi^i \\ m_s^i &= e_s^i + z^i + R_s \theta^i + w_s \ell_s^i - p_s c_s^i, \quad s = g, b \end{aligned} \quad (5)$$

where z^i is the bond holding and θ^i the ownership share of the firm purchased by agent i and

$$\begin{aligned} \bullet \quad & \xi^i = 0, \quad \text{if } i = w, c, & \xi^i &= q_e - a \quad \text{if } i = k \\ \bullet \quad & c_s^i = 0, \quad \text{if } i = w, k, & c_s^i &= c_s \quad \text{if } i = c \\ \bullet \quad & \ell^i = 0, \quad \text{if } i = c, k, & \ell_s^i &= \ell_s \quad \text{if } i = w \end{aligned} \quad (6)$$

Thus capitalists as initial owners of the firm finance the investment a and get income from the sale of their ownership shares ($\xi^k = q_e - a$), while only the consumers purchase the produced good ($c_s^c = c_s$) and only workers sell their labor services ($\ell_s^w = \ell_s$). While capitalists are assumed to finance the investment, any mode of financing whether by debt or by issuing new shares would lead to the same equilibrium in view of the Miller-Modigliani theorem. All agents are assumed to know the firm's choice of investment a at date 0 and to correctly anticipate future spot prices and the firm's profit R_s in each outcome s at date 1.

Given the linearity of the agents' preferences in the numeraire composite commodity the first-order conditions for the optimal choice of bond and equity holdings are

$$\frac{1}{1+r} = \delta, \quad q_e = \delta \sum_{s=g,b} \pi_s(a) R_s = \sum_{s=g,b} \frac{\pi_s(a)}{1+r} R_s \quad (7)$$

so that pricing is risk neutral. Since the date 1 payoff of the bond is $(1, 1)$, if $R_g \neq R_b$, the bond and equity have linearly independent payoff streams, so that the financial markets are complete and the sequential budget constraints (5) are equivalent to the single intertemporal (present value) budget constraint

$$m_0^i + \sum_{s=g,b} \frac{\pi_s(a)}{1+r} m_s^i = e_0^i + \frac{e_1^i}{1+r} + \xi^i + \sum_{s=g,b} \frac{\pi_s(a)}{1+r} (w_s \ell_s^i - p_s c_s^i), \quad i = w, c, k \quad (8)$$

where (ξ^i, c^i, ℓ^i) are given by (6). In view of the linearity of the agents' preferences in $m^i = (m_0^i, m_g^i, m_b^i)$ any m^i satisfying (8) is equivalent for agent i , and when the budget constraint

(8) is satisfied the utility of agent i is

$$\begin{aligned}
\bullet \quad & e_0^w + \frac{e_1^w}{1+r} + \sum_{s=g,b} \frac{\pi_s(a)}{1+r} (w_s \ell_s - v(\ell_s)) && \text{for a worker} && \text{(a)} \\
\bullet \quad & e_0^c + \frac{e_1^c}{1+r} + \sum_{s=g,b} \frac{\pi_s(a)}{1+r} (u(c_s) - p_s c_s) && \text{for a consumer} && \text{(b)} \\
\bullet \quad & e_0^k + \frac{e_1^k}{1+r} + q_e - a && \text{for a capitalist} && \text{(c)}
\end{aligned} \tag{9}$$

Thus a worker will choose ℓ to maximize 9(a), a consumer will choose c to maximize 9(b) and a capitalist has no other choice than to spend his income on the composite good, his utility being maximized when the firm's manager maximizes $q_e - a$ which, given (7), is equal to the shareholder value SV . Summing the budget equations (8), assuming that (6) holds, gives

$$\sum_{i=w,c,k} m_0^i + \sum_{s=g,b} \frac{\pi_s(a)}{1+r} m_s^i = e_0 + \frac{e_1}{1+r} + \sum_{s=g,b} \frac{\pi_s(a)}{1+r} (-p_s c_s + w_s \ell_s) + q_e - a$$

If the markets clear for the produced good ($c_s = f(l_s)$) and labor ($l_s = \ell_s$) then in view of (7) the terms involving the firm's market value cancel giving

$$\sum_{i=w,c,k} m_0^i + \sum_{s=g,b} \frac{\pi_s(a)}{1+r} m_s^i = e_0 + \frac{e_1}{1+r} - a.$$

Given the indeterminacy in the choice of m^i , we can assume that when agents choose m^i to satisfy (8) they in addition choose money holding such that

$$\sum_{i=w,c,k} m_0^i + a = e_0, \quad \sum_{i=w,c,k} m_s^i = e_1, \quad s = g, b \tag{10}$$

so that the market for the composite good clears at date 0 and in each outcome s at date 1.

Since our objective is to compare the consumption, labor and investment choices which arise in a market equilibrium with those at the social optimum, we focus directly on a succinct reduced form definition of an equilibrium involving these three choices: from this reduced form of equilibrium a complete description of the equilibrium on the spot markets for the produced good, money and labor, and on the financial markets for the bond and equity can be reconstructed easily using (5)-(7) and (10).

Definition 1. A (*reduced form*) *capitalist equilibrium* of the economy \mathcal{E} is a pair $\left((\bar{\ell}, \bar{c}, \bar{a}, \bar{l}), (\bar{w}, \bar{p}) \right)$ consisting of actions and prices such that

- (i) the labor choice $\bar{\ell} = (\bar{\ell}_g, \bar{\ell}_b) \geq 0$ maximizes worker's utility 9(a) given (\bar{a}, \bar{w}) ;
- (ii) the consumption choice $\bar{c} = (\bar{c}_g, \bar{c}_b) \geq 0$ maximizes consumer's utility 9(b) given (\bar{a}, \bar{p}) ;
- (iii) the firm's production plan $(\bar{a}, \bar{l}) = (\bar{a}, \bar{l}_g, \bar{l}_b) \geq 0$ maximizes shareholder value (4) given (\bar{w}, \bar{p}) ;

(iv) the markets clear: $\bar{\ell}_s = \bar{l}_s$, $\bar{c}_s = f_s(\bar{l}_s)$, $s = g, b$.

In a capitalist equilibrium the optimal labor choice $\bar{\ell}$ for the workers satisfies

$$v'(\bar{\ell}_s) = \bar{w}_s, \quad s = g, b \quad (11)$$

and the consumers' optimal choice \bar{c} satisfies

$$u'(\bar{c}_s) = \bar{p}_s, \quad s = g, b \quad (12)$$

while the firm's profit maximizing choice of labor \bar{l} implies that for each outcome at date 1 the real wage equals the marginal product of labor

$$\bar{p}_s f'_s(\bar{l}_s) = \bar{w}_s, \quad s = g, b \quad (13)$$

Combining (11), (12) and (13) gives the pair of equations

$$u'(f_s(\bar{l}_s))f'_s(\bar{l}_s) = v'(\bar{l}_s), \quad s = g, b \quad (14)$$

which defines the equilibrium \bar{l} on the labor market. Since (14) is identical to (1), $\bar{l} = l^*$, so that the choice of labor in each outcome is optimal. Let $\bar{R}_s = \bar{p}_s \bar{f}(\bar{l}_s) - \bar{w}_s \bar{l}_s$ denote the firm's current profit in outcome s , then the equilibrium choice of investment \bar{a} is the solution of the first-order condition

$$\frac{\pi'(\bar{a})}{1+r} (\bar{R}_g - \bar{R}_b) = 1 \text{ if } \bar{R}_g > \bar{R}_b, \quad \bar{a} = 0 \text{ otherwise} \quad (15)$$

and the equation in (15) has a unique solution since $\pi'(a)$ decreases from ∞ to 0. To compare the first-order conditions for the choice of investment (15) and (3), for a given spot price p_s paid by the consumers and wage w_s given to the workers, define the consumer surplus and worker surplus by

$$CS_s(p_s) = \max_{c_s \geq 0} \{u(c_s) - p_s c_s\}, \quad WS_s(w_s) = \max_{\ell_s \geq 0} \{w_s \ell_s - v(\ell_s)\}. \quad (16)$$

Since $u(0) = 0$ and $v(0) = 0$, $CS(p_s)$ is the net gain in utility for the representative consumer from being able to buy the produced good at price p_s , while $WS_s(w_s)$ is the net gain in utility for a representative worker from being able to sell labor at price w_s . When, as in a capitalist equilibrium, agents trade on spot markets at price (\bar{w}_s, \bar{p}_s) and spot markets clear ($\bar{\ell}_s = \bar{l}_s$, $\bar{c}_s = f_s(\bar{l}_s)$), then the social welfare in outcome s at equilibrium can be expressed as a sum

$$\begin{aligned} \bar{W}_s = u(\bar{c}_s) - v(\bar{\ell}_s) &= \left(u(\bar{c}_s) - \bar{p}_s \bar{c}_s \right) + \left(\bar{w}_s \bar{\ell}_s - v(\bar{\ell}_s) \right) + \left(\bar{p}_s f(\bar{l}_s) - \bar{w}_s \bar{l}_s \right) \\ &= \bar{CS}_s + \bar{WS}_s + \bar{R}_s \end{aligned} \quad (17)$$

consisting of consumer surplus, worker surplus and profit at the spot equilibrium prices. Since the equilibrium consumption choices are optimal

$$\bar{W}_s = u(\bar{c}_s) - v(\bar{\ell}_s) = u(c_s^*) - v(\ell_s^*) = W_s^* \quad (18)$$

so that

$$W_g^* - W_b^* = \left(\overline{CS}_g - \overline{CS}_b \right) + \left(\overline{WS}_g - \overline{WS}_b \right) + \left(\bar{R}_g - \bar{R}_b \right). \quad (19)$$

In the next Proposition we show that $W_g^* - W_b^* > \bar{R}_g - \bar{R}_b$, so that the profit criterion always leads to less investment than would be socially optimal.

Proposition 1. *There is underinvestment in the capitalist equilibrium: $\bar{a} < a^*$.*

Proof: We first show that $W_g^* > W_b^*$, which implies that a^* is positive and is defined by the equation in (3). Consider the parametrized family of production functions

$$f(t, l) = tf_g(l) + (1-t)f_b(l), \quad t \in [0, 1] \quad (20)$$

Note that³ $f_1 > 0$, $f_2 > 0$, $f_{12} > 0$, $f_{22} < 0$, for all $(t, l) \in [0, 1] \times \mathbb{R}_+$. For $t \in [0, 1]$ let $l(t)$ denote the solution of the equation

$$u'(f(t, l(t)))f_2(t, l(t)) = v'(l(t)) \quad (21)$$

which generalizes equation (14) determining the optimal choice of labor and let

$$W(t) = u(f(t, l(t))) - v(l(t))$$

be the generalization of the social welfare function W_s , $s = g, b$. In view of (21)

$$\begin{aligned} W'(t) &= u'(f(t, l(t))) \left(f_1(t, l(t)) + f_2(t, l(t)) l'(t) \right) - v'(l(t)) l'(t) \\ &= u'(f(t, l(t))) f_1(t, l(t)) > 0 \end{aligned}$$

so that $W_g^* = W(1) > W(0) = W_b^*$. Thus g is indeed the ‘good’ social outcome. Since $\pi'(a)$ decreases from ∞ to 0, equation (3) has a unique solution $a^* > 0$ which is the socially optimal level of investment. Since the outcome g is better than the outcome b and $\pi'(0) = \infty$, it is always worthwhile for the economy as a whole to make a positive investment at date 0 to make the ‘good’ outcome more likely.

