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Incentives and risk sharing in a stock market equilibrium*

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Abstract. This paper aims to integrate the literature on portfolio choice and security pricing with the literature on agency costs and capital structure. It introduces the concept of a *stock market equilibrium with rational, competitive price perceptions* (RCPP). Using this equilibrium concept, the paper finds that incentive considerations induce entrpeneurs (i) to retain a larger share of their own firm and a smaller share of the equity of other firms, and (ii) to make much more extensive use of debt than would be predicted by the standard general equilibrium model of finance. These differences translate into higher interest rates and lower risk premia on the risky securities.

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1 Introduction

Economists have long been ambivalent on the merits of the stock market. On the one hand, the capital asset pricing model (CAPM), which is the basis for the modern theory of finance, emphasizes the merit of the stock market for diversifying the idiosyncratic risks and sharing the aggregate risks of productive activity. On the other hand, the traditional view of the classical economists, revived in modern times by Berle and Means (1932),

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Jensen and Meckling (1976) and the ensuing agency-cost literature, emphasized the negative effect on incentives of the separation of ownership and control implied by the corporate form of ownership. This paper provides a framework for reconciling these two perspectives and shows the circumstances under which the stock market can provide an optimal trade-off between the beneficial effects of risk sharing and the distortive effects on incentives.

To study the efficiency properties of the stock market it is natural to use the framework of general equilibrium. We adopt the simplest model which permits the simultaneous analysis of production, risk-sharing and financing decisions—namely the two-period general equilibrium model of Diamond (1967). In the spirit of Knight (1921) we model the firm as an entity arising from the organizational ability, foresight and initiative of an *entrepreneur*. The activity of a firm consists in combining entrepreneurial effort and physical input (the value of capital and non-managerial labor) at an initial date: this gives rise to a random profit stream at the next date. In addition to entrepreneurs there is another class of agents which we call *investors*: they have initial wealth at date 0 but no productive opportunities. In the spirit of the principal-agent literature, we assume that the effort of entrepreneurs is not observable and that the risks to which firms are exposed are sufficiently complex to make the writing and enforcement of contracts contingent on states unfeasible (states of nature are unverifiable). Under these assumptions, markets for channeling capital from investors to firms and for sharing risks must either be non-contingent or based on the realized outputs of firms. In this paper we concentrate on the simplest (linear) contracts: default-free debt and equity. Entrepreneurs can thus obtain funds for financing their capital investment by drawing on their own initial wealth, by selling shares of their firms or by issuing debt; they can diversify their risks by buying shares of other firms. Since arrangements for financing typically have to be made before production can take place, we assume that the trades on the debt and equity markets are made *before* the entrepreneurs choose the level of effort to invest in their firms.

Under these circumstances trade on the financial markets will influence the effort that entrepreneurs invest in their firms. If an entrepreneur finances his venture by selling most of the shares of his firm, he will not have much incentive to invest effort in his firm, since most of the payoff from his effort goes directly to outside shareholders. On the other hand if the financing is done principally by debt, then typically a high level of effort will be required to ensure that the firm does not go bankrupt. The effect on incentives is not however the only consequence of the choice of capital structure: for the choice of debt and equity also determines the way the productive risks of the economy are shared. To take an extreme example, if equity were not traded at all, and if all financing were made by debt, then no share of the productive risks would be carried by investors — the full burden would fall on the entrepreneurs, who would have undiversified and leveraged profit streams.

The trade-off between incentives and risk sharing is the problem that is studied in the principal-agent literature: the difference is that in the setting that we consider there is no principal who directly designs a contract to induce agents (entrepreneurs) to behave in an optimal way. Whatever incentive schemes there are must somehow be created by the markets. It is thus natural to ask whether the stock and bond markets can create incentive schemes which lead to a socially optimal balance between incentives and risk sharing.

The moral hazard problem posed by the nonobservability of entrepreneurial effort only arises when equity is sold, for then the benefit of an entrepreneur's effort is shared between the entrepreneur and the outside shareholders, while the cost is born solely by the entrepreneur. If a price system is to provide appropriate incentives, then it must discourage entrepreneurs from selling too much equity of their firms. Intuitively this will only happen if entrepreneurs are aware that the market will "punish" them by a low price for their firms' shares, if they attempt to sell too much of their equity.

In Section 2 we propose a concept of equilibrium in which markets play such a disciplining role. It is based on two ideas: first, it assumes that investors are well informed — they can observe all the financial decisions of entrepreneurs — and use this information to deduce the effort that entrepreneurs will exert. Second, it assumes that entrepreneurs are aware of this fact: this is formalized by the concept of *price perceptions.* To decide whether an investment-financing plan is optimal, an entrepreneur needs to evaluate what would happen if he were to change this plan: his price perceptions describe how he perceives that the price of his equity would react to any such change of plan. The price perceptions are assumed to be *rational* (i.e., entrepreneurs think that investors will correctly deduce from their investment-financing decision what their effort and the associated output of their firm will be) and *competitive* (an entrepreneur cannot affect the span of the financial markets and thus the risk premium that investors require to invest in the risky income stream that he sells). Putting these ideas together leads to the concept of a *stock market equilibrium with rational, competitive price perceptions* (an RCPP equilibrium).

This concept of equilibrium describes markets functioning at their best: does it suffice to induce a socially optimal outcome? First best optimality is clearly too demanding a criterion to use in this setting: what is needed is a extension of the concept of *constrained efficiency* introduced by Diamond (1967) which respects both the limited available set of financial securities and the incentive constraints imposed by the nonobservability of effort. The associated constrained social optimum problem is in fact equivalent to a principal-agent problem. In Section 3 we show that an RCPP equilibrium is constrained efficient: markets can thus be thought of as designing an incentive contract which is the solution of a principal-agent problem. More precisely, it is the rational price perceptions which provide the incentive schemes (nonlinear prices) that induce entrepreneurs to choose an optimal capital structure.

