Economics Letters 19 (1985) 221–225
North-Holland

INVESTMENT IN INFORMATION ACQUISITION *

Jean-Pierre DANTHINE

Université de Lausanne, CH-1015 Lausanne-Dorigny, Switzerland

Michael J.P. MAGILL

University of Southern California, Los Angeles, CA 90089-0152, USA

Received 10 April 1985

This paper presents a simple model in which a community must decide on the amount of real investment and investment in information so as to maximise the expected utility of consumption over two periods. It is shown that under conditions involving the product of the elasticity of the marginal utility of consumption and the elasticity of the production function that optimal real investment is either a monotone function of the information or is independent of it. In the former case information is of value and it is optimal to invest a positive amount in information acquisition. In the latter case information has no value and society devotes no resources to information acquisition.

1. Introduction

Consider the simplest aggregated model of an economy in which the return on investment is uncertain. How much of its resources should society devote to acquiring prior information about the uncertain return? How does this information affect the society's investment program? A related question was raised by R. Kihlstrom (1974a,b,1976) in a series of articles. Kihlstrom's concern was the demand for information about product quality and about price and technology. More importantly his was a Bayesian setting where information alters prior probability distributions. Our approach is non-Bayesian; we deal with information about technology and we aim at obtaining specific results as did Kihlstrom (1974a) (rather than a 'general theory'). Like Kihlstrom (1974a) we rely on an assumption of normality of the underlying random variable. With our approach we are able to get qualitative results similar to those of Kihlstrom (1974a) without placing any parametric restrictions on the utility function. We show precisely how the characteristics of the utility function (in terms of usual measures of risk aversion) together with those of the production function determine the reaction of rational investors to information. We further demonstrate that a rational agent will not gather information purely for the sake of reducing prior uncertainty, nor will he find it worthwhile to use his resources to completely eliminate uncertainty.

2. The model

We formalise the problem by considering the following one-good, two-period centralised model of an economy. A community inherits an initial stock of capital $y_0 > 0$ which can be used either for

* Research support from the National Science Foundation (SES-8200432) is gratefully acknowledged.

0165-1765/85/$3.30

consumption in the first period or for investment. Investment is of two kinds: real investment and investment in information acquisition. Real investment (k) leads to an uncertain output $f(k, r)$ next period, where r is a random variable, all of this output being used for consumption in the second period. $f: R^+ \times R \to R^+$ is twice continuously differentiable with $f_k \geqq 0$, $f_r > 0$, $f_{kk} \leqq 0$ and $f_{kr} \geqq 0$. The community has a utility function $u(c)$ for consumption in each period, $u: R^+ \to R$, which is twice continuously differentiable and satisfies $u' > 0$, $u'' < 0$ for $c > 0$. In the special case where there is no process for acquiring prior information about r, the community chooses real investment k so as to maximise the expected discounted utility of consumption

$$\max_{k \in [0, y_0]} u(y_0 - k) + \beta \int_{-\infty}^{\infty} u(f(k, r)) \mathrm{d}G(r), \tag{1}$$

where $G(\cdot)$ is the distribution function of the random variable r and β is a discount factor.

To model the process of information acquisition we make the additional assumption that r is normally distributed with mean $\bar{r}$ and variance $\sigma_r^2 > 0$ and is decomposable into a sum $r = \eta + \epsilon$, where η is the observable and ϵ the unobservable component. (Alternatively, η could be interpreted as a measurement of r with error ϵ. Some formal changes would be required by this interpretation.) η is observed in period 1 prior to the real investment decision but ϵ remains unknown. η and ϵ are independent and normally distributed with mean and variance $(\bar{r}, \sigma_r^2 - \sigma^2)$ and $(0, \sigma^2)$ respectively. Since the standard deviation of the residual uncertainty ϵ is σ, we say that the community obtains information of quality σ (or gathers the amount of information $\sigma_r - \sigma$). With two-parameter probability densities, the quality (or quantity) of information is naturally identified with its reliability. The same is achieved, more generally, by the concept of 'sufficiency' used by Kihlstrom. Let Δ denote investment in information acquisition – to obtain information of quality σ requires investment $\Delta = c(\sigma_r - \sigma)$, where $c(0) = 0$, $c' > 0$, $c'' \geqq 0$, and $0 \leqq \sigma < \sigma_r$.