If $\bar{R}_g - \bar{R}_b \leq 0$ then $\bar{a} = 0$, and the proposition holds since $a^* > 0$. If $\bar{R}_g - \bar{R}_b > 0$, then in view of (3) and (15), showing that $\bar{a} < a^*$ is equivalent to showing that $W_g^* - W_b^* > \bar{R}_g - \bar{R}_b$, and by (19) this is equivalent to showing

$$\overline{CS}_g + \overline{WS}_g > \overline{CS}_b + \overline{WS}_b \quad (22)$$

The proof will be complete if we show that the surplus function

$$\psi(t) \equiv CS(t) + WS(t) \equiv [u(f(t, l(t))) - u'(f(t, l(t))) f(t, l(t))] + [v'(l(t)) l(t) - v(l(t))]$$

is increasing on the interval $[0, 1]$: note that

$$\psi' = -u'' f_1 f + (v'' l - u'' f_2 f) l' \quad (23)$$

³Sub-indices $i = 1, 2$ denote partial derivatives. For example $f_1 = \frac{\partial f}{\partial t}$. There is no ambiguity with f_s since $s \in \{g, b\}$. Ordinary derivatives are denoted by primes.

where the arguments of the functions are omitted to simplify the notation. Since $l(t)$ is defined by equation (21), applying the Implicit Function Theorem gives

$$l' = -\frac{u'' f_1 f_2 + u' f_{21}}{u''(f_2)^2 + u' f_{22} - v''} \quad (24)$$

where the denominator is negative. Inserting (24) into (23) and collecting terms leads to a numerator which can be written as

$$u' u'' f [f_2 f_{21} - f_1 f_{22}] + v'' u'' f_1 [f - f_2 l] - v'' u' f_{21} l < 0$$

where the middle term is negative since $f(t, \cdot)$ is concave and satisfies $f(t, 0) = 0$ so that $f(t, l) - f_2(t, l)l > 0$. Thus $\psi' > 0$ for $t \in [0, 1]$, so that $\psi(1) > \psi(0)$, implying (22) and the proof is complete. \square

The reason why a capitalist equilibrium leads to underinvestment is now clear: by (17) social welfare in outcome s consists not only of profit R_s but also of consumer and worker surplus ($CS_s + WS_s$). Investment based on the profit criterion underestimates the benefits obtained from investment at date 0 by ignoring the benefits obtained by consumers and workers if the ‘good’ outcome rather than the ‘bad’ outcome arises. Thus firms need to change their criterion for the choice of investment, taking into account not just the profit to their shareholders but also the benefits accruing to their consumers and workers from the possibility of finding improved techniques of production.

2.3 Arrow-Debreu Equilibrium

Although the price taking behavior postulated in a capitalist equilibrium—the firm takes (p_s, w_s) as given in choosing its investment—resembles the price-taking behavior in an Arrow-Debreu equilibrium, and the objective of maximizing the present value of profit is the same, a capitalist equilibrium is clearly not the same as an Arrow-Debreu equilibrium since it is not Pareto optimal. To decide whether the above model or the Arrow-Debreu model constitutes a better approximation of the real world, it is therefore important to understand where the difference between the two models lies.

The Arrow-Debreu model is based on the existence of exogenous states of nature whose probability of occurrence cannot be influenced by the actions of economic agents. To describe the uncertainty facing the firm using the Arrow-Debreu framework it would need to be possible to itemize a complete collection of “exogenous circumstances” with fixed probabilities which, when combined with the firm’s action (its investment) explain whether a good or bad outcome will arise. For example consider the problem faced by an automobile company like Toyota which needs to design and implement the production of a new model or improve on the design and production of an existing model. It can hire engineers to design the various components of the car, test the prototypes, and set up a factory to produce and assemble all the components. At the end of the period of design and production, cars get produced which are either ‘good’ (no flaws) or ‘bad’ (have flaws in the functioning of some parts leading for example to unintended acceleration). It is difficult to pinpoint exactly the circumstances that lead to good or bad cars—one design concept rather than another which comes to the minds of the engineers, the

choice of tests for the prototype which may or may not catch the possible malfunctioning of some components, in short all the myriad circumstances which can occur in the design, testing and production of cars. The model must then describe how these exogenous circumstances (states of nature) combine with a given investment expenditure to lead to ‘good’ or ‘bad’ cars: a possible design flaw on the part of an engineer which could lead to a ‘bad’ car may be corrected if Toyota hires two engineers rather than one to do the job, or if the quality control department increases the length or thoroughness of the test of its prototypes.

It should be clear from the above description that the contingencies which condition the outcome of the production process are numerous and difficult to describe. Furthermore, whatever the difficulties involved in their enumeration, they are very much ‘internal’ to the firm, and hence unlikely to be observable and verifiable by outsiders. However the precise description of these contingencies is essential to the Arrow-Debreu model since it assumes that prices are based on these contingencies. Here we see the dramatic difference between the Arrow-Debreu model and the model outlined above in which there are just two prices, the price of a ‘Toyota without flaws’ and the price of a ‘Toyota with flaws’. In the Arrow-Debreu model there is a price in each state of nature: given these prices, the firm can make an investment decision which is efficient. However using the Toyota example as a template, it is difficult to give any sense of realism to these Arrow-Debreu prices, since prices in the “real world” would be based on the known quality of the cars and not on events internal to the firm which consumers cannot observe.

Coming back to the abstract model, an Arrow-Debreu representation of the economy presented above thus hinges on the existence of an underlying probability space $(\Omega, \mathcal{B}, \mathbf{P})$. For convenience let us assume that Ω is a large but finite set of states of nature; \mathcal{B} denotes the collection of subsets of Ω and $\mathbf{P}(B)$ the probability of any subset $B \in \mathcal{B}$. All agents in the economy are assumed to know this basic probability space. For each level of investment a there is a subset $\Omega(a) \subset \Omega$ with $\mathbf{P}(\Omega(a)) = \pi(a)$ which leads to the good technology f_g ; $a' > a$ implies $\Omega(a) \subset \Omega(a')$ so that $\pi(a') > \pi(a)$. Consider all the investment levels which lead to f_g if ω occurs

$$A(\omega) = \{a \in \mathbb{R}_+ \mid \omega \in \Omega(a)\}$$

Given the monotonicity assumption, $A(\omega)$ is a half line: if we let $\underline{a}(\omega) = \inf\{a \mid a \in A(\omega)\}$ (with $\underline{a}(\omega) = \infty$ if $A(\omega) = \emptyset$) and if we assume that $A(\omega)$ is closed, then $A(\omega) = [\underline{a}(\omega), \infty)$. The state-dependent production function⁴ for the firm is

$$F_\omega(a, l) = \begin{cases} f_g & \text{if } a \geq \underline{a}(\omega) \\ f_b & \text{if } a < \underline{a}(\omega) \end{cases} \quad (25)$$

Consistent with the form of preferences given above, the workers’ preferences are given by

$$U^w(m, \ell) = m_0 + \delta \sum_{\omega \in \Omega} \mathbf{P}_\omega(m_\omega - v(\ell_\omega)) \quad (26)$$

⁴In order that the production set be closed we could define the production correspondence by (25) if $a \neq \underline{a}$ and by $F_\omega(a, l) \in \{tf_g(l) + (1-t)f_b(l), t \in [0, 1]\}$ if $a = \underline{a}(\omega)$, but this would not change any result in the analysis below.

the consumers' preferences by

$$U^c(m, c) = m_0 + \delta \sum_{\omega \in \Omega} \mathbb{P}_\omega (m_\omega + u(c_\omega)) \quad (27)$$

and the capitalists preferences by

$$U^k(m) = m_0 + \delta \sum_{\omega \in \Omega} \mathbb{P}_\omega m_\omega \quad (28)$$

Agents have deterministic endowments (e_0^i, e_1^i) , $i = w, c, k$ and the capitalists own the firm.

A complete set of contingent contracts promising the delivery of one unit of money, or of the consumption good, or of labor at date 0 and in each state of nature are traded at date 0. We normalize the price of money to be 1 at date 0. Given the agents' preferences, the price of a promise to deliver 1 unit of money in state ω must be $\delta \mathbb{P}_\omega$ so we do not introduce a separate notation for this price. In the same way the price of a promise to deliver one unit of labor and the produced good in state ω are $\delta \mathbb{P}_\omega w_\omega$ and $\delta \mathbb{P}_\omega p_\omega$ respectively: factoring out $\delta \mathbb{P}_\omega$ from the prices makes it easier to write the equilibrium.

A worker chooses $(m^w, \ell) = (m_\omega^w, \ell_\omega)_{\omega \in \Omega}$ to maximize (26) subject to the budget constraint

$$m_0^w + \delta \sum_{\omega \in \Omega} \mathbb{P}_\omega m_\omega^w = e_0^w + \delta e_1^w + \delta \sum_{\omega \in \Omega} \mathbb{P}_\omega w_\omega \ell_\omega \quad (29)$$

which is equivalent to choosing ℓ to maximize

$$\delta \sum_{\omega \in \Omega} \mathbb{P}_\omega (w_\omega \ell_\omega - v(\ell_\omega)) \quad (30)$$

the choice among money streams then being indeterminate among those satisfying (29). In the same way a consumer chooses $(m^c, c) = (m_\omega^c, c_\omega)_{\omega \in \Omega}$ to maximize (27) subject to the budget constraint

$$m_0^c + \delta \sum_{\omega \in \Omega} \mathbb{P}_\omega (m_\omega^c + p_\omega c_\omega) = e_0^c + \delta e_1^c \quad (31)$$

which is equivalent to choosing c to maximize

$$\delta \sum_{\omega \in \Omega} \mathbb{P}_\omega (u(c_\omega) - p_\omega c_\omega) \quad (32)$$

and the agent is indifferent among the money streams satisfying (31). Finally a capitalist chooses m^k to maximize maximize (28) subject to the budget constraint

$$m_0^k + \delta \sum_{\omega \in \Omega} \mathbb{P}_\omega m_\omega^k = e_0^k + \delta e_1^k + \left[\delta \sum_{\omega \in \Omega} \mathbb{P}_\omega (p_\omega F_\omega(a, l_\omega) - w_\omega l_\omega) - a \right] \quad (33)$$

All capitalists agree that the firm's manager should choose (a, l) so to maximize the (present value of) profit

$$R = \delta \sum_{\omega \in \Omega} \mathbb{P}_\omega (p_\omega F_\omega(a, l_\omega) - w_\omega l_\omega) - a \quad (34)$$

and are indifferent among all money streams satisfying (33). The indeterminacy of agents' money streams implies that if the markets for the produced good and labor clear in every state ω , the money streams can be chosen so that the market for money clear at each date and each state of nature: thus we can omit the markets for money in the description of the equilibrium.

Definition 2. $\left((\tilde{\ell}, \tilde{c}), (\tilde{a}, \tilde{l}), (\tilde{w}, \tilde{p}) \right)$ is a (simplified) *Arrow-Debreu (AD) equilibrium* of the economy \mathcal{E} if

- (i) $\tilde{\ell}$ maximizes (30) given \tilde{w}
- (ii) \tilde{c} maximizes (32) given \tilde{p}
- (iii) (\tilde{a}, \tilde{l}) maximizes (34) given (\tilde{w}, \tilde{p})
- (iv) markets clear: $\tilde{c}_\omega = F_\omega(\tilde{a}, \tilde{l}_\omega)$, $\tilde{\ell}_\omega = \tilde{l}_\omega$, for all $\omega \in \Omega$.

In the informal discussion preceding the description of the AD model we expressed doubts about the realism of the market structure based on states of nature. We now show even if we accepted the strong assumption that such markets can be put in place, it would not suffice to solve the inefficiency, since this economy has no Arrow-Debreu equilibrium.

Proposition 2. (Non-existence) *The economy \mathcal{E} has no Arrow-Debreu equilibrium.*

Proof: Suppose $\left((\tilde{\ell}, \tilde{c}), (\tilde{a}, \tilde{l}), (\tilde{w}, \tilde{p}) \right)$ is an AD equilibrium. By the First Theorem of Welfare Economics, $\tilde{a} = a^* > 0$. In all the states $\omega \in \Omega(\tilde{a})$, $F_\omega(\tilde{a}, \cdot) = f_g$ and the demand and supply conditions are the same. Thus $(\tilde{w}_\omega, \tilde{p}_\omega) = (\bar{w}_g, \bar{p}_g)$ where (\bar{w}_g, \bar{p}_g) are the spot prices of the capitalist equilibrium. If $\omega \notin \Omega(\tilde{a})$, then $(\tilde{w}_\omega, \tilde{p}_\omega) = (\bar{w}_b, \bar{p}_b)$.

Suppose the firm considers increasing the investment from \tilde{a} to $a > \tilde{a}$, *taking the prices $(\tilde{w}_\omega, \tilde{p}_\omega)$ as given*. In the states of the subset $\omega \in \Omega(a) \setminus \Omega(\tilde{a})$ of measure $\pi(a) - \pi(\tilde{a})$ the firm would operate f_g facing the prices (\bar{w}_b, \bar{p}_b) leading to a change in (spot) profit

$$\Delta R^b = \max_{l \geq 0} \{ \bar{p}_b f_g(l) - \bar{w}_b l \} - \max_{l \geq 0} \{ \bar{p}_b f_b(l) - \bar{w}_b l \}$$

In all other states a and \tilde{a} give the same profit. Thus the difference in the present value of the profit net of investment is

$$\delta (\pi(a) - \pi(\tilde{a})) \Delta R^b - (a - \tilde{a})$$

A necessary condition for \tilde{a} to be optimal is that the increase in cost is more than the additional profit i.e.

$$\delta \frac{\pi(a) - \pi(\tilde{a})}{a - \tilde{a}} \Delta R^b \leq 1, \quad \forall a > \tilde{a}$$

which requires that

$$\pi'(\tilde{a}) \Delta R^b \leq (1 + \bar{r}) \tag{35}$$

where \bar{r} is the implicit interest rate in equilibrium given by $\delta = \frac{1}{1+\bar{r}}$.