The model that we study makes it possible to integrate two branches of the literature: the classical literature on portfolio choice and security pricing (the standard general equilibrium model of finance) and the literature on agency costs and their relation to capital structure, which following Jensen-Meckling (1976), have been studied in partial equilibrium models. Having a model with incentives which contains the classical risk-sharing model as a special case, permits one to study how the predictions of the standard model are modified by the presence of incentive effects. In Section 4 we give examples of RCPP-equilibria and compare the resulting capital structure and security prices with those of the standard finance model: we find that in an RCPP equilibrium diversification is less extensive for entrepreneurs, since incentive considerations induce them to retain a larger share of their own firm and a smaller share of the equity of other firms than would be required solely on the basis of risk diversification; furthermore, incentives induce entrepreneurs to make much more extensive use of debt than would be predicted by the standard model. These differences translate into higher interest rates and lower risk premia on the risky securities.

Related Literature. The study of the way ownership structure in business enterprise affects incentives has a long tradition in economics. The classical economists were uncompromisingly in favor of sole proprietorship, arguing that shared ownership has a negative effect on incentives (Smith[1] (1776), Mill[2] (1848), Marshall[3] (1890)). The idea that share systems can be explained as a compromise between risk sharing and incentives was introduced in the sharecropping literature by Cheung (1969) and Stiglitz (1974): for a more recent discussion of shared ownership (and the stock market) versus sole proprietorship see Hammond (1993). The paper by Stiglitz was an early contribution to the literature on the principal-agent problem which subsequently gave rise to an extensive literature (see for example Sappington (1991) for a survey). Although our paper is not set up as a principal-agent problem, as we pointed out above the social optimum problem defining a constrained Pareto optimum can be expressed as a principal-agent problem, with the planner acting as a "benevolent" principal.

The idea that financial decisions of agents transmit information about characteristics or actions of agents that are not directly observable or knowable by the market, has been extensively explored in the finance literature. Concepts of equilibrium based on this idea and the idea of rational expectations have been used in many partial equilibrium models: for *adverse selection* in the signaling models of Ross (1977), and Leland and Pyle (1977), and the subsequent literature [see Harris and Raviv (1992) for a survey]; for problems of *moral hazard* by Jensen and Meckling (1976), Grossman and Hart (1982), and Brander and Spencer (1989). This paper differs from these latter contributions in that it makes explicit in a general equilibrium setting with moral hazard how the market can resolve (or at least mitigate) the incentive problems created by asymmetry of information; it also provides a framework in which the risk-sharing function of financial markets and their disciplining role in attenuating the agency costs of firms can be studied simultaneously. This permits the agency costs and benefits of equity and debt to be balanced against the risk-sharing benefits and costs of these securities.

A simpler concept of rational expectations is present in all the literature on general equilibrium with incomplete markets (GEI) which began with the papers of Arrow (1953) and Diamond (1967), and subsequently gave rise to an extensive literature [for a survey of results in this area see Magill and Shafer (1991)]. We have chosen the simplest version of the GEI model with production, namely Diamond's model, to study how the agency theory of the firm can be incorporated into a general equilibrium analysis. It is well-known that a stock market equilibrium in Diamond's model is constrained efficient but that such a result can not in general be expected to hold in more complex GEI models. Since the problem of constrained inefficiency arising in an incomplete markets model with many goods or many periods, or in a production economy without partial spanning, is not directly related to the problems posed by incentives, we have chosen to take as a benchmark the simplest model of a production economy in which financial markets lead to constrained efficiency in the absence of incentive effects.

An alternative approach to incorporating asymmetric information into general equilibrium, which is tantamount to extending Arrow-Debreu theory directly to a world with moral hazard and adverse selection, has been proposed by Prescott and Townsend (1984a, 1984b). The contracts they consider are lotteries on an abstract consumption space. For the moment it is not clear to us how the two approaches are related: the contracts they study seem very different from the standard debt and equity contracts which are the focus of our analysis.

More recently a number of papers have studied how moral hazard within the firm affects the pricing of its equity contract [Kahn (1990), Kocherlakota (1995), Shorish and Spear (1996)]: these are representative agent models modified to incorporate the effect of unobservable effort on produc-

[1] See Book III, Chapter II of the *Wealth of Nations* for a criticism of the *metayer* system, the share system used in Continental Europe, by which the farmer and the landowner each obtained one half (*metarius*) of the output of the farm. See Book V, Part III for a vehement criticism of joint stock companies.

[2] See Book II, Chapters VI-VIII of *Principles of Political Economy* for a more balanced assessment of the metayer system and Book I, Chapter IX for a discussion of joint stock companies.

[3] See Book VI, Chapter X and Book IV, Chapter XII of *Principles of Economics.*

tion. The findings of Kocherlakota are similar in spirit to those of Section 4 — namely that, when trades are observable, moral hazard does not help to solve the equity premium puzzle.

The paper is organized as follows. Section 2 presents the basic model of a stock market economy with moral hazard and introduces the concept of an RCPP equilibrium. Section 3 analyzes it normative properties, while Section 4 presents examples of RCPP equilibria, contrasting them with the equilibria of a standard finance model.