The society's problem of determining the optimal amount of real investment and investment in information acquisition is decomposed into two parts. The first is to find $k^*(\eta; \sigma, \Delta)$ such that

$$\max_{k \in [0, y_0 - \Delta]} u(y_0 - \Delta - k) + \beta \int_{-\infty}^{\infty} u(f(k, \eta + \sigma z)) \mathrm{d}F(z), \tag{2}$$

where we have written $r = \eta + \sigma z$ and used the fact that η has been observed (at cost Δ), z being normally distributed with zero mean and unit variance. Now let η be written as $\eta = \bar{r} + (\sigma_r^2 - \sigma^2)^{1/2} z_\eta$, where z_η is normal with zero mean and unit variance and define

$$\begin{aligned} W(\sigma, \Delta) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} & u\Big(y_0 - \Delta - k^*\Big(\bar{r} + \sqrt{\sigma_r^2 - \sigma^2}\, z_\eta;\ \sigma, \Delta\Big)\Big) \\ & + \beta u\Big(f\Big(k^*\Big(\bar{r} + \sqrt{\sigma_r^2 - \sigma^2}\, z_\eta;\ \sigma, \Delta\Big), \bar{r} + \sqrt{\sigma_r^2 - \sigma^2}\, z_\eta + \sigma z\Big)\Big) \mathrm{d}F(z) \mathrm{d}F(z_\eta). \end{aligned} \tag{3}$$

The optimal investment in information acquisition (Δ^*) and the implied quality of information (σ^*) are the solution of

$$\max_{(\sigma, \Delta) \in K} W(\sigma, \Delta), \qquad K = \{(\sigma, \Delta) \,|\, 0 \leqq \sigma \leqq \sigma_r, \ \Delta = c(\sigma_r - \sigma)\}. \tag{4}$$

Thus the first step in (2) involves finding the optimal real investment k^* given the prior information η of quality σ (obtained at cost Δ). The second step in (4) determines the optimal investment in information Δ^*.

That a solution to (2) exists follows from the continuity of $u(\cdot)$ and $f(\cdot)$ and compactness of $[0, y_0 - \Delta]$. By the strict concavity of $u(\cdot)$ and $f(\cdot, r)$, $k^*(\cdot)$ is unique and hence $\mathscr{W}(\sigma) = W(\sigma, c(\sigma_r - \sigma))$ is continuous. Since (4) is equivalent to $\max_{\sigma \in [0, \sigma_r]} \mathscr{W}(\sigma)$, a solution to (4) exists.

3. Qualitative properties

What can we say about the qualitative properties of (k^*, Δ^*)? Consider first properties of k^*. If $0 < k^* < y_0 - \Delta$, then the first-order condition for (2) implies

$$u'(y_0 - \Delta - k^*) = \beta \int_{-\infty}^{\infty} u'(f(k^*, \eta + \sigma z)) f_k(k^*, \eta + \sigma z) \mathrm{d}F(z), \tag{5}$$

namely that the supply price u' of capital must equal its expected demand price $\beta \int u' f_k \, \mathrm{d}F$. If we define the following elasticities in consumption and production:

$$R(k, r) = -\frac{u''(f(k, r)) f(k, r)}{u'(f(k, r))}, \qquad S(k, r) = \frac{f_r(k, r) f_k(k, r)}{f(k, r) f_{kr}(k, r)}, \qquad (k, r) \in R^+ \times R,$$

then a straightforward application of the implicit function theorem to (5) leads to the following result:

Proposition 1. Let $k^* \in (0, y_0 - \Delta)$ *be the optimal solution to (2). (i) If* $f_k > 0$, $f_{kr} > 0$, *then* $\partial k^*/\partial \eta > (<) 0$ *if* $RS < (>)$ *and* $\partial k^*/\partial \eta < 0$ *if* $f_{kr} = 0$. *(ii)* $\partial k^*/\partial \eta = 0$ *if* $RS = 1$ *or* $f_k = 0$ $\forall (k, r) \in R^+ \times R$.