A similar reasoning for a deviation $a < \tilde{a}$ shows that the loss in profit

$$\Delta R^g = \max_{l \geq 0} \{\bar{p}_g f_g(l) - \bar{w}_g l\} - \max_{l \geq 0} \{\bar{p}_g f_b(l) - \bar{w}_g l\}$$

in the states $\omega \in \Omega(\tilde{a}) \setminus \Omega(a)$ where the firm operates f_b and faces prices (\bar{w}_g, \bar{p}_g) must be higher than the saving in the investment cost:

$$\delta (\pi(\tilde{a}) - \pi(a)) \Delta R^g \geq \tilde{a} - a, \quad \forall a < \tilde{a}$$

which requires that

$$\pi'(\tilde{a}) \Delta R^g \geq (1 + \bar{r}) \quad (36)$$

Let us show that (35) and (36) cannot hold at the same time, because $\Delta R^b > \Delta R^g$. This will show that there is no positive investment which maximizes the profit (34) and thus that there is no AD equilibrium.

To show this property we use the function $f(t, l)$ defined in (20) and the hypothetical equilibrium $(c(t), l(t), p(t), w(t))$ which would hold in a spot economy in which the characteristics of the consumers and workers are those of this section and the firm has technology $f(t, \cdot)$.

$$c(t) = f(t, l(t)), \quad w(t) = v'(l(t)), \quad p(t) = u'(f(t, l(t)))$$

where $l(t)$ is defined by (21). Consider the function

$$R(t, t') = \max_{l \geq 0} \{p(t) f(t', l) - w(t) l\}$$

which gives the profit obtained by operating the technology $f(t', \cdot)$ when prices are those corresponding to the the equilibrium with technology $f(t, \cdot)$. We want to show that

$$\Delta R^b = R(0, 1) - R(0, 0) > R(1, 1) - R(1, 0) = \Delta R^g$$

A sufficient condition for this is that $\frac{\partial^2 R}{\partial t \partial t'}(t, t') < 0$. The following Lemma thus concludes the proof of Proposition 2:

Lemma 1. $R_{12}(t, t') = \frac{\partial^2 R}{\partial t \partial t'}(t, t') < 0$ for all $(t, t') \in [0, 1] \times [0, 1]$.

Proof: see Appendix. □

The simple stochastic two-outcome (or more generally finite-outcome) economy falls into the class of stochastic economies mentioned by Arrow (1971) for which no Arrow-Debreu equilibrium exists due to the inherent non-convexity of the production set when translated to the state-of-nature setting. And yet this success/failure type of uncertainty with the probability of success influenced by some action on the part of the firm is a common and pervasive type of uncertainty, which is handled in a natural way by the basic probability model presented at the beginning of this this section.

One way of restoring the existence of an AD equilibrium is to modify the “success or failure” model by allowing for a continuum of outcomes. But then the problem of finding a rich enough set of states of nature with endogenous probabilities that permit us to disentangle the “work” done by nature and the “work” done by the firm’s investment in explaining every possible outcome is greatly enhanced and the attendant richness of the market structure required makes such an approach even more implausible.

Another more subtle difficulty faced by the Arrow-Debreu model of our simple economy is revealed in the course of proving Proposition 2. It concerns the “price-taking” assumption for the firm. In the probability model (capitalist equilibrium) the firm anticipates that the spot prices will be (p_s, w_s) if it produces with technology f_s . In the Arrow-Debreu version of \mathcal{E} the price-taking assumption requires that the firm “believes” that since prices are determined by the state $\omega \in \Omega$, they do not depend on the realization of its technology f_s , and this property is a crucial cause of non-existence of an AD equilibrium. The latter price-taking assumption, which might be reasonable if the states of nature were well-identified economy-wide shocks, is no longer plausible when the states of nature refer to circumstances which are internal to the firm.

There is a more natural way of changing the structure of our basic economy in which each firm has a finite number of outcomes and the probability of the outcome can be influenced by its investment, to obtain an equilibrium with profit-maximizing firms which is also Pareto optimal. In this modified economy there is a continuum of firms identical to the firm described above with i.i.d risks for the outcomes at date 1 so that an appropriate variant of the Law of Large Numbers can be applied (see Zame (2008) for models of this type). More precisely we may consider a modified economy with a continuum of ex-ante similar firms, where each firm makes an investment at date 0 which influences the probability of its outcome f_g or f_b at date 1. If each firm’s outcome is independent of the outcomes of the other firms and all firms choose the same investment a , then a proportion $\pi(a)$ of firms will produce with f_g , a proportion $1 - \pi(a)$ will produce with f_b , and the average output produced and the spot prices are non random. It is easy to show (see Appendix) that there exist a symmetric capitalist equilibrium in which each firm’s investment maximizes the present value of its profit and the equilibrium investment is Pareto optimal.

However in this modified model, which is elegant and well behaved from a theoretical point of view, a firm has been transformed into an *infinitesimal* entity, far removed from the large corporate firm that we seek to model: the infinitesimal firms that populate this economy aptly fit what Berle and Means (1932) in their classic study described as the small sole proprietorships originally envisaged by Adam Smith. As they argued with great clarity, such firms have little or nothing in common with the large corporate firms whose securities are traded on the stock market and which, even in their day, had come to have a significant footprint on the economic landscape.⁵

In the benchmark model presented in this section, the spot prices vary with the outcome f_g

⁵“When Adam Smith talked of “enterprise” he had in mind as the typical unit the small individual firm in which the owner perhaps with the aid of a few . . . workers, labored to produce goods for market. . . . These units have been supplanted . . . by great aggregations in which tens or even hundreds of thousands of workers and property . . . belonging to tens or even hundred of thousands of individuals are combined through the corporate mechanism into a single producing organization under unified control”, Berle and Means (1932, pp.4 &303).

or f_b for the firm, so that the firm has a non-negligible impact on the economy. This provides a more appropriate model of the large corporations traded on the stock market than the perfectly competitive model with a continuum of negligible firms. To concentrate on the consequence of this type of impact for the optimal choice of investment in risky projects, we abstract from the firm’s potential exploitation of its market power in choosing prices or quantities on the spot markets at date 1.

3 Stakeholder Equilibrium

In any stochastic economy in which the structure of uncertainty is such that firms can affect the probabilities of their outcomes, non-negligible firms will have an external effect on their consumers and workers. In the economy \mathcal{E} when consumption c_s and labor ℓ_s vary with the outcome $s = g, b$, the utilities U^c and U^w of the representative consumer and worker depend on the investment of the firm through the probabilities $\pi_s(a)$ of the ‘good’ and ‘bad’ outcomes. The inefficiency of the capitalist equilibrium comes from the fact that when a firm maximizes its profit—acting in the best interest of its shareholders—it does not take into account its external effect on consumers and workers.

Economists⁶ have essentially proposed three ways of resolving externalities: government intervention (through regulation or Pigouvian taxes), internalization within larger entities (by integration of all the parties involved in the externality) or creating tradeable property rights associated with the externality (Coase, 1960). In the case that we are studying, to intervene efficiently the government (regulator) would need a great deal of information on the technology of the firm, and this in a setting where the firm has inevitably privileged information on its chance for improving the production of its commodity through internal investment. As is well known in mechanism design, the firm is likely to have little incentive to truthfully reveal information on its technological possibilities when this information is to be used to design regulation, taxes or subsidies applied to the firm. In addition, although our model focuses for simplicity on a large firm in a particular sector, in a typical economy there are many large firms which can create externalities of the type studied here, in different sectors, with different technologies, which would seriously compound the informational problem faced by a regulator with a systematic policy of intervention.

The externalities studied in this paper are in a precise sense “local” to the firm, involving its different stakeholders—shareholders, consumers, workers, and possibly other suppliers of inputs. Thus a more natural and direct approach would be to change the charter of the corporation to ensure that the benefits to the different parties concerned are taken into account in making the investment decision: this modified charter in essence involves enlarging the boundary of the corporation to include not only the shareholders but all other parties which have a stake in its investment decision. To formalize the idea of a modified charter we introduce the concept of a stakeholder equilibrium in which the corporation takes into account the interests of all its stakeholders. To this end we define the *expected discounted surplus of the (representative) consumer* as a function of the firm’s investment a when the spot price of

⁶See for example the discussion in Laffont (1989).

the good is $p = (p_g, p_b)$

$$CS(a; p) = \delta \sum_{s=g,b} \pi_s(a) \max_{c_s \geq 0} (u(c_s) - p_s c_s)$$

and the *expected discounted surplus of the (representative) worker* as a function of the firm's investment a when the wage is $w = (w_g, w_b)$

$$WS(a; w) = \delta \sum_{s=g,b} \pi_s(a) \max_{\ell_s \geq 0} (w_s \ell_s - v(\ell_s))$$

Definition 3. A (*reduced form*) *stakeholder equilibrium* of the economy \mathcal{E} is a pair $\left((\hat{\ell}, \hat{c}, \hat{a}, \hat{l}), (\hat{w}, \hat{p}) \right)$ consisting of actions and prices such that

- (i) the labor choice $\hat{\ell} = (\hat{\ell}_g, \hat{\ell}_b) \geq 0$ maximizes worker's utility 9(a) given (\hat{a}, \hat{w}) ;
- (ii) the consumption choice $\hat{c} = (\hat{c}_g, \hat{c}_b) \geq 0$ maximizes consumer's utility 9(b) given (\hat{a}, \hat{p}) ;
- (iii) the firm's production plan $(\hat{a}, \hat{l}) = (\hat{a}, \hat{l}_g, \hat{l}_b) \geq 0$ is such that
 - (a) the labor choice \hat{l}_s maximizes the spot profit $\hat{p}_s f_s(l) - \hat{w}_s l$, for $s = g, b$;
 - (b) the investment \hat{a} maximizes the total surplus of all the stakeholders

$$CS(a; \hat{p}) + WS(a; \hat{w}) + SV(a, \hat{l}; \hat{w}, \hat{p})$$

given the spot prices (\hat{w}, \hat{p}) , where the shareholder value SV defined in (4) is the present value of the profit net of investment.

- (iv) the markets clear: $\hat{\ell}_s = \hat{l}_s, \quad \hat{c}_s = f_s(\hat{l}_s), \quad s = g, b.$

As explained in Section 2 we have omitted the market for money and financial assets, since the agents' money holdings and portfolios can be readily reconstructed using (5)-(7) and (10). A stakeholder equilibrium differs from a capitalist equilibrium in that the firm makes its investment decision taking into account the benefits accruing to all of its stakeholders—consumers, workers and shareholders—and as a result its investment decision is socially optimal.