2 Stock market equilibrium

The Model. Consider a two-period model of an economy with production, in which there is one good (income), and in which an investment of capital and effort at date 0 gives rise to an uncertain income stream at date 1, the uncertainty being modelled by states of nature ($s = 1, \ldots, S$). There are I agents: each agent i has an initial wealth w_0^i at date 0 and if agent i is an entrepreneur, by investing capital (an amount of the good (income)) and effort e^i at date 0 he can obtain the uncertain stream of income at date 1 given by

$$\boldsymbol{F}^i(z^i, e^i) = (F_1^i(z^i, e^i), \ldots, F_S^i(z^i, e^i)),$$

where $\boldsymbol{F}^i(\cdot)$ is an increasing function of (z^i, e^i) on $\mathcal{R}_+^2$. When agent i is an investor, we set $\boldsymbol{F}^i = 0$. Each agent has a utility function U^i where $U^i(\boldsymbol{x}^i, e^i)$ is the utility associated with the consumption stream $\boldsymbol{x}^i = (x_0^i, x_1^i, \ldots, x_S^i)$ and the effort level e^i. U^i, which is defined on the domain $\mathcal{R}_+^{S+1} \times \mathcal{R}_+$, is increasing in $\boldsymbol{x}^i$ and decreasing in e^i. Since the effort e^i of an investor is not productive, it will always be set equal to zero. Each agent is thus characterized by $(U^i, w_0^i, \boldsymbol{F}^i)$ and we let $\mathcal{E}(\boldsymbol{U}, \boldsymbol{w}_0, \boldsymbol{F})$ denote the resulting economy with characteristics $\boldsymbol{U} = (U^1, \ldots, U^I)$, $\boldsymbol{w}_0 = (w_0^1, \ldots, w_0^I)$, $\boldsymbol{F} = (\boldsymbol{F}^1, \ldots, \boldsymbol{F}^I)$.

The characteristics of the economy $\mathcal{E}(\boldsymbol{U}, \boldsymbol{w}_0, \boldsymbol{F})$ satisfy the following additional assumptions. Agents' utility functions are separable

$$U^i(\boldsymbol{x}^i, e^i) = u_0^i(x_0^i) + u_1^i(x_1^i, \ldots, x_S^i) - c^i(e^i),$$

where the functions u_0^i, u_1^i are strictly concave increasing, and c^i is convex increasing. These functions are differentiable on their domains and satisfy the boundary conditions[4]

$$u_0^{i\prime}(x_0^i) \longrightarrow \infty \text{ if } x_0^i \longrightarrow 0,$$

$$\| \nabla u_1^i(\boldsymbol{x}_1^i) \| \longrightarrow \infty \text{ if } \boldsymbol{x}_1^i \longrightarrow \partial R_+^S, \text{ and } c^{i\prime}(0) = 0.$$

[4] $\nabla u_1^i = \left(\frac{\partial u_1^i}{\partial x_1^i}, \ldots, \frac{\partial u_1^i}{\partial x_S^i}\right)$ denotes the gradient of u_1^i and $\partial \mathcal{R}_+^S$ is the boundary of the non-negative orthant of $\mathcal{R}^S$.

In short, consumption is essential in all states and effort is essentially costless for small levels of effort.

On the production side, we assume that the production functions have the multiplicative form

$$\boldsymbol{F}^i(z^i, e^i) = f^i(z^i, e^i)\boldsymbol{\eta}^i, \tag{1}$$

the function f^i expressing the specific ability of agent i for transforming an initial investment of capital and effort (z^i, e^i) into a profit stream at date 1. To permit the same notation to be used for both investors and entrepreneurs, we adopt the convention that $\boldsymbol{\eta}^i \equiv 0$ if agent i is an investor. $f^i(z^i, e^i)$ is assumed to be a differentiable, increasing function of (z^i, e^i) which satisfies $f^i(0, e^i) = f^i(z^i, 0) = 0$ (both inputs are essential). While f^i is concave in z^i reflecting decreasing returns to capital, concavity in e^i is not needed as long as the marginal cost of effort increases faster than its marginal product (see Assumption MCMP(a) below).

The multiplicative factor structure[5] in (1) was first introduced by Diamond (1967). Its principal advantage is that *it leads to a competitive pricing of the firms' risks which is well-defined even if the financial markets are incomplete.* By altering his actions (z^i, e^i), entrepreneur i can influence the expected value of the profit stream of his firm, but he cannot influence the risk profile $\boldsymbol{\eta}^i$ of the income stream that he sells, and thus the "risk price" of this basic income stream. More general risk structures for the production functions $\boldsymbol{F}^i$ would require more markets than the basic debt-equity markets to ensure that an entrepreneur has no influence on the structure of the financial markets. We leave this case for further analysis and adopt here the simplest framework in which the assumption of competitive pricing of risks is appropriate, concentrating on the new element introduced by incentives.

We accept as a fact that the complexity of business risks, when combined with the unobservability of entrepreneurial effort, makes the writing and enforcement of contracts contingent on states unfeasible. The opportunities for sharing the production risks in the economy are those that can be obtained through shared ownership of the firms. Thus, there is a stock market on which entrepreneurs, who have the initial property rights to the profit streams of their firms (since this is the result of their effort and initiative) can sell a part of their ownership shares to obtain funds for capital investment, and can buy shares in other firms in order to diversify their risks. We assume that after selling ownership shares of their firms, entrepreneurs remain the sole managers of their firms even though they hold less than 100% of the shares: they are thus "owner-managers" in the sense of Jensen-Meckling (1976). In addition to obtaining funds by issuing

[5] This is in essence a nonlinear version of *activity analysis*, the vector $\boldsymbol{\eta}^i$ constituting the "activity" (income stream) of firm i (see Tjalling Koopmans (1951)).

equity, firms can also issue debt. To simplify the analysis the penalty for bankruptcy is assumed to be infinite: there is thus a single instrument traded on the bond market, which is the "default-free" bond.