Proof. Let $v(k, \eta)$ denote the expression being maximised in (2), then (5) reduces to $v_k(k^*, \eta) = 0$ so that $\partial k^*/\partial \eta = -(v_{k\eta}(k^*, \eta)/v_{kk}(k^*, \eta))$. Since $v_{kk} = u'' + \beta \int_{-\infty}^{\infty} u'' f_k^2 \mathrm{d}F < 0$, $\operatorname{sgn} \partial k^*/\partial \eta = \operatorname{sgn} v_{k\eta}(k^*, \eta)$ and $v_{k\eta}(k^*, \eta) = \beta \int_{-\infty}^{\infty} (\partial/\partial \eta)(u' f_k) \mathrm{d}F(z) = \beta \int_{-\infty}^{\infty} (u'' f_r f_k + u' f_{kr}) \mathrm{d}F(z) = -\beta \int_{-\infty}^{\infty} u' f_{kr} (RS - 1) \mathrm{d}F(z)$ if $f_{kr} > 0$. Q.E.D.

To examine the properties of Δ^* or equivalently of σ^* note that

$$\mathscr{W}'(\sigma) = -W_\Delta(\sigma, c(\sigma_r - \sigma)) c'(\sigma_r - \sigma) + W_\sigma(\sigma, c(\sigma_r - \sigma)), \tag{6}$$

where, in view of (5), if we let $k^*(\cdot) = k^*(\bar{r} + \sqrt{\sigma_r^2 - \sigma^2} z_\eta; \sigma, \Delta)$,

$$W_\Delta(\sigma, \Delta) = -\int_{\infty}^{\infty} u'(y_0 - \Delta - k^*(\cdot)) \mathrm{d}F(z_\eta) < 0,$$

$$W_\sigma(\sigma, \Delta) = \beta \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} u'\Big(f\Big(k^*(\cdot), \bar{r} + \sqrt{\sigma_r^2 - \sigma^2} z_\eta + \sigma z\Big)\Big)$$

$$\times f_r\Big(k^*(\cdot), \bar{r} + \sqrt{\sigma^2 - \sigma^2} z_\eta + \sigma z\Big) \left[z - \frac{\sigma z_\eta}{\sqrt{\sigma_r^2 - \sigma^2}} \right] \mathrm{d}F(z) \mathrm{d}F(z_\eta), \tag{7}$$

for all $(\sigma, \Delta) \in [0, \sigma_r] \times R^+$.

Proposition 2. *(i)* $\sigma^* > 0$. *(ii) If* $RS = 1$ *or* $f_k = 0$ *for all* $(k, r) \in R^+ \times R$, *then* $\sigma_r - \sigma^* = 0$. *(iii) Let* $c'(0) = 0$. *If* $f_k > 0$, $f_{kr} > 0$ *and either* $RS < 1$ *or* $RS > 1$ *for all* $(k, r) \in R^+ \times R$ *or if* $f_k > 0$, $f_{kr} = 0$ *for all* $(k, r) \in R^+ \times R$, *then* $\sigma_r - \sigma^* > 0$.

Proof. (i) $W_\sigma(0, \Delta) = 0$ since $E(z) = 0$. Thus $W_\sigma(0, c(\sigma_r)) = 0$ and $\mathscr{W}'(0) = -W_\Delta(0, c(\sigma_r))c'(\sigma_r) > 0$ implies $\sigma^* > 0$. (ii) Introduce the new random variable $w = z - (\sigma z_\eta/(\sigma_r^2 - \sigma^2)^{1/2})$. Note that $E(w) = 0$ and (w, r) are independent since they are joint normal and $\text{cov}(w, r) = \sigma(E(z^2) - E(z_\eta^2)) = 0$. Note also that $r - \sigma w = \bar{r} + (\sigma_r^2 z_\eta/(\sigma_r^2 - \sigma^2)^{1/2})$ so that we can write $k^*(\eta; \sigma, \Delta) = \bar{k}(\bar{r} - \sigma w; \sigma, \Delta)$. Let $H(\cdot)$ denote the distribution function for w, then (7) becomes

$$W_\sigma(\sigma, \Delta) = \beta \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} u'(f(\bar{k}(r - \sigma w; \sigma, \Delta), r)) f_r(\bar{k}(r - \sigma w; \sigma, \Delta), r) w \, dH(w) \, dG(r). \tag{8}$$