Proposition 3. *A stakeholder equilibrium is Pareto optimal.*

Proof: As we saw in Section 2 when workers, consumers and the firm maximize on the spot markets at date 1 taking spot prices as given, and markets for labor and output clear, the choice of labor and consumption is socially optimal: $\hat{\ell} = \hat{l} = \ell^* = l^*, \hat{c} = c^*, \hat{w}_s = v'(\ell_s^*), \hat{p}_s = u'(c_s^*)$. The choice of investment in (iii) thus maximizes

$$\delta \sum_{s=g,b} \pi_s(a) (u(c_s^*) - u'(c_s^*) c_s^*) + \delta \sum_{s=g,b} \pi_s(a) (v'(\ell_s^*) \ell_s^* - v(\ell_s^*)) + \delta \sum_{s=g,b} \pi_s(a) (u'(c_s^*) c_s^* - (v'(\ell_s^*) \ell_s^*) - a$$

which is equal to

$$\delta \sum_{s=g,b} \pi_s(a)(u(c_s^*) - v(\ell_s^*)) - a = \delta \sum_{s=g,b} \pi_s(a)W_s^* - a \quad (\text{expected social welfare})$$

so that $\hat{a} = a^*$ and the equilibrium is Pareto optimal. \square

Thus although we have changed the notation from ‘bar’ to ‘hat’ to distinguish a stakeholder equilibrium from a capitalist equilibrium, the two equilibria have the same date 1 spot prices, the same choices of consumption and labor, and these choices are optimal

$$\hat{\ell} = \bar{\ell} = \ell^*, \quad \hat{c} = \bar{c} = c^*, \quad \hat{w} = \bar{w} = v'(\ell^*), \quad \hat{p}_s = \bar{p}_s = u'(c^*).$$

The only difference is in the choice of investment

$$\hat{a} = a^* > \bar{a}$$

which comes from the change in the criterion (iii)(b) which is used to choose the firm’s investment, from profit maximization to total surplus maximization. The “reform of capitalism” that this proposition calls for is a change in the objective that guides the corporation, from maximizing shareholder value to maximizing the joint surplus of its stakeholders. The idea that a corporation should serve the interest of a broader segment than just its shareholders has often been advocated and formed the basis for Berle and Means (1932) final chapter on the “new concept of the corporation”. The implementation of such a broader objective has proved to be difficult, in part because of the lack of precision in what it means to take into account the joint interests of workers, consumers and shareholders. The criterion (iii)(b’) provides a way of formalizing the idea, by providing precise content to the “joint interests of the shareholders”. Providing a precise definition of the stakeholders’ interests does not however guarantee that the criterion can or will be used as the basis of decision making by the firm’s manager: ways of measuring the “surpluses” CS and WS, as well as incentives for the management to maximize (iii)(b), must also exist. Thus the implementation of a stakeholder equilibrium raises three issues:

- *Incentives*: incentives must be given to the firm’s manager to apply the stakeholder criterion.
- *Information* : to apply the stakeholder criterion the manager needs information on the characteristics of the consumers and workers to evaluate their surpluses.
- *Financing*: if the shareholder value at the stakeholder equilibrium is negative, an additional source of funds beyond equity and debt must be found, since otherwise the shareholders would dispose of their ownership shares rather than being forced to finance a project with a negative net present value.

3.1 Issuing worker and consumer rights

Our objective is to study how a market system can resolve these three issues. We show that, in the spirit of Coase (1960), creating explicit tradeable property rights associated with the externalities created by the firm provides the appropriate extension of the markets required to implement a stakeholder equilibrium. Suppose therefore that at date 0, in addition to the market for equity on which ownership shares are traded, there is a market for “consumer rights”—or more briefly c-rights—on which agents exchange the right to buy the good produced by the firm at date 1 at the spot price $p = (p_g, p_b)$. In addition there is a market for “worker rights”—or more briefly w-rights—on which agents exchange the right to sell labor to the firm at date 1 at the spot price $w = (w_g, w_b)$. We first assume that the time when the firms issues these rights has passed and that every consumer has an endowment of one c-right and every worker has an endowment of one w-right. That is, we do not consider the initial stage at which the firm issues these rights (the equivalent of an IPO for equity). To understand how the market prices of these rights are determined it is useful to first assume that only a mass $1 - \varepsilon$ of consumers and workers is endowed with rights, the remaining mass ε having no such right initially, and then let ε go to 0. A worker with no initial w-right who observes the investment a and anticipates date 1 wage $w = (w_g, w_b)$ would be willing to pay up to

$$\delta(\pi(a)WS_g(w_g) + (1 - \pi(a))WS_b(w_b)) \quad (37)$$

to obtain the right to work for the firm, where $WS_s(w_s) = \max_{\ell_s \geq 0} \{w_s \ell_s - v(\ell_s)\}$ is the surplus utility that a worker derives from selling labor at price w_s . A worker who owns a w-right will accept to sell it if its price is equal or exceeds (37). Thus if $\varepsilon > 0$, equilibrium on the market for w-rights occurs at the price

$$q_w(a, w) = \delta\left(\pi(a)WS_g(w_g) + (1 - \pi(a))WS_b(w_b)\right) \quad (38)$$

If $\varepsilon = 0$ and every worker is endowed with a w-right, then no worker needs to buy a right, so that any price between 0 and $q_w(a, w)$ (at which every worker wants to keep the initial w-right) is an equilibrium price. To keep the symmetry of the model we assume that every worker is endowed with a w-right and that the market price of a w-right is given by (38), since any scarcity, no matter how small, will immediately force the price to $q_w(a, w)$. By a similar argument, the market price $q_c(a, p)$ of a c-right is taken to be the surplus utility derived by a consumer from buying the produced good at price p

$$q_c(a, p) = \delta\left(\pi(a)CS_g(p_g) + (1 - \pi(a))CS_b(p_b)\right) \quad (39)$$

With the market valuations (38) and (39) in hand we now have a way of implementing a stakeholder equilibrium. If the firm’s manager makes the labor choice which maximizes the date 1 profit $R_s(\hat{p}_s, \hat{w}_s)$ and chooses the investment a to maximize the total market value of the rights of its stakeholders

$$q_w(a, \hat{w}) + q_c(a, \hat{p}) + q_e(a, \hat{w}, \hat{p}) - a \quad (40)$$

net of the cost of investment, then the firm criterion for choosing the investment coincides with the net surplus criterion (iii)(b) of a stakeholder equilibrium and leads to the socially

optimal investment a^* . The advantage of having an explicit market for w-rights and c-rights in addition to equity is that the firm's manager maximizes an objective, observable market value rather than an unobservable surplus. However to provide the manager with the incentive to maximize the stakeholder value (40), workers and consumers must be able to influence the investment decision of the firm. The reform of capitalism that we have in mind requires that when w-rights and c-rights are issued by the firm, the owners of these rights acquire legal voting rights in the decision process for investment. If unanimity is required to approve a change of management, then the management will maximize the net stakeholder value (40) or be replaced: for if a manager fails to maximize (40) a "raider" could choose an investment with a higher stakeholder value and in the process transfer enough value to workers, consumers and shareholders to buy their votes.

In addition to providing the manager with incentives to apply the stakeholder criterion, the existence of markets for w-rights and c-rights provides the required information on the worker and consumer surpluses: knowledge of the price functions $q_w(a, \hat{w})$ and $q_c(a, \hat{p})$, which may be acquired from repeated observations of market prices, is sufficient information to be able to maximize the total surplus in the economy.

Since, in the above analysis we have assumed that the w-rights and c-rights have already been issued, neither consumers nor workers contribute to the funding of the firm's investment which must be paid by the shareholders, either directly as assumed in Section 2, or indirectly through the issue of bonds, which is equivalent. Such financing is possible only if $q_e(a^*, \hat{w}, \hat{p}) - a^* \geq 0$. Otherwise the shareholders will prefer to dispose of their equity shares rather than finance a project with a negative net present value. If $q_e(a^*, \hat{w}, \hat{p}) - a^* < 0$, the stakeholder equilibrium can still be implemented through stakeholder value maximization, provided that the model is taken at the stage where the firm issues the rights. Since the expected total surplus is positive

$$\pi(a^*)W_g^* + (1 - \pi(a^*))W_b^* - a^* > \pi(0)W_g^* + (1 - \pi(0))W_b^* \geq 0$$

the net market value of these surpluses is positive

$$q_w(a^*, \hat{w}) + q_c(a^*, \hat{p}) + q_e(a^*, \hat{w}, \hat{p}) - a^* > 0. \quad (41)$$

If the firm issues the rights and chooses a^* to maximize the market value of the rights plus the net profit, then the proceeds $q_w(a^*, \hat{w}) + q_c(a^*, \hat{p})$ from the sale of the rights is sufficient to ensure that the shareholder value is positive since (41) can be written as

$$q_e(a^*, \hat{w}, \hat{p}) - \left(a^* - q_w(a^*, \hat{w}) + q_c(a^*, \hat{p}) \right) > 0$$

Thus the issue of rights can resolve the problem of financing when the net expected profit at the optimal investment is negative.

4 Heterogeneous Agents

In this section we explore how the results of the previous sections are altered when the underlying economy $\mathcal{E}(U, e, f, \pi)$ is changed to allow for heterogeneity in the characteristics (U, e)

of the agents. Two properties carry over to this more general setting: first there is always under-investment in a capitalist equilibrium and second, a stakeholder equilibrium is Pareto optimal. However a single market for worker rights and a single market for consumer rights do not permit differences in worker surpluses and in consumer surpluses to be taken into account, so that an equilibrium in which investment is chosen to maximize the total value of the stakeholder rights is no longer Pareto optimal. We will however show that introducing markets for w-rights and c-rights and maximizing the total value of the rights leads to an improvement over the capitalist equilibrium.

The firm, the firm technology (f, π) , the three goods—the produced good, the composite good (money) and labor—are unchanged, and, as before, there are still three groups of agents, the workers, the consumers and the capitalists. The workers are now indexed by a parameter α , worker α having the utility function

$$U^w(m, \ell, \alpha) = m_0 + \delta \sum_{s=g,b} \pi_s(a)(m_s + v(\ell, \alpha))$$

where the parameter $\alpha \in (0, 1)$ is drawn from a continuous distribution G with density g . For fixed $\alpha \in (0, 1)$, $v(\cdot, \alpha)$ has the same properties as the function v in Section 2, and without loss of generality the workers can be ordered so that $v_\alpha(\ell, \alpha) = \frac{\partial v}{\partial \alpha}(\ell, \alpha) < 0, \forall \ell > 0, \forall \alpha \in (0, 1)$: a worker with a smaller α has a larger disutility of working.

The consumers are indexed by a parameter β drawn from a distribution H on $(0, 1)$ with density h . The utility of consumer β is

$$U^c(m, c, \beta) = m_0 + \delta \sum_{s=g,b} \pi_s(a)(m_s + u(c_s, \beta))$$

where for each β the function $u(\cdot, \beta)$ has the same property as the function u in Section 2, and consumers are ordered so that $u_\beta(c, \beta) = \frac{\partial u}{\partial \beta}(c, \beta) > 0, \forall c > 0, \forall \beta \in (0, 1)$. Since there is no interest in introducing heterogeneity among the capitalists, we continue to assume that there is a continuum of mass 1 of identical capitalists who own the firm and only consume money.

The money endowment of the three groups of agents at date 0 and at date 1 are given by

$$e = \left((e_0^w(\alpha), e_1^w(\alpha)), (e_0^c(\beta), e_1^c(\beta)), (e_0^k, e_1^k) \right)$$

so that the aggregate endowment of money at dates $t = 0, 1$ is given by

$$\int_0^1 e_t^w(\alpha) dG(\alpha) + \int_0^1 e_t^c(\beta) dH(\beta) + e_t^k, \quad t = 0, 1$$

Let $\tilde{\mathcal{E}} = (U, e, G, H, f, \pi)$ denote the resulting economy with a distribution of worker and consumer characteristics given by (G, H) .

4.1 Socially optimal investment

In view of the quasi-linearity of the agents' preferences, a Pareto optimum is an allocation $(\ell^*, c^*, a^*) = (\ell^*(\alpha))_{\alpha \in (0,1)}, (c^*(\beta))_{\beta \in (0,1)}, a^*$ which maximizes the sum of agents utilities

$$e_0 - a + \delta \sum_{s=g,b} \pi_s(a) \left(e_1 + \int_0^1 u(c_s(\beta), \beta) dH(\beta) - \int_0^1 v(\ell_s(\alpha), \alpha) dG(\alpha) \right) \quad (42)$$

subject to the resource constraints

$$\int_0^1 c_s(\beta) dH(\beta) = f_s \left(\int_0^1 \ell_s(\alpha) dG(\alpha) \right), \quad s = g, b \quad (43)$$

In view of the additive separability of the social welfare function in (42) the problem of finding a Pareto optimum can be decomposed into the problem of finding the labor-consumption allocation (ℓ_s^*, c_s^*) which solves for $s = g, b$

$$\max_{(\ell_s, c_s) \geq 0} \int_0^1 u(c_s(\beta), \beta) dH(\beta) - \int_0^1 v(\ell_s(\alpha), \alpha) dG(\alpha) \quad (44)$$

subject to the corresponding resource constraint in (43), and then finding the optimal investment. Given the strict concavity of the objective, the solution to the maximum problem (44) is unique, and (ℓ_s^*, c_s^*) are defined by the first-order conditions

$$\begin{aligned} u_c(c_s(\beta), \beta) f'_s(\ell_s) &= v_\ell(\ell_s(\alpha), \alpha), \quad \alpha \in (0, 1), \beta \in (0, 1), s = g, b \\ \ell_s &= \int_0^1 \ell_s(\alpha) dG(\alpha), \quad y_s = \int_0^1 c_s(\beta) dH(\beta), \quad y_s = f(\ell_s) \end{aligned} \quad (45)$$

Thus we can define the social welfare in state s

$$W_s^* = \int_0^1 u(c_s^*(\beta), \beta) dH(\beta) - \int_0^1 v(\ell_s^*(\alpha), \alpha) dG(\alpha), \quad s = g, b$$

and the socially optimal investment a^* which maximizes

$$\pi(a)W_g^* + (1 - \pi(a))W_b^* - a.$$

is characterized by the first-order condition

$$\delta \pi'(a^*)(W_g^* - W_b^*) = 1 \text{ if } W_g^* > W_b^*, \quad a^* = 0 \text{ otherwise} \quad (46)$$

The equation in (46) has a solution since $\pi'(a)$ decreases from ∞ to 0.