To make clear how the timing of agents decisions takes place, date 0 is divided into two subperiods $0_1, 0_2$. In subperiod 0_1 entrepreneurs use the financial markets to obtain the capital required to set up their firms and to diversify their risks: in the second subperiod 0_2, after the investment and financing decisions have been made, firms become "operative" and entrepreneurs decide on the appropriate effort to invest in the running of their firms. At date 1 "nature" chooses a state of the world (shock): production takes place and profit is realized.

In subperiod 0_1 entrepreneur i decides on the amount of capital z^i to invest in his firm, on the amount to borrow b^i (if $b^i > 0$, lend if $b^i < 0$), on the share $(1-\theta_i^i)$ of his firm to sell and on the shares θ_k^i of other firms $k \neq i$ to buy: let $\boldsymbol{\theta}^i = (\theta_1^i, \ldots, \theta_I^i)$ denote the agent's portfolio of equity contracts. Since we study the case in which financial markets are still relatively simple (debt and equity only), we assume that there are no short sales[6] so that $\boldsymbol{\theta}^i \in \mathcal{R}_+^I$. Let q_0 denote the price of the bond and let $\boldsymbol{Q} = (Q_1, \ldots, Q_I)$ denote the vector of prices of the firms' shares: thus Q_i is the price of full ownership of firm i, and if agent i is not an entrepreneur i.e., if $\boldsymbol{F}^i(z^i, e^i) = 0$, then $Q_i = 0$. The accountability of agent i requires that the following budget equations be satisfied

$$x_0^i = w_0^i + q_0 b^i - \sum_{k\neq i} \theta_k^i Q_k + (1-\theta_i^i) Q_i - z^i, \quad (2)$$

$$x_s^i = -b^i + \sum_{k\neq i} \theta_k^i f^k(z^k, e^k)\eta_s^k + \theta_i^i f^i(z^i, e^i)\eta_s^i, \quad s = 1, \ldots, S, \quad (3)$$

the consumption in each state being non-negative. If $\boldsymbol{x}_1^i = (x_1^i, \ldots, x_S^i)$ denotes the date 1 consumption stream, and if $\mathbf{1} = (1, \ldots, 1)$ denotes the riskless income stream at date 1, then the S equations in (3) can be written in the more condensed vector form

$$\boldsymbol{x}_1^i = -b^i \mathbf{1} + \sum_{k\neq i} \theta_k^i f^k(z^k, e^k)\boldsymbol{\eta}^k + \theta_i^i f^i(z^i, e^i)\boldsymbol{\eta}^i. \quad (4)$$

The agents' financial transactions $(z^i, b^i, \boldsymbol{\theta}^i)_{i=1}^I$ carried out in subperiod 0_1 are assumed to be mutually observable. Thus an investor who spends money buying shares of firm i, knows exactly how this money is used by entrepreneur i: how much is invested in the firm (z^i), how much goes to private consumption (x_0^i), etc.; he also knows agent i's sources of income at date 1, his debt payment $-b^i$, and the dividends he will receive from the different firms in the economy. What the investor cannot observe when buying his shares in firm i is the effort entrepreneur i will invest in his firm: this decision will be made by the entrepreneur in subperiod 0_2, and the best the investor can do is to form an expectation about what e^i will be.

[6] This is not essential. If short sales were allowed, to carry out the anlysis we would need to add the assumption that the income streams $\boldsymbol{\eta}^i$, for $\boldsymbol{\eta}^i \neq 0$, are linearly independent.

Optimal Effort Function. Consider how entrepreneur i chooses his optimal effort in subperiod 0_2. Given that this decision is made after the financing decision $(z^i, b^i, \boldsymbol{\theta}^i)$ has been chosen, the entrepreneur will choose the effort level e^i which maximizes $u_1^i(\boldsymbol{x}_1^i) - c^i(e^i)$, the date 1 consumption stream $\boldsymbol{x}_1^i$ being given by (4). If agent i correctly anticipates the effort of other entrepreneurs ($k \neq i$), then he will correctly anticipate what his date 1 *outside income stream* $\boldsymbol{m}^i$ will be, where

$$\boldsymbol{m}^i = \boldsymbol{m}^i(b^i, (\theta_k^i)_{k\neq i}) = -b^i\mathbf{1} + \sum_{k\neq i} \theta_k^i f^k(z^k, e^k)\boldsymbol{\eta}^k \quad (5)$$

The agent's choice of effort is thus the solution of the problem

$$\max_{e^i \geq 0} \left\{ u_1^i(\boldsymbol{m}^i + \theta_i^i f^i(z^i, e^i)\boldsymbol{\eta}^i) - c^i(e^i) \right\} \quad (\mathrm{E})$$

where the parameters $(\boldsymbol{m}^i, z^i, \theta_i^i) \in \mathcal{R}^S \times \mathcal{R}_+^2$ must be such that $\boldsymbol{m}^i + \theta_i^i f^i(z^i, e^i)\boldsymbol{\eta}^i \geq 0$ for some $e^i \geq 0$: let $\mathcal{D}$ denote this domain.

Assumption MCMP (marginal cost-marginal product).

(a) For all $z^i > 0, c^{i\prime}(\cdot)/\frac{\partial f^i(z^i,\cdot)}{\partial e^i}$ is increasing and tends to ∞ when $e^i \longrightarrow \infty$.