Since

$$\frac{\partial}{\partial k}(u'(f(k, r)) f_r(k, r)) = \frac{\partial}{\partial r}(u'(f(k, r)) f_k(k, r)) = u'' f_k f_r + u' f_{rk} = -u' f_{rk}(RS - 1), \tag{9}$$

if $RS - 1 = 0$ or $f_k = 0$ (and hence $f_{kr} = 0$), then $u' f_r$ does not depend on k and hence not on w. But then $W_\sigma(\sigma, \Delta) = 0$ for all $(\sigma, \Delta) \in [0, \sigma_r] \times R^+$ since $E(w) = 0$. Thus $\mathscr{W}'(\sigma) = -W_\Delta(\sigma, c(\sigma_r - \sigma))c' \times (\sigma_r - \sigma) > 0$ for all $\sigma \in [0, \sigma_r)$ so that $\sigma^* = \sigma_r$. (iii) Suppose $f_k > 0$, $f_{kr} > 0$ and $RS - 1 < 0$. By Proposition 1 (i) k^* is an increasing function of η and hence $\bar{k}$ is a decreasing function of w. By (9) since $u' > 0$, $f_{kr} > 0$, $u' f_r$ is a decreasing function of k. Thus $u'(f(\bar{k}(\cdot), r)) f_r(\bar{k}(\cdot), r)$ is a decreasing function of w. Since for a decreasing function $g(\cdot)$, $\int_{-\infty}^{\infty} g(w) w \, dH(w) < 0$, it follows that $W_\sigma(\sigma, \Delta) < 0$ $\forall (\sigma, \Delta) \in (0, \sigma_r] \times R^+$. Since $c'(0) = 0$ it follows from (6) that $\mathscr{W}'(\sigma_r) < 0$. Since $\mathscr{W}'(0) > 0$ by the continuity of $\mathscr{W}'$ there exists $\sigma^* \in (0, \sigma_r)$ such that $\mathscr{W}'(\sigma^*) = 0$. If $RS - 1 > 0$ or if $f_k > 0$ and $f_{kr} = 0$ it follows by a similar argument that $u'(f(\bar{k}(\cdot), r)) f_r(\bar{k}(\cdot), r)$ is a decreasing function of w. Q.E.D.

These two propositions can be given a straightforward economic interpretation. Proposition 1 shows how investment varies with the prior information η on r under a condition on the product of the elasticities in consumption (R) and production (S) – a condition which may be explained as follows. By the first-order condition (5), $k^*(\eta)$ is chosen so that the supply price of investment u' is equated to the demand price $\beta \int u' f_k \, dF$. When $f_k > 0$ an increase in η (for a given realisation z) increases output f and hence reduces u', but, when $f_{kr} > 0$ the increase in η raises the marginal product of capital f_k. The first effect decreases the demand price tending to lower k^* while the second effect by raising the demand price increases k^*. Under the elasticity condition $RS < 1$ the first effect dominates; under the condition $RS > 1$ the second effect dominates. When $RS = 1$ the two effects balance out and k^* remains unchanged even when a change in η signals a likely change in r. Proposition 1 shows that different individuals may react differently – even in opposite directions – to the same information. As such it casts some doubt on the proposition that prices will transmit all available information in a world of heterogeneous agents.

Proposition 2 uses this systematic dependence of k^* on η to determine the extent to which it is optimal for the community to invest in information gathering. The first-order condition for investment in information $\mathscr{W}'(\sigma^*) \leq 0$, requires that the demand price for information $(-W_\sigma)$ be at least as great as its supply price $(-W_\Delta c')$. If having prior information does not affect the society's investment decision ($RS = 1$ or $f_k = 0$) then the demand price for information is zero. In this case the society devotes no resources to information gathering – information is not gathered purely for the sake of reducing prior uncertainty. The same point is made by Kihlstrom (1974a, p. 117) in the context of information about product quality. In the case where the investment decision is influenced by the prior information ($RS < 1$, $RS > 1$ or $f_{k_r} = 0$ with $f_k > 0$), if the cost of information gathering is not too great (which occurs in particular when $c'(0) = 0$), then it is optimal for the community to invest *some* of its resources in information gathering. However, since the demand price for information is zero at full information, if the marginal cost of gathering information is positive, then it is not optimal to gather complete information.

References

Kihlstrom, Richard E., 1974a, A Bayesian model of demand for information about product quality, International Economic Review 15, 99–118.

Kihlstrom, Richard E., 1974b, A general theory of demand for information about product quality, Journal of Economic Theory 8, 413–440.

Kihlstrom, Richard E., 1976, Firm demand for information about price and technology, Journal of Political Economy 84, 1335–1341.