4.2 Capitalist equilibrium

The concept of a reduced form capitalist equilibrium extends in a straightforward way to the economy $\tilde{\mathcal{E}}$ with heterogeneous agents and can be defined succinctly as follows:

Definition 4. A (*reduced form*) *capitalist equilibrium* of the economy $\tilde{\mathcal{E}}$ is a pair $\left((\bar{\ell}, \bar{c}, \bar{a}, \bar{l}), (\bar{w}, \bar{p}) \right)$ consisting of actions and prices such that

$$(i) \quad \bar{\ell} = (\bar{\ell}(\alpha))_{\alpha \in (0,1)}, \quad \bar{\ell}(\alpha) = (\bar{\ell}_s(\alpha))_{s=g,b}$$

$$\bar{\ell}_s(\alpha) = \operatorname{argmax}_{\ell_s \geq 0} \{ \bar{w}_s \ell_s - v(\ell_s, \alpha) \}, \quad s = g, b$$

$$(ii) \quad \bar{c} = (\bar{c}(\beta))_{\beta \in (0,1)}, \quad (\bar{c}(\beta)) = (\bar{c}_s(\beta))_{s=g,b}$$

$$\bar{c}_s(\beta) = \operatorname{argmax}_{c_s \geq 0} \{u(c_s, \beta) - \bar{p}_s c_s\}, \quad s = g, b$$

$$(iii) \quad (a) \quad \bar{l} = (\bar{l}_s)_{s=g,b}$$

$$\bar{l}_s = \operatorname{argmax}_{l_s \geq 0} \{\bar{p}_s f_s(l_s) - \bar{w}_s l_s\}, \quad s = g, b$$

$$(b) \quad \bar{a} = \operatorname{argmax}_{a \geq 0} \left\{ \frac{1}{1+r} \sum_{s=g,b} \pi_s(a) R_s(\bar{w}_s, \bar{p}_s) - a \right\}, \quad \text{where } \delta = \frac{1}{1+r} \text{ and } R_s(\bar{w}_s, \bar{p}_s) = \bar{p}_s f_s(\bar{l}_s) - \bar{w}_s \bar{l}_s$$

$$(iv) \quad \text{markets clear: } \bar{l}_s = \int_0^1 \bar{l}_s(\alpha) dG(\alpha), \quad f_s(\bar{l}_s) = \int_0^1 \bar{c}_s(\beta) dH(\beta), \quad s = g, b.$$

Note that, as in the homogeneous case, the agents and the firm's optimal choices $(\bar{l}, \bar{c}, \bar{l})$ on the spot markets coincide with the socially optimal choices (ℓ^*, c^*, l^*) since the first-order conditions at equilibrium coincide with the first-order conditions (45) for a Pareto optimum. However the choice of investment \bar{a} is not optimal since maximizing the present value of profit implies that \bar{a} is characterized by

$$\frac{\pi'(\bar{a})}{1+r} (\bar{R}_g - \bar{R}_b) = 1 \text{ if } \bar{R}_g > \bar{R}_b, \quad \bar{a} = 0 \text{ otherwise}$$

where $\bar{R}_s = R_s(\bar{w}_s, \bar{p}_s)$, while a^* is characterized by (46).

Proposition 4. *For the heterogeneous agent economy $\tilde{\mathcal{E}}$ there is under-investment in a capitalist equilibrium: $\bar{a} < a^*$.*

Proof: It suffices to replace the utility function u in the proof of Proposition 1 by the maximized total utility Φ of the consumers defined by

$$\Phi(\chi) = \max_{(c(\beta))_{\beta \in (0,1)}} \left\{ \int_0^1 u(c(\beta), \beta) dH(\beta) \mid \int_0^1 c(\beta) dH(\beta) = \chi \right\} \quad (47)$$

and the disutility if work v by the minimized total disutility Ψ of the workers defined by

$$\Psi(l) = \min_{(\ell(\alpha))_{\alpha \in (0,1)}} \left\{ \int_0^1 v(\ell(\alpha), \alpha) dG(\alpha) \mid \int_0^1 \ell(\alpha) dG(\alpha) = l \right\} \quad (48)$$

$\Phi(\chi)$ is the maximum total utility of the consumers that can be obtained by optimally distributing a quantity χ of produced good, while $\Psi(l)$ is the minimum total disutility of the workers of providing a total quantity l of labor. The functions Φ and Ψ inherit the properties of concavity/convexity, differentiability and monotonicity of the functions $u(\cdot, \beta)$ and $v(\cdot, \alpha)$,⁷ permitting the proof of Proposition 1 to be carried over to the heterogeneous-agent setting. \square

⁷See Magill-Quinzii (1996, pp 189-194).

4.3 Stakeholder equilibrium

A stakeholder equilibrium is essentially a modification of a capitalist equilibrium, which continues to make use of the spot markets to allocate labor and consumption, and appropriately modifies the criterion that the firm uses to make its investment decision so that it takes into account the external effect of its investment on workers and consumers. To appropriately modify the firm's criterion we need to define the total worker surplus and the total consumer surplus when the spot prices are (w, p) .

Let $\ell(w_s, \alpha)$ denote the decision of a worker of type α which maximizes his worker surplus as in (i) of Definition 4, when faced with the spot wage w_s . The worker surplus is then

$$\text{ws}(w_s, \alpha) = w_s \ell(w_s, \alpha) - v(\ell(w_s, \alpha), \alpha), \quad \alpha \in (0, 1).$$

In the same way, let $c(p_s, \beta)$ denote the demand function of consumer of type β . The consumer surplus faced with the spot price p_s is then

$$\text{cs}(p_s, \beta) = u(c(p_s, \beta), \beta) - p_s c(p_s, \beta), \quad \beta \in (0, 1)$$

so that the total worker surplus and the total consumer surplus in each outcome s are given by

$$WS(w_s) = \int_0^1 \text{ws}(w_s, \alpha) dG(\alpha), \quad CS(p_s) = \int_0^1 \text{cs}(p_s, \beta) dH(\beta) \quad (49)$$

Definition 5. A (*reduced form*) *stakeholder equilibrium* of the economy $\tilde{\mathcal{E}}$ is a pair $\left((\bar{\ell}, \bar{c}, \hat{a}, \bar{l}), (\bar{w}, \bar{p}) \right)$ such that (i), (ii), (iii)(a) and (iv) of Definition 4 are satisfied, and the firm's criterion (iii)(b) is replaced by

$$(iii)(b') \quad \hat{a} = \arg \max_{a \geq 0} \left\{ \frac{1}{1+r} \sum_{s=g,b} \pi_s(a) \left(WS(\bar{w}_s) + CS(\bar{p}_s) + R_s(\bar{w}_s, \bar{p}_s) \right) - a \right\}$$

With this criterion the firm correctly takes into account its external effect on workers and consumers, so that its investment is Pareto optimal.

Proposition 5. A *stakeholder equilibrium of the heterogeneous-agent economy $\tilde{\mathcal{E}}$ is Pareto optimal.*

4.4 η -rights equilibrium

In the homogeneous-agent case we saw that introducing a market for worker rights and a market for consumer rights permits the stakeholder equilibrium to be implemented, since the price of a w-right correctly reflects the expected surplus of the representative worker and the price of a c-right correctly reflects the expected surplus of the representative consumer. When the agents are heterogeneous a single market for w-rights and c-rights cannot implement the stakeholder equilibrium since the market value of these rights will reflect the expected surplus of the *marginal* buyer, so that the total value of the rights will not coincide with the total

expected surplus. However we show that, in the limit where only an infinitesimal proportion of workers or consumers is excluded from participating in the spot markets, if the firm's manager maximizes the sum of the market values of the w-rights, c-rights and shareholder rights, then the resulting equilibrium improves on the capitalist equilibrium.

Let $\eta = (\eta_w, \eta_c)$ with $0 < \eta_w \leq 1$, $0 < \eta_c \leq 1$, denote the supply of worker and consumer rights issued by the firm at an earlier stage, which are now held by workers and consumers. Only a mass η_w of workers can sell their labor to the firm and only a mass η_c of consumers can buy the produced good, so that the spot prices $(w_s(\eta), p_s(\eta))$ will depend on η . A worker of type α anticipating (a, w_g, w_b) will be willing to buy a w-right (or keep it if he owns one) if

$$q_w \leq \delta \left(\pi(a) \text{ws}(w_g, \alpha) + (1 - \pi(a)) \text{ws}(w_b, \alpha) \right)$$

The assumption $v_\alpha < 0$ implies that $\text{ws}(w_g, \alpha)$ is increasing in α , so that if worker α wants to buy a right, all workers with $\alpha' > \alpha$ will want to do the same.⁸ With supply η_w of w-rights, investment a and wages $w = (w_g, w_b)$, the equilibrium market price must thus occur at price $q_w(w, a)$ such that

$$q_w(w, a) = \delta \left(\pi(a) \text{ws}(w_g(\eta), \hat{\alpha}(\eta)) + (1 - \pi(a)) \text{ws}(w_b(\eta), \hat{\alpha}(\eta)) \right) \quad (50)$$

where the marginal buyer $\hat{\alpha}(\eta)$ is defined by

$$\eta_w = \int_{\hat{\alpha}(\eta)}^1 dG(\alpha).$$

In the same way the equilibrium price of a c-right will be

$$q_c(p, a) = \delta \left(\pi(a) \text{cs}(p_g(\eta), \hat{\beta}(\eta)) + (1 - \pi(a)) \text{cs}(p_b(\eta), \hat{\beta}(\eta)) \right), \quad \eta_c = \int_{\hat{\beta}(\eta)}^1 dH(\beta). \quad (51)$$

Definition 6. For a given supply of rights $\eta = (\eta_w, \eta_c)$, a (*reduced form*) *stakeholder rights equilibrium* (or more briefly an η -rights equilibrium) of the economy $\tilde{\mathcal{E}}$ is a pair of actions and prices $\left((\ell(\eta), c(\eta), a(\eta), l(\eta)), (w(\eta), p(\eta), q(\eta)) \right)$, with $q(\eta) = (q_w(\eta), q_c(\eta), q_e(\eta))$ such that

$$(i) \quad \ell(\eta) = (\ell(\alpha, \eta))_{\alpha \in (0,1)}, \quad \ell(\alpha, \eta) = (\ell_s(\alpha, \eta))_{s=g,b}$$

$$\ell_s(\alpha, \eta) = \begin{cases} \ell_s(w_s(\eta), \alpha) & \text{if } \alpha \geq \hat{\alpha}(\eta) \\ 0 & \text{otherwise} \end{cases}$$

$$(ii) \quad c(\eta) = (c(\beta, \eta))_{\beta \in (0,1)}, \quad c(\beta, \eta) = (c_s(\beta, \eta))_{s=g,b}$$

$$c_s(\beta, \eta) = \begin{cases} c_s(p_s(\eta), \beta) & \text{if } \beta \geq \hat{\beta}(\eta) \\ 0 & \text{otherwise} \end{cases}$$

⁸ $\frac{d}{d\alpha} [\text{ws}(w_s, \alpha)] = \frac{d}{d\alpha} [w_s \ell(w_s, \alpha) - v(\ell(w_s, \alpha), \alpha)] = (w_s - v_\ell) \frac{\partial \ell}{\partial \alpha} - v_\alpha = -v_\alpha > 0$.