(b) There is a smooth path $e^i : [0,1] \longrightarrow \mathcal{R}_+$ with $e^i(0) = 0$ and $e^{i\prime}(t) > 0$ such that

$$\lim_{t\longrightarrow 0} \frac{\partial f}{\partial z^i}\left(t, e^i(t)\right) = \infty, \quad \lim_{t\longrightarrow 0} c^{i\prime}(e^i(t))e^{i\prime}(t) < \infty.$$

Assumption MCMP(a) ensures that the problem (E) has a unique solution, while MCPM(b) ensures that each entrepreneur's technology is sufficiently productive relative to his cost of effort to make it worthwhile to put his firm into operation: if the entrepreneur were to operate at $(z^i, e^i) = (0,0)$, there would be a way of slightly increasing capital ($z^i = t$) and effort ($e^i = e^i(t)$) so that the increase in marginal utility arising from the increase in output exceeds the marginal increase in the cost of effort.

Example. If $f^i(z^i, e^i) = (z^i)^\beta (e^i)^\gamma$ and $c^i(e^i) = (e^i)^\delta$, then MCMP(a) is satisfied if $\delta > \gamma$ and MCMP(b) is satisfied if $\delta > \frac{\gamma}{1-\beta}$. The higher the power δ, the flatter is the cost curve at zero, and the more readily MCMP is satisfied.

Proposition 1: *(i) If Assumption MCMP(a) is satisfied, then for each* $(\boldsymbol{m}^i, z^i, \theta_i^i) \in \mathcal{D}$ *the problem (E) has a unique solution*

$$\widetilde{e}^i(\boldsymbol{m}^i, z^i, \theta_i^i) = \arg\max_{e^i \geq 0} \{u_1^i(\boldsymbol{m}^i + \theta_i^i f^i(z^i, e^i)\boldsymbol{\eta}^i) - c^i(e^i)\} \qquad (6)$$

and $\widetilde{e}^i$ *is differentiable whenever* $\widetilde{e}^i(\boldsymbol{m}^i, z^i, \theta_i^i) > 0$.

(ii) If Assumption MCMP(b) holds, then for all $\boldsymbol{x}^i = (x_0^i, \boldsymbol{x}_1^i) \in \mathcal{R}_+^{S+1}$ *with* $x_0^i > 0$, *there exist* $(z^i, e^i) \gg 0$ *such that*

$$u_0^i(x_0^i - z^i) + u_1^i(\boldsymbol{x}_1^i + f^i(z^i, e^i)\boldsymbol{\eta}^i) - c^i(e^i) > u^i(x_0^i) + u^i(\boldsymbol{x}_1^i). \qquad (7)$$

Proof. (i) The first-order condition for the problem (E) is given by

$$\frac{c^{i\prime}(e^i)}{\frac{\partial f^i}{\partial e^i}(z^i, e^i)} \geq \theta_i^i \sum_{s=1}^{S} \frac{\partial u_1^i}{\partial x_s^i}\left(\boldsymbol{m}^i + \theta_i^i f^i(z^i, e^i)\boldsymbol{\eta}^i\right)\eta_s^i \qquad (8)$$

with equality if $e^i > 0$. Since $f^i(z^i, \cdot)$ is increasing, and $\frac{\partial u_1^i(\cdot)}{\partial x_s^i}$ is decreasing by concavity of u_1^i, the RHS of (8) is a decreasing function of e^i, while LHS is increasing. If at $e^i = 0$, LHS exceeds RHS then $e^i = 0$ is the solution: in the opposite case, since LHS goes to ∞ there is a unique $e^i > 0$ satisfying (8) with equality, and the differentiability of this solution follows by applying the Implicit Function Theorem, noting that the hypothesis of Proposition 1 implies $c^{i\prime\prime}/c^{i\prime} > \frac{\partial^2 f^i}{\partial^2 e^i} / \frac{\partial f^i}{\partial e^i}$.

(ii) Let ΔU^i denote the difference in utility in (7) between investing $(z^i, e^i(z^i))$ in activity i and investing $(0,0)$, where $e^i(\cdot)$ is the function defined in MCMP(b) and $z^i \leq \min\{x_0^i/2, 1\}$. Then

$$\begin{aligned}\Delta U^i &= -\int_0^{z^i} u_0^{i\prime}(x_0^i - t)dt - \int_0^{z^i} c^{i\prime}(e^i(t))e^{i\prime}(t)dt \\ &\quad + \int_0^{z^i} \nabla u_1^i(\boldsymbol{x}_1^i + f^i(t, e^i(t))\boldsymbol{\eta}^i) \times \\ &\quad \left[\frac{\partial f^i}{\partial z^i}(t, e^i(t)) + \frac{\partial f^i}{\partial e^i}(t, e^i(t))e^{i\prime}(t)\right]\boldsymbol{\eta}^i dt.\end{aligned}$$

Set $k = u_0^{i\prime}(x_0^i/2)$, $K = \nabla u_1^i(\boldsymbol{x}_1^i + f^i(1, e^i(1))\boldsymbol{\eta}^i) \cdot \boldsymbol{\eta}^i$. Then, since $\frac{\partial f}{\partial z^i} > 0$, $\frac{\partial f}{\partial e^i} > 0$, $e^{i\prime} > 0$ and u_0^i and u_1^i are concave

$$\Delta U^i \geq \int_0^{z^i} \left(K\frac{\partial f^i}{\partial z^i}(t, e^i(t)) - k - c^{i\prime}(e^i(t))e^{i\prime}(t)\right) dt.$$

By MCMP(b), for $z^i > 0$ sufficiently small this expression is positive. □

Note that by (5), the entrepreneur's outside income $\boldsymbol{m}^i$ is a function of his borrowing and of his equity shares in other firms $(b^i, (\theta_k^i)_{k \neq i})$, so that his optimal effort is well-defined once he has chosen his financial variables. We may thus use either the notation $\widetilde{e}^i(\boldsymbol{m}^i, z^i, \theta_i^i)$ as in (6) or $\widetilde{e}^i(z^i, b^i, \boldsymbol{\theta}^i)$ to denote an entrepreneur's optimal effort function.

Stock Market Equilibrium. Consider an investor who is thinking of buying shares of entrepreneur i's firm and can observe his financial decisions $(z^i, b^i, \boldsymbol{\theta}^i)$. It would be "irrational" for the investor not to use this information to deduce what the most likely effort of entrepreneur i will be. To be able to deduce $\widetilde{e}^i(z^i, b^i, \boldsymbol{\theta}^i)$, however, the investor would need to know in addition to the entrepreneurs financial decisions, his characteristics $(u_1^i, c^i, f^i, \boldsymbol{\eta}^i)$. In the analysis that follows we make the strong assumption that the agents' characteristics are common knowledge. Thus the investor can deduce from the financial variables $(z^i, b^i, \boldsymbol{\theta}^i)$ the effort that entrepreneur i will choose: in short, we suppose that every investor knows the entrepreneur's effort function $\widetilde{e}^i(z^i, b^i, \boldsymbol{\theta}^i)$. In practice agents will probably not have such a precise knowledge of other agents' characteristics — however they are likely to have a good idea of "what makes entrepreneurs tick". Experienced investors are not readily fooled: they are likely to predict that an entrepreneur who retains only a small share of his firm and has a lot of outside income will not exert much effort to make his firm productive.

If investors correctly anticipate, through the price they are prepared to pay for each firm i, the effect of the financial decisions of entrepreneur i on the effort that he invests in his firm, then it seems reasonable to suppose that each entrepreneur will come to understand this. Hence our second assumption: entrepreneurs know that investors will use their financial decisions as "signals" of the effort that they will exert in their firms. The next step is to incorporate these two assumptions into a concept of equilibrium.

The description of an equilibrium consists of two parts. The first is the standard part which enumerates the *actions* of the I agents, the *prices* of the $I+1$ securities and the mutual compatibility of their actions under these prices. The second part describes the entrepreneurs' *perceptions* of the way their financial decisions affect the price that the "market" will pay for the shares of their firms, and ensures that these perceptions are compatible with the equilibrium prices. Let

$$\widetilde{Q}_i : \mathcal{R}_+ \times \mathcal{R} \times \mathcal{R}_+^I \longrightarrow \mathcal{R}_+, \; i = 1, \ldots, I$$

denote the price perception of each entrepreneur i and let $\widetilde{\boldsymbol{Q}} = (\widetilde{Q}_1, \ldots, \widetilde{Q}_I)$. Thus $\widetilde{Q}_i(z^i, b^i, \boldsymbol{\theta}^i)$ denotes the price that entrepreneur i expects to receive if he sells the share $1 - \theta_i^i$ of his firm, when his other financial decisions are given by $(z^i, b^i, (\theta_k^i)_{k \neq i})$.

Definition 1 A *stock market equilibrium with price perceptions* $\widetilde{Q}$ is a triple

$$((\bar{\boldsymbol{x}}, \bar{\boldsymbol{e}}, \bar{\boldsymbol{z}}, \bar{\boldsymbol{b}}, \bar{\boldsymbol{\theta}}), (\bar{q}_0, \bar{\boldsymbol{Q}}); \widetilde{\boldsymbol{Q}})$$

consisting of actions, prices and price perceptions such that

(i) for each agent $i, (\bar{\boldsymbol{x}}^i, \bar{e}^i)$ maximizes $U^i(\boldsymbol{x}^i, e^i)$ among consumption-effort streams such that[7]

$$\begin{aligned} x_0^i &= w_0^i + q_0 b^i - \sum_{k \neq i} \bar{Q}_k \theta_k^i + \widetilde{Q}_i(z^i, b^i, \boldsymbol{\theta}^i)(1 - \theta_i^i) - z^i \\ \boldsymbol{x}_1^i &= -b^i \mathbf{1} + \sum_{k \neq i} \theta_k^i f^k(\bar{z}^k, \bar{e}^k) \boldsymbol{\eta}^k + \theta_i^i f^i(z^i, e^i) \boldsymbol{\eta}^i. \end{aligned}$$

for some $(z^i, b^i, \boldsymbol{\theta}^i) \in \mathcal{R}_+ \times \mathcal{R} \times \mathcal{R}_+^I$;

(ii) $\bar{Q}_i = \widetilde{Q}_i(\bar{z}^i, \bar{b}^i, \bar{\boldsymbol{\theta}}^i)$, $i = 1, \ldots, I$;

(iii) $\sum_{i=1}^I \bar{b}^i = 0$;

(iv) $\sum_{i=1}^I \bar{\theta}_k^i = 1, \quad k = 1, \ldots, I$

Thus in an equilibrium with price perceptions $\widetilde{\boldsymbol{Q}}$, each entrepreneur takes the prices and production plans of the other entrepreneurs as given, and correctly anticipates the effort they invest in their firms; he chooses his own actions, anticipating that those which are observable (his financial decisions) will influence the price that outside investors are prepared to pay for their shares in his venture, in the way indicated by the function $\widetilde{Q}_i(z^i, b^i, \boldsymbol{\theta}^i)$. By (ii), the price perceptions are consistent with the observed equilibrium prices $\bar{\boldsymbol{Q}}$, and by (iii) and (iv), the bond and equity markets clear.