- (iii) (a) $l(\eta) = (l_s(\eta))_{s=g,b}$ $l_s(\eta) = \arg \max_{l_s \geq 0} \{p_s(\eta)f_s(l_s) - w_s(\eta)l_s\}$
 (b) $a(\eta) = \arg \max_{a \geq 0} \{\eta_w q_w(a, \eta) + \eta_c q_c(a, \eta) + q_e(a, \eta) - a\}$
- (iv) $q(\eta) = q(a(\eta), \eta)$
- (v) $l_s(\eta) = \int_{\hat{\alpha}(\eta)}^1 \ell_s(\alpha, \eta) dG(\alpha)$, $f_s(l_s(\eta)) = \int_{\hat{\beta}(\eta)}^1 c_s(\beta, \eta) dH(\beta)$, $s = g, b$.

Instead of reflecting the average valuation of the workers and consumers as in the homogeneous case, in a stakeholder rights equilibrium with heterogeneous agents the market prices $(q_w(\eta), q_c(\eta))$ reflect the valuation of the lowest types of buyers $(\hat{\alpha}(\eta), \hat{\beta}(\eta))$. Thus swamping the market with rights $\varepsilon_w = \eta_c = 1$ may lead to very low market prices for the rights, inducing a possibly serious under-estimate of the total worker and consumer surplus. On the other hand, if restricting the supply of rights raises their market prices so that they more accurately reflect the social surplus, it also brings with it an imperfection, since some workers and consumers are restricted from participating in the spot markets at date 1. Our objective is to show that under normal conditions, although it does not lead to first-best efficiency, the introduction of markets for worker and consumer rights with a supply which is not “too restricted” nevertheless leads to an improvement over a capitalist equilibrium.

The social welfare achieved at an η -rights equilibrium is given by

$$\mathcal{W}(\eta) = -a(\eta) + \frac{1}{1+r} \sum_{s=g,b} \pi_s(a(\eta)) \left(\int_{\hat{\alpha}(\eta)}^1 ws(w_s(\eta), \alpha) dG(\alpha) + \int_{\hat{\beta}(\eta)}^1 cs(p_s(\eta), \beta) dH(\beta) + R_s(w_s(\eta), p_s(\eta)) \right)$$

while the social welfare in a capitalist equilibrium is

$$\bar{\mathcal{W}} = -\bar{a} + \frac{1}{1+r} \sum_{s=g,b} \pi_s(\bar{a}) \left(WS(\bar{w}_s) + CS(\bar{p}_s) + R_s(\bar{w}_s, \bar{p}_s) \right)$$

where the total surpluses WS and CS are defined by (49). When the supply of w-rights and c-rights tends to $\eta_w = 1$ and $\eta_c = 1$, the spot prices in an η -rights equilibrium tend to the spot prices in a capitalist equilibrium since all workers and consumers have rights. We can then compare $\mathcal{W}(\eta)$ and $\bar{\mathcal{W}}$ showing that the η -rights equilibrium leads to an improvement, in the case where the wage \bar{w}_g in the good state exceeds \bar{w}_b . Under the current assumptions it is always true that $\bar{p}_g > \bar{p}_b$, but the same inequality for wages requires the following additional assumption.

Assumption A

A1. $\frac{-\Phi''(\chi)}{\Phi'(\chi)} \chi \leq 1, \forall \chi > 0$, where Φ is the social utility of consuming an aggregate quantity χ of the produced good defined in (47);

A2. $\frac{f'_g(l)}{f_g(l)} > \frac{f'_b(l)}{f_b(l)}, \forall l > 0$.

A1 is an assumption on consumer preferences, A2 an assumption on technology. Since $p(\chi) = \Phi'(\chi)$, A1 requires that the price elasticity of demand is low, $\frac{-p'(\chi)}{p(\chi)}\chi \leq 1$. A2 requires that the marginal product of labor increases sufficiently between outcome ‘g’ and outcome ‘b’. These two properties ensure that $\bar{w}_g > \bar{w}_b$, so that the workers are better off in outcome ‘g’ than in outcome ‘b’. We add the assumption that workers with smaller disutility also have smaller marginal disutility of labor, $v_{\ell\alpha} < 0$, and that consumers with greater utility also have greater marginal utility for the good, $u_{c\beta} > 0$. This leads to the following monotonicity properties of the agents’ surplus functions.

Lemma 2. *If $v_{\ell\alpha} < 0, u_{c\beta} > 0$ and Assumption A holds then*

- (i) $ws(\bar{w}_g, \alpha) - ws(\bar{w}_b, \alpha) > 0$, and is increasing in α , $\alpha \in (0, 1)$
- (ii) $cs(\bar{p}_g, \beta) - cs(\bar{p}_b, \beta) > 0$, and is increasing in β , $\beta \in (0, 1)$.

Proof: See Appendix.

We can now compare $\mathcal{W}(\eta)$ and $\bar{\mathcal{W}}$ when $\eta \rightarrow (1, 1)$.

Proposition 6. (Stakeholder rights equilibrium improves on capitalism).

Under the assumptions of Lemma 2 and $\bar{a} > 0$,

- either* (i) $\lim_{\eta \rightarrow (1,1)} \mathcal{W}(\eta) > \bar{\mathcal{W}}$
or (ii) $\lim_{\eta \rightarrow (1,1)} \mathcal{W}(\eta) = \bar{\mathcal{W}}$ and $\frac{\partial \mathcal{W}}{\partial \eta_w}(1, 1) < 0, \frac{\partial \mathcal{W}}{\partial \eta_c}(1, 1) < 0$
so that for η sufficiently close to $(1, 1)$, $\mathcal{W}(\eta) > \bar{\mathcal{W}}$.

Proof: If $\eta = (\eta_w, \eta_c) \rightarrow (1, 1)$ then at the limit no worker and no consumer is excluded from the spot markets so that the spot prices coincide with those at the capitalist equilibrium

$$(w(\eta), p(\eta)) \rightarrow (\bar{w}, \bar{p}) \text{ when } \eta \rightarrow (1, 1).$$

The marginal worker $\hat{\alpha}(\eta) \rightarrow 0$, and the marginal consumer $\hat{\beta}(\eta) \rightarrow 0$. By Lemma 2 the difference in worker surplus is decreasing in α so when $\hat{\alpha} \rightarrow 0$ the difference tends to a limit which is either positive or zero

$$\lim_{\hat{\alpha} \rightarrow 0} ws(w_g, \hat{\alpha}) - ws(w_b, \hat{\alpha}) = \begin{cases} > 0 \\ \text{or} \\ = 0 \end{cases} \quad (52)$$

The first case occurs if, when $\alpha \rightarrow 0$, the function $v(\cdot, \alpha)$ converges to an increasing convex function with the same properties as $v(\cdot, \alpha)$ for $\alpha \in (0, 1)$; the second case arises when $v(\cdot, \alpha)$ converges to the discontinuous convex function $v(0, 0) = 0, v(\ell, 0) = \infty$ for $\ell > 0$. In the

same way the difference in consumer surplus for the marginal consumer $\hat{\beta}$ either converges to a positive number or to zero

$$\lim_{\hat{\beta} \rightarrow 0} \text{cs}(p_g, \hat{\beta}) - \text{cs}(p_b, \hat{\beta}) = \begin{cases} > 0 \\ \text{or} \\ = 0 \end{cases} \quad (53)$$

In the first case the limit function $u(\cdot, 0)$ has the same properties as $u(\cdot, \beta)$ for $\beta \in (0, 1)$, in the second case the limit function is $u(c, 0) = 0$ for all $c \geq 0$. If one or both limits in (52) and (53) are positive we are in case (i) of Proposition 6; if both limits are zero we are in case (ii). Consider each case separately.

case (i) When $\eta \rightarrow (1, 1)$ the choice of investment $a(\eta)$ in an η -rights equilibrium converges to the solution \tilde{a} of the first-order condition

$$\pi'(a) \left(\lim_{\hat{\alpha} \rightarrow 0} \left(\text{ws}(\bar{w}_g, \hat{\alpha}) - \text{ws}(\bar{w}_b, \hat{\alpha}) \right) + \lim_{\hat{\beta} \rightarrow 0} \left(\text{cs}(\bar{p}_g, \hat{\beta}) - \text{cs}(\bar{p}_b, \hat{\beta}) \right) + \left(R_g(\bar{w}_g, \bar{p}_g) - R_b(\bar{w}_b, \bar{p}_b) \right) \right) = 1$$

This equation has a solution since we have assumed $\bar{a} > 0$, which is equivalent to $\bar{R}_g - \bar{R}_b > 0$ and, since at least one of the two limits is positive, $\tilde{a} > \bar{a}$. Since by Lemma 2 the differences in surplus are increasing in α and β , the difference for the lowest type is less than the differences in the total surplus defined by (49)

$$\begin{aligned} \lim_{\hat{\alpha} \rightarrow 0} \left(\text{ws}(\bar{w}_g, \hat{\alpha}) - \text{ws}(\bar{w}_b, \hat{\alpha}) \right) &< WS(\bar{w}_g) - WS(\bar{w}_b) \\ \lim_{\hat{\beta} \rightarrow 0} \left(\text{cs}(\bar{p}_g, \hat{\beta}) - \text{cs}(\bar{p}_b, \hat{\beta}) \right) &< CS(\bar{p}_g) - CS(\bar{p}_b) \end{aligned}$$

These inequalities imply that $\tilde{a} < a^*$. Since social welfare viewed as a function of a

$$-a + \frac{1}{1+r} \sum_{s=g,b} \pi_s(a) (WS(\bar{w}_s) + CS(\bar{p}_s) + R_s(\bar{w}_s, \bar{p}_s))$$

is a concave function which takes its maximum at a^* , it is increasing on $[\bar{a}, a^*]$ so that $\bar{a} < \tilde{a} < a^*$ implies $\lim_{\eta \rightarrow (1,1)} \mathcal{W}(\eta) > \bar{W}$.

case (ii) In this case “worker 0” who has the limit behavior of a type α worker when $\alpha \rightarrow 0$, supplies no labor ($\ell(w, 0) = 0$) and gets no surplus ($\text{ws}(w, 0) = 0$) for all $w > 0$. In the same way “consumer 0” with the limit behavior of consumer β when $\beta \rightarrow 0$, does not consume ($c(p, 0) = 0$) and gets no surplus ($\text{cs}(p, 0) = 0$) for all $p > 0$. We know that at the limit $\eta = (1, 1)$ the spot prices are those of the capitalist equilibrium. We now show that these limit behaviors imply that marginally decreasing the supply of w-rights and c-rights from $(1, 1)$ does not change the spot prices (to terms of first order): since worker 0 does not work and consumer 0 does not consume, excluding workers and consumers with characteristics close to 0 does not change the aggregate supply of labor or the aggregate demand for the good (to terms of first order). As a result the equilibrium prices do not change.

Let us prove this by analyzing the equations defining a spot market equilibrium in an η -rights equilibrium. Since the equations have the same form in each of the outcome $s = g, b$ we omit the subscript s . Let $L(w, \eta)$ denote the aggregate supply of labor and let $\chi(p, \eta)$ denote the aggregate demand for the good

$$L(w, \eta) = \int_{\hat{\alpha}(\eta)}^1 \ell(w, \alpha) dG(\alpha), \quad \chi(p, \eta) = \int_{\hat{\beta}(\eta)}^1 c(p, \beta) dH(\beta) \quad (54)$$

The pair of equations defining the spot price $(w(\eta), p(\eta))$ in either outcome is given by

$$p(\eta) f'(L(w(\eta), \eta)) = w(\eta), \quad \chi(p(\eta), \eta) = f(L(w(\eta), \eta))$$

(where f can be either f_g or f_b). Differentiating with respect to η_k

$$\begin{aligned} dp f'(L) + p f''(L) \left(\frac{\partial L}{\partial w} dw + \frac{\partial L}{\partial \eta_k} d\eta_k \right) - dw &= 0 \\ \frac{\partial \chi}{\partial p} dp + \frac{\partial \chi}{\partial \eta_k} d\eta_k - f'(L) \left(\frac{\partial L}{\partial w} dw + \frac{\partial L}{\partial \eta_k} d\eta_k \right) &= 0 \end{aligned} \quad (55)$$

Suppose $k = w$. Then $\frac{\partial \chi}{\partial \eta_k} = 0$ since χ does not depend on η_w .

$$\frac{\partial L}{\partial \eta_k} = -\frac{\partial \hat{\alpha}(\eta)}{\partial \eta_w} \ell(w(\eta), \hat{\alpha}(\eta)) g(\hat{\alpha}(\eta)) = 0 \text{ when } \eta = (1, 1)$$

since worker 0 does not work. Thus all the terms in $d\eta_w$ are zero. When $k = c$, the same reasoning leads to the conclusion that all terms in $d\eta_c$ are zero when $\eta = (1, 1)$ since consumer 0 does not consume. Thus in both cases the system of equations (55) defining (dw, dp) is an homogeneous linear system with a non zero determinant (easy to check) so that the only solution is $(dw, dp) = (0, 0)$.

To analyze the effect of a change $d\eta = (d\eta_w, d\eta_c)$ in the supply of rights on a η -rights equilibrium, consider the social welfare achieved in outcome s

$$W_s(\eta) = \int_{\hat{\alpha}(\eta)}^1 ws(w_s(\eta), \alpha) dG(\alpha) + \int_{\hat{\beta}(\eta)}^1 cs(p_s(\eta), \beta) dH(\beta) + R_s(w_s(\eta), p_s(\eta)), \quad s = g, b$$

The social welfare function $\widetilde{\mathcal{W}}(a, \eta) = -a + \sum_{s=g,b} \frac{\pi_s(a)}{1+r} W_s(\eta)$ then permits us to decompose the effect of a change in η into the change in welfare induced by excluding agents from the spot markets, and the change in welfare induced by the change in investment. Let $a(\eta)$ the optimal choice of investment in Definition 6 (iii)b, then

$$\mathcal{W}(\eta) = \widetilde{\mathcal{W}}(a(\eta), \eta) \quad \text{so that} \quad \frac{\partial \mathcal{W}}{\partial \eta_k} = \frac{\partial \widetilde{\mathcal{W}}}{\partial a} \frac{\partial a}{\partial \eta_k} + \frac{\partial \widetilde{\mathcal{W}}}{\partial \eta_k}, \quad k = w, c.$$

To show that $\frac{\partial \mathcal{W}}{\partial \eta_k} < 0$, $k = w, c$, we show successively:

$$\text{Claim 1. } \left. \frac{\partial \widetilde{\mathcal{W}}}{\partial \eta_k} \right|_{(\eta=(1,1))} = 0, \quad \text{Claim 2. } \left. \frac{\partial \widetilde{\mathcal{W}}}{\partial a} \right|_{(\eta=(1,1))} > 0, \quad \text{Claim 3. } \left. \frac{\partial a(\eta)}{\partial \eta_k} \right|_{(\eta=(1,1))} < 0$$

Proof of Claim 1. This is equivalent to showing $\left. \frac{\partial W_s}{\partial \eta_k} \right|_{(\eta=(1,1))} = 0$, $s = g, b$ and $k = w, c$. In evaluating this partial derivative all terms which come from a change in the spot prices are zero since we have shown that at $\eta = (1, 1)$, the price derivatives are zero. The other terms come from the change in the marginal buyer of rights

$$\frac{\partial W_s(\eta)}{\partial \eta_k} = -\frac{\partial \hat{\alpha}}{\partial \eta_k} \text{ws}(w_s(\eta), \hat{\alpha}(\eta))g(\hat{\alpha}(\eta)) - \frac{\partial \hat{\beta}}{\partial \eta_k} \text{cs}(p_s(\eta), \hat{\beta}(\eta))h(\hat{\beta}(\eta))$$

and this derivative is zero at $\eta = (1, 1)$ since the surpluses of worker 0 and consumer 0 are equal to zero.

Proof of Claim 2. When $\eta(1, 1)$ the limiting η -equilibrium coincides with the capitalist equilibrium. Thus

$$\left. \frac{\partial \widetilde{\mathcal{W}}}{\partial a} \right|_{(\eta=(1,1))} = -1 + \pi'(\bar{a})(\bar{W}_g - \bar{W}_b) > -1 + \pi'(\bar{a})(\bar{R}_g - \bar{R}_b) = 0$$

Proof of Claim 3. Let $V_s(\eta)$ denote the contribution of outcome s to the value of the firm's securities

$$V_s(\eta) = \eta_w \text{ws}(w_s(\eta), \hat{\alpha}(\eta)) + \eta_c \text{cs}(p_s(\eta), \hat{\beta}(\eta)) + R_s(w_s(\eta), p_s(\eta)), \quad s = g, b$$

so that the total value of the securities is

$$\eta_w q_w(\eta) + \eta_c q_c(\eta) + q_e(\eta) = \sum_{s=g,b} \frac{\pi_s(a)}{1+r} V_s(\eta)$$

The investment $a(\eta)$ which maximizes the stakeholder value is defined by the first-order condition

$$\frac{\pi'(a(\eta))}{1+r} (V_g(\eta) - V_b(\eta)) = 1$$

By the implicit function theorem

$$\pi''(a(\eta)) \frac{\partial a(\eta)}{\partial \eta_k} (V_g(\eta) - V_b(\eta)) + \pi'(a(\eta)) \frac{\partial}{\partial \eta_k} (V_g(\eta) - V_b(\eta)) = 0$$

Since $\pi' > 0$, $\pi'' < 0$, $V_g(\eta) - V_b(\eta) > 0$, $\frac{\partial a(\eta)}{\partial \eta_k} < 0$ is equivalent to $\frac{\partial}{\partial \eta_k} (V_g(\eta) - V_b(\eta)) < 0$. In evaluating the partial derivatives of $V_g - V_b$ at $(1, 1)$ all the terms involving changes in spot prices are equal to zero. Thus, since $\text{ws}(\bar{w}_s, 0) = 0$

$$\left. \frac{\partial}{\partial \eta_w} (V_g(\eta) - V_b(\eta)) \right|_{\eta=(1,1)} = \frac{\partial}{\partial \hat{\alpha}} (\text{ws}(\bar{w}_g, 0) - \text{ws}(\bar{w}_b, 0)) \frac{\partial \hat{\alpha}}{\partial \eta_w} < 0$$

since $\frac{\partial \hat{\alpha}}{\partial \eta_w} < 0$ and by Lemma 2 the difference in surplus is increasing in α . Evaluating the partial with respect to η_c leads to a similar expression for $\frac{\partial}{\partial \eta_c} (V_g(\eta) - V_b(\eta))$, β replacing α and the difference in surpluses being replaced by $(cs(\bar{p}_g, 0) - cs(\bar{p}_b, 0))$ which is increasing in β . \square

When $\eta = (\eta_w, \eta_c)$ converges to $(1,1)$ there is no loss in social welfare from excluding agents from participating on the spot markets, since every worker and every consumer can participate. If at the limit the investment is greater than at the capitalist equilibrium ($\lim_{\eta \rightarrow (1,1)} a(\eta) = \tilde{a} > \bar{a}$) then there is a gain in expected social welfare from the greater likelihood of ending up at the outcome ‘g’ where social welfare is higher. This is the case when the worker with the highest disutility of labor, who becomes the marginal buyer of worker rights ($\hat{\alpha}(1, 1) = 0$), and/or the consumer with the lowest utility for the good, who becomes marginal buyer of consumer rights ($\hat{\beta}(1, 1) = 0$), have a positive surplus from participating in the spot markets. For if the marginal buyer has a positive surplus, by Lemma 2 this surplus is higher at ‘g’ than at ‘b’: taking into account in the firm’s criterion not only the value to shareholders (profit) but also the value to workers and consumers assessed by the prices (q_w, q_c) of the rights leads to a greater perceived benefit of getting outcome ‘g’ rather than ‘b’, and induces a higher investment than in the capitalist equilibrium. This is essentially the statement (i) of Proposition 6.

On the other hand if at the limit when $\hat{\alpha} \rightarrow 0$ and $\hat{\beta} \rightarrow 0$ worker 0 and consumer 0 get no surplus from working and consuming, then adding the zero value of the w-rights and c-rights does not change the investment decision from that at the capitalist equilibrium. Proposition 6 (ii) asserts that in this case the capitalist equilibrium, which coincides with the η -rights equilibrium when $\eta = (1, 1)$, is a local minimum of the η -rights equilibrium welfare: infinitesimally restricting the issue of rights gives them a positive price leading to an increase in investment, the gain in welfare from increasing investment being greater than the loss incurred by excluding some workers and consumers from participating on the spot markets. This is intuitive since the agents who are excluded have negligible surplus, while the rest of society gains from the improved signal to the value of investment given by the positive price of the rights.

APPENDIX

Proof of Lemma 1. Let $L(t, t')$ denote the optimal labor choice which maximizes $R(t, t')$. It is defined by the first-order condition

$$p(t)f_2(t', L(t, t')) = w(t)$$

By the envelope theorem

$$R_2(t, t') = p(t)f_1(t', L(t, t'))$$

so that

$$R_{21}(t, t') = p'(t)f_1(t', L(t, t')) + p(t)f_{12}(t', L(t, t'))L_1(t, t')$$

Since $f_1 > 0, p > 0, f_{12} > 0$, showing that $R_{12} < 0$ amounts to showing that (i) $p'(t) < 0$, and (ii) $L_1(t, t') < 0$. In proving (i) and (ii) we often omit the arguments of the functions in order to simplify notation.

(i) $p'(t) = \frac{d}{dt}u'(f(t, l(t))) = u''(f_1 + f_2 l')$. Inserting the expression for l' calculated in (24), it is easy to see that $p'(t) < 0$: a better technology decreases the equilibrium price of the output.

(ii) Let $\rho(t) = \frac{w(t)}{p(t)}$ be the relative price of labor with respect to output in the ' t ' equilibrium. The FOC defining L can be written as

$$f_2(t', L(t, t')) = \rho(t) \implies f_{22}(t', L(t, t')) L_1(t, t') = \rho'(t)$$

Since $f_{22} < 0$, the proof of (ii) consists in showing that $\rho'(t) > 0$: when the technology improves the price of labor relative to the price of output increases.

$$\rho'(t) = \frac{d}{dt} \left(\frac{v'(l(t))}{u'(f(t, l(t)))} \right) = \frac{u'v''l' - v'u''(f_1 + f_2 l')}{(u')^2}$$

Inserting the value of l' calculated in (24) leads to

$$\rho' = \frac{-u'v''(u''f_1f_2 + u'f_{21}) - v'u''(u'f_{22}f_1 - v''f_1 - u'f_{21}f_2)}{(u')^2(u''(f_2)^2 + u'f_{22} - v'')} \equiv \frac{N}{D}$$

D is negative and after simplification

$$N = v'u'u''(f_{21}f_2 - f_1f_{22}) - (u')^2v''f_{21} + v''u''f_1(v' - u'f_2)$$

The first two terms are negative and the last one is zero since $v' - u'f_2 = 0$ is the first-order condition defining $l(t)$. Thus $N < 0$ and ρ' is positive, which concludes the proof of the lemma. \square

Capitalist Equilibrium with a Continuum of Firms with i.i.d. Risks. Consider an economy as described in Section 2 except that the firm is replaced by a continuum of mass 1 of ex-ante identical firms with i.i.d risks. If a particular firm invests a units of money at date 0, with probability $\pi(a)$ it produces at date 1 with the production function f_g and with

probability $1 - \pi(a)$ it produces with f_b . The risks of the firms are independent and each capitalist owns a share of each firm. As is typical in the literature we consider only symmetric equilibria in which all firms choose the same actions (a, l_g, l_b) consisting of investment at date 0 and levels of production contingent on the realized production possibilities, and we invoke and (appropriately extended) Law of Large number to assume that if all firms invest a , a proportion $\pi(a)$ of them operate the technology f_g at date 1 and a proportion $1 - \pi(a)$ operate f_b . Only the “names” of the firms which operate f_g or f_b change with the different outcomes. The aggregate supply side is thus deterministic, so that the date 1 prices (w, p) of the labor and the produced good are non random. Leaving out the markets for money, bond and equity as in Section 2, a reduced form capitalist equilibrium $\left((\bar{\ell}, \bar{c}), (\bar{a}, \bar{l}_g, \bar{l}_b), (\bar{w}, \bar{p})\right)$ is such that:

- the representative worker chooses $\bar{\ell}$ which maximizes $\bar{w}\ell - v(\ell)$;
- the representative consumer chooses \bar{c} which maximizes $u(c) - \bar{p}c$;
- the representative firm chooses $(\bar{a}, \bar{l}_g, \bar{l}_b)$ which maximizes

$$\frac{1}{1+r} \left(\pi(a)(\bar{p}f_g(l_g) - \bar{w}l_g) + (1 - \pi(a))(\bar{p}f_b(l_b) - \bar{w}l_b) \right) - a$$

where, as in Section 2, $\delta = \frac{1}{1+r}$.