Without more precise assumptions on the price perceptions $\widetilde{Q}_i(z^i, b^i, \boldsymbol{\theta}^i)$, this concept of equilibrium only incorporates the first assumption that we discussed above — namely that investors have correct expectations — but it does not yet explicitly incorporate the second — namely that entrepreneurs are fully aware of this fact. For example, the equilibrium concept in Definition 1 would be compatible with myopic expectations of the form $\widetilde{Q}_i(z^i, b^i, \boldsymbol{\theta}^i) = \bar{Q}_i, i = 1, \ldots, I$. At first glance this might seem like the natural candidate for a concept of "competitive" equilibrium. However, this is not a legitimate use of the assumption of price-taking behavior, since Q_i is not a "per-unit" price, but rather is the price of the whole firm. Competition means that agents take per-unit prices as given, independent of the amount that they supply to the market. The "good" sold by entrepreneur i to investors is the risk profile $\boldsymbol{\eta}^i$ that they can use for taking or diversifying risks, and we assume that entrepreneur i takes it s price as given. The notion of competition does not however explain how an entrepreneur should perceive that the "market" will evaluate the *personalized* part $f^i(z^i, e^i)$, namely the "amount" of $\boldsymbol{\eta}^i$ that we will supply when e^i is not observable. To answer this part, the concept of rational expectations is more appropriate than the concept of competition. We are thus led to the following concept of equilibrium.

[7]Whenever $\boldsymbol{k}$ is not an entrepreneur, since $\boldsymbol{F}^k(z^k, e^k) = \mathbf{0}$, the shares θ_k^i are fictitious: they are shares of the zero vector. In this case we set $\theta_i^i = 1$, $\theta_k^i = 0, i \neq k$, so that the market clearing condition (iv) can be written symmetrically for all agents.

Definition 2 A *stock market equilibrium with rational, competitive price perceptions* (RCPP) is an equilibrium $((\bar{\boldsymbol{x}}, \bar{\boldsymbol{e}}, \bar{\boldsymbol{b}}, \bar{\boldsymbol{\theta}})), (\bar{q}_0, \bar{\boldsymbol{Q}}; \widetilde{\boldsymbol{Q}})$ with price perceptions in which the perception functions satisfy the following condition: there exist prices $(\bar{q}_1, \ldots, \bar{q}_I)$ for the firms' basic income streams $\boldsymbol{\eta}^i$, $i = 1, \ldots, I$ such that for $i = 1, \ldots, I$,

$$\widetilde{Q}_i(z^i, b^i, \boldsymbol{\theta}^i) = \bar{q}_i f^i(z^i, \widetilde{e}^i(\boldsymbol{m}^i, z^i, \theta_i^i)), \tag{9}$$

$$\text{where } \boldsymbol{m}^i = -b^i \mathbf{1} + \sum_{k \neq i} \theta_k^i f^k(\bar{z}^k, \bar{e}^k) \boldsymbol{\eta}^k. \tag{10}$$

Thus to check if his financial decision $(\bar{z}^i, \bar{b}^i, \bar{\boldsymbol{\theta}}^i)$ at equilibrium is optimal, entrepreneur i forms expectations about what the price $\widetilde{Q}_i$ would be if he were to make an alternative financial decision $(z^i, b^i, \boldsymbol{\theta}^i)$. To form these expectations he takes the price $\bar{q}_i$ of one unit of his income stream $\boldsymbol{\eta}^i$ as given[8] , and calculates that the market price of his firm will be $t^i \bar{q}_i$, if the market anticipates his profit will be $t^i \boldsymbol{\eta}^i$. To evaluate $\boldsymbol{m}^i$ in (10) he takes as given the effort $\bar{e}^k$ that other entrepreneurs $(k \neq i)$ make given their financial choices $(\bar{e}^k = \widetilde{e}^k(\bar{\boldsymbol{m}}^k, \bar{b}^k, \bar{\theta}_k^k))$. This is the competitive part of his calculation.

To evaluate what the market anticipates his "output" t^i will be, he draws on his knowledge of investor rationality: he anticipates that the market will deduce from $(\boldsymbol{m}^i, z^i, \theta_i^i)$ what his optimal effort will be, and thus anticipates that t^i will be equal to $f^i(z^i, \widetilde{e}^i(\boldsymbol{m}^i, z^i, \theta_i^i))$. This is the rational expectations part of his calculation.

An RCPP equilibrium describes a situation where entrepreneurial effort is not observable, but where all participants on the market use all available information to deduce the likely values of the hidden (moral hazard) variables — and all agents know this: in short, there is common knowledge of

[8]Note that the "competitive" price $\bar{q}_i$ can be deduced from the observable market prices $\bar{Q}_i$, only if the firm of entrepreneur i is active. For if $f^i(\bar{z}^i, \bar{e}^i) > 0$ then (ii) in Definition 1 and (9) imply that $\bar{q}_i = \bar{Q}_i / f^i(\bar{z}^i, \bar{e}^i)$. However if $f^i(\bar{z}^i, \bar{e}^i) = 0$, then (ii) and (9) imply $\bar{Q}_i = 0$, so that $\bar{q}_i$ is indeterminate. In this latter case, the concept of equilibrium does not guarantee that the price $\bar{q}_i$ used by entrepreneur i to reach the decision $(\bar{z}^i, \bar{e}^i) = 0$ is "reasonable", since it does not correspond to an objective market signal. Assumption MCMP(b) avoids the conceptual difficulties that arise in these cases.

rationality.