- markets clear: $\pi(\bar{a})\bar{l}_g + (1 - \pi(\bar{a}))\bar{l}_b = \bar{\ell}$, $\pi(\bar{a})f_g(\bar{l}_g) + (1 - \pi(\bar{a}))f_b(\bar{l}_b) = \bar{c}$

On the other hand the socially optimal choice of investment and production (a^*, l_g^*, l_b^*) maximizes the welfare sum

$$e_0 - a + \delta(e_1 + u(c) - v(\ell))$$

subject to the feasibility constraints

$$\ell \geq \pi(a)l_g + (1 - \pi(a))l_b, \quad c \leq \pi(a)f_g(l_g) + (1 - \pi(a))f_b(l_b)$$

Proposition A. A symmetric capitalist equilibrium $(\bar{\ell}, \bar{c}), (\bar{a}, \bar{l}_g, \bar{l}_b), (\bar{w}, \bar{p})$ of the economy with a continuum of firms exists and is Pareto optimal.

Proof: (i) *Existence of equilibrium*

STEP 1. Show that for any π there is a unique equilibrium on the labor and produced good markets. Since a proportion π of firms produce with f_g and maximize profit on the spot markets taking prices as given, while a proportion $1 - \pi$ of firms produce with f_b , the prices (w, p) are solutions of the system of equations

$$\begin{aligned} \ell(w) - \pi l_g(w, p) - (1 - \pi)l_b(w, p) &= 0 \\ c(p) - \pi f_g(l_g(w, p)) - (1 - \pi)f_b(l_b(w, p)) &= 0 \end{aligned} \tag{56}$$

where $l_g(w, p)$ (resp. $l_b(w, p)$) is the demand of labor of a firm with technology f_g (resp. f_b), $\ell(w)$ is the supply of labor and $c(p)$ is the demand for the produced good. These supplies and demands are defined implicitly by

$$pf'_s(l_s(w, p)) = w, \quad s = g, b, \quad v'(l(w)) = w, \quad u'(c(p)) = p \tag{57}$$

(56) is a system of 2 equations with 2 unknowns (w, p) parameterized by $\pi \in [0, 1]$, which can be written as $\phi(w, p; \pi) = 0$, where $\phi(w, p; \pi) = (\phi_1(w, p; \pi), \phi_2(w, p; \pi))$ is the function defined by the LHS of (56). $\phi : \mathbb{R}_{++}^2 \times [0, 1] \rightarrow \mathbb{R}^2$ is a smooth function. For $\pi = 0$, the system has a unique solution defined by

$$u'(f_b(l_b))f'_b(l_b) = v'(l_b), \quad p = u'(f_b(l_b)), \quad w = v'(l_b)$$

Let us show that, for all $\pi \in [0, 1]$, if (w, p) solves (56), then the Jacobian of the system of equations at (w, p) has a negative determinant. Since the degree of $\phi(w, p; \pi)$ — i.e. the sum of the signs of the determinants of the Jacobian— at the solutions to $\phi(w, p; \pi) = 0$ does not vary⁹ with π (see Mas Colell (1985, p. 46)), this will prove that the solution to (56) is unique. The determinant of the Jacobian is

$$\begin{vmatrix} \ell'(w) - \pi \frac{\partial l_g}{\partial w} - (1 - \pi) \frac{\partial l_b}{\partial w} & -\pi \frac{\partial l_g}{\partial p} - (1 - \pi) \frac{\partial l_b}{\partial p} \\ -\pi f'_g \frac{\partial l_g}{\partial w} - (1 - \pi) f'_b \frac{\partial l_b}{\partial w} & c'(p) - \pi f'_g \frac{\partial l_g}{\partial p} - (1 - \pi) f'_b \frac{\partial l_b}{\partial p} \end{vmatrix}$$

where the arguments of the functions have been omitted to simplify notation. Since by (57), at a solution of (56) $f'_g = f'_b = \frac{w}{p}$, the determinant is of the form

$$\begin{vmatrix} \ell'(w) - A & -B \\ -A \frac{w}{p} & c'(p) - B \frac{w}{p} \end{vmatrix}$$

where

$$A = \pi \frac{\partial l_g}{\partial w} + (1 - \pi) \frac{\partial l_b}{\partial w} < 0, \quad B = \pi \frac{\partial l_g}{\partial p} + (1 - \pi) \frac{\partial l_b}{\partial p} > 0$$

Since $c'(p) < 0$, $\ell'(w) > 0$, it is easy to check that the determinant is negative.

STEP 2. Let us show that there exist a solution to the optimal choice of the representative firm. From Step 1 we know that for all $a \geq 0$ there is a corresponding spot market equilibrium $(w(\pi(a)), p(\pi(a)))$. Let $R_s(a) = p(a)f'_g(l_g(w(a), p(a))) - w(\pi(a))l'_g(w(a), p(a))$, $s = g, b$. There is a symmetric equilibrium if the FOC for optimal investment

$$\pi'(a) \left(R_g(a) - R_b(a) \right) - 1 \leq 0, \quad = 0 \text{ if } a > 0$$

has a solution. Suppose $R_g(0) - R_b(0) \leq 0$. Then $a = 0$ is optimal at prices $(p(0), w(0))$. Suppose $R_g(0) - R_b(0) > 0$. Since $\pi'(a) \rightarrow \infty$ when $a \rightarrow 0$, $\pi'(a) \left(R_g(a) - R_b(a) \right) - 1 > 0$ when $a \rightarrow 0$. Since $\pi'(a) \rightarrow 0$ when $a \rightarrow \infty$, $\pi'(a) \left(R_g(a) - R_b(a) \right) - 1 < 0$ when $a \rightarrow \infty$. By the intermediate value theorem, there is a solution $a > 0$ to the FOC, which gives the optimal investment at prices $(w(a), p(a))$ (the labor choices $(l_g(a), l_b(a))$ being optimal by definition of $(w(a), p(a))$).

(ii) *Pareto optimality*. Let $\left((\bar{\ell}, \bar{c}), (\bar{a}, \bar{l}_g, \bar{l}_b), (\bar{w}, \bar{p}) \right)$ be a symmetric capitalist equilibrium. Suppose it is not Pareto optimal. Then there exists a feasible allocation $\left((\tilde{\ell}, \tilde{c}), (\tilde{a}, \tilde{l}_g, \tilde{l}_b) \right)$ such

⁹All the functions $\phi(w, p; \pi) = 0$, $\pi \in [0, 1]$ are homotopic.

that

$$-\tilde{a} + \delta(u(\tilde{c}) - v(\tilde{\ell})) > -\bar{a} + \delta(u(\bar{c}) - v(\bar{\ell})) \quad (58)$$

Feasibility implies

$$\tilde{\ell} = \pi(\tilde{a})\tilde{l}_g + (1 - \pi(\tilde{a}))\tilde{l}_b, \quad \tilde{c} = \pi(\tilde{a})f_g(\tilde{l}_g) + (1 - \pi(\tilde{a}))f_b(\tilde{l}_b) \quad (59)$$

and the maximizing property of the equilibrium implies

$$\begin{aligned} u(\tilde{c}) - \bar{p}\tilde{c} &\leq u(\bar{c}) - \bar{p}\bar{c}, & \bar{w}\tilde{\ell} - v(\tilde{\ell}) &\leq \bar{w}\bar{\ell} - v(\bar{\ell}) \\ -\tilde{a} + \delta\left(\pi(\tilde{a})(\bar{p}f_g(\tilde{l}_g) - \bar{w}\tilde{l}_g) + (1 - \pi(\tilde{a}))(\bar{p}f_b(\tilde{l}_b) - \bar{w}\tilde{l}_b)\right) &\leq \\ -\bar{a} + \delta\left(\pi(\bar{a})(\bar{p}f_g(\bar{l}_g) - \bar{w}\bar{l}_g) + (1 - \pi(\bar{a}))(\bar{p}f_b(\bar{l}_b) - \bar{w}\bar{l}_b)\right) &\leq \end{aligned} \quad (60)$$

Combined with (59), (61) implies that

$$-\tilde{a} + \delta(\bar{p}\tilde{c} - \bar{w}\tilde{\ell}) \leq -\bar{a} + \delta(\bar{p}\bar{c} - \bar{w}\bar{\ell})$$

which, when combined with (60), leads to

$$-\tilde{a} + \delta(u(\tilde{c}) - v(\tilde{\ell})) \leq -\bar{a} + \delta(u(\bar{c}) - v(\bar{\ell}))$$

which contradicts (58). □

Proof of Lemma 2. There are two parts to the proof: first show that worker and consumer surplus at ‘g’ is larger than at ‘b’ for any worker and consumer; second show that the difference in surplus is increasing in the type parameter. The key to the proof is to consider the one-parameter family of production functions $f(t, l)$ introduced in (20) which begins at $t = 0$ with f_b and ends at $t = 1$ with f_g . For the identical-agent economy of Section 2, for any $t \in [0, 1]$, the spot market equilibrium of the ‘t’ economy is summarized by the labor allocation $l(t)$ satisfying equation (21). The functions Φ and Ψ defined in (47) and (48) permit the spot market equilibrium in the heterogeneous-agent economy to be summarized by the labor allocation $l(t)$ satisfying the analogous equation

$$\Phi'(f(t, l(t))f_2(t, l(t))) = \Psi'(l(t)), \quad t \in [0, 1] \quad (62)$$

the spot prices being given by

$$p(t) = \Phi'(f(t, l(t))), \quad w(t) = \Psi'(l(t)), \quad t \in [0, 1]$$

Using the same argument as in the proof of Lemma 1(i), replacing (u, v) by (Φ, Ψ) shows that $p'(t) < 0$, so that $\bar{p}_g < \bar{p}_b$. To show that $\bar{w}_g > \bar{w}_b$ we show $w'(t) > 0$. Since $w'(t) = \Psi''(l(t))l'(t)$ and $\Psi''(l) > 0$, $w'(t) > 0$ is equivalent to $l'(t) > 0$. Differentiating (62) leads to

$$l'(t) = -\frac{\Phi''f_1f_2 + \Phi'f_{21}}{\Phi''(f_2)^2 + \Phi'f_{22} - \Psi''}$$

which is the analogue of equation (24), the denominator being negative. We need to show that $\Phi'' f_1 f_2 + \Phi' f_{21} > 0$, and since $\Phi' > 0$ this is equivalent to

$$\frac{f_{21}(t, l(t)) f(t, l(t))}{f_1(t, l(t)) f_2(t, l(t))} > -\frac{\Phi''(t, l(t))}{\Phi'(t, l(t))} f(t, l(t)) \quad (63)$$

By Assumption A1, RHS is less than or equal to 1. Thus the result follows if we show that LHS is greater than 1. A little calculation shows that

$$\frac{\partial f_1(t, l)}{\partial l f(t, l)} > 0 \iff \frac{f'_g(l)}{f_g(l)} > \frac{f'_b(l)}{f_b(l)}, \quad \forall l > 0$$

Since $\frac{\partial f_1}{\partial l f} = \frac{f_{12} f - f_1 f_2}{f^2}$, A2 ensures that LHS of (63) is more than 1 and thus A1, A2 imply that $w'(t) > 0$ and $\bar{w}_g > \bar{w}_b$.

Since

$$\frac{\partial}{\partial w} \text{ws}(w, \alpha) = \ell(w, \alpha) > 0, \quad \frac{\partial}{\partial p} \text{cs}(p, \beta) = -c(p, \beta) < 0$$

$\bar{w}_g > \bar{w}_b$ and $\bar{p}_g < \bar{p}_b$ imply that the surplus of any worker or consumer is greater in 'g' than in 'b'. To show that the difference in surplus is increasing in the type note that

$$\frac{\partial}{\partial \alpha} \left(\text{ws}(\bar{w}_g, \alpha) - \text{ws}(\bar{w}_b, \alpha) \right) = -v_\alpha(\ell(\bar{w}_g, \alpha), \alpha) + v_\alpha(\ell(\bar{w}_b, \alpha), \alpha) > 0$$

since $\ell(\bar{w}_g, \alpha) > \ell(\bar{w}_b, \alpha)$ and $v_{\alpha\ell} < 0$. In the same way

$$\frac{\partial}{\partial \beta} \left(\text{cs}(\bar{p}_g, \beta) - \text{cs}(\bar{p}_b, \beta) \right) = u_\beta(c(\bar{p}_g, \beta), \beta) - u_\beta(c(\bar{p}_b, \beta), \beta) > 0$$

since $c(\bar{p}_g, \beta) > c(\bar{p}_b, \beta)$ and $u_{\beta c} > 0$. □

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