3 Constrained efficiency

A well-known result of Diamond (1967) asserts that in a model similar to the one considered in this paper, but in which there are no incentive effects, the stock market leads to efficient investment and risk sharing, the efficiency being relative to the existing structure of securities — in short, he proved that a stock market equilibrium is constrained efficient. When the firms' profit functions $f^i(z^i, e^i)$ are independent of e^i, so that the effort variables are omitted, the model we are studying reduces to Diamond's model of the stock market. *Does the constrained efficiency result carry over to the more general version of the model in which entrepreneurs' incentives are explicitly taken into account?* Since the stock market cannot achieve risk sharing without distorting incentives, the question arises whether this trade-off is achieved in an optimal way at an equilibrium. In their attempt to diversify their risks, do outside shareholders acquire excessively large holdings in the firms, leading to undue distortion of the entrepreneurs' incentives to invest effort in their firms? Or, on the contrary, are the entrepreneurs unduly reluctant to sacrifice ownership shares in their profit streams, thus robbing other agents of potential opportunities for risk sharing? To answer these questions we need to generalize the concept of constrained efficiency introduced by Diamond to the context of this model. This means introducing a concept of constrained feasible allocations, which respects the limited trading opportunities achievable by a system of bond and equity markets, and in addition respects the incentive constraints imposed by the nonobservability of effort. Applying the Pareto ranking criterion to this constrained feasible set leads to the concept of a constrained Pareto optimum.

Definition 3 An allocation $(\boldsymbol{x}, \boldsymbol{e}) = (\boldsymbol{x}^i, e^i)_{i=1}^I$ is *constrained feasible* if there exist inputs and portfolios $(\boldsymbol{z}, \boldsymbol{b}, \boldsymbol{\theta}) = (z^i, b^i, \boldsymbol{\theta}^i)_{i=1}^I \in \mathcal{R}_+^I \times \mathcal{R}^I \times R_+^{II}$ such that

$$\sum_{i=1}^I x_0^i = \sum_{i=1}^I w_0^i - \sum_{i=1}^I z^i \quad (11)$$

$$\sum_{i=1}^I b^i = 0 \quad (12)$$

$$\sum_{i=1}^I \theta_k^i = 1, \ k = 1, \dots, I \quad (13)$$

and for each agent $i = 1, \dots, I$

$$\boldsymbol{x}_1^i = -b^i \mathbf{1} + \sum_{k=1}^I \theta_k^i f^k(z^k, e^k)\boldsymbol{\eta}^k \quad (14)$$

$$e^i = \tilde{e}^i(\boldsymbol{m}^i, z^i, \theta_i^i), \qquad \boldsymbol{m}^i = -b^i\mathbf{1} + \sum_{k \neq i} \theta_k^i f^k(z^k, e^k)\boldsymbol{\eta}^k \quad (15)$$

An allocation $(\boldsymbol{x}, \boldsymbol{e})$ is *constrained Pareto optimal* (CPO), if it is constrained feasible, and if there does not exist any alternative constrained feasible allocation $(\widehat{\boldsymbol{x}}, \widehat{\boldsymbol{e}})$ such that $U^i(\widehat{\boldsymbol{x}}^i, \widehat{e}^i) \geq U^i(\boldsymbol{x}^i, e^i), i = 1, \dots, I$ with strict equality for at least one i.

Constrained Efficiency of Stock Market. We can think of a CPO allocation as being achieved by a "planner" who chooses the variables which, in equilibrium, are determined by trade on markets, with the objective of maximizing social welfare. Here the planner chooses the variables[9] $(x_0^i, z^i, b^i, \boldsymbol{\theta}^i)$. The implicit assumption which limits the planners instruments to $(x_0^i, z^i, b^i, \boldsymbol{\theta}^i)$ is that he cannot remove the observational constraints of the model, which limit the instruments for risk sharing and make entrepreneurs' effort impossible to control directly: in particular, the planner has to respect the fact that entrepreneurs will personally choose their effort levels based on the incentives created by his choice of investment-portfolio variables $(z^i, b^i, \boldsymbol{\theta}^i)$. Proving that an equilibrium is CPO thus amounts to showing that, given the observational constraints of the model, there is no way of improving the trade-off between risk sharing and incentives that results from decentralized trade on the markets. In short, even if a "planner" replaces "markets", he cannot improve on the allocation.

Proposition 2: *If $\mathcal{E}(\boldsymbol{u}, \boldsymbol{w}_0, \boldsymbol{F})$ is an economy satisfying the assumptions of Section 2, then every RCPP equilibrium is constrained Pareto optimal.*

Proof. If the equilibrium $((\bar{\boldsymbol{x}}, \bar{\boldsymbol{e}}, \bar{\boldsymbol{z}}, \bar{\boldsymbol{b}}, \bar{\boldsymbol{\theta}}), (\bar{q}_0, \bar{\boldsymbol{Q}}); \widetilde{\boldsymbol{Q}})$ is not CPO, then there is a constrained feasible allocation $(\boldsymbol{x}, \boldsymbol{e}, \boldsymbol{z}, \boldsymbol{b}, \boldsymbol{\theta})$ satisfying (11)-(15) such that $u^i(\boldsymbol{x}^i, e^i) \geq u^i(\bar{\boldsymbol{x}}^i, \bar{e}^i), i = 1, \dots, I$ with strict inequality for at least one i. By Proposition 1, Assumption MCMP(b) implies that in an equilibrium all entrepreneurs invest a positive amount of capital and effort in the sector in which they are productive: as a result all income streams $\boldsymbol{\eta}^i$, with $\boldsymbol{\eta}^i \neq 0$, are traded in an equilibrium. Thus the date 1 consumption stream

$$\boldsymbol{x}_1^i = -b^i\mathbf{1} + \sum_{k \neq i} \theta_k^i f^k(z^k, e^k)\boldsymbol{\eta}^k + \theta_i^i f^i(z^i, e^i)\boldsymbol{\eta}^i$$

would have been available to agent i, (when he in fact chose the equilibrium consumption $\bar{\boldsymbol{x}}_1^i$) had he chosen the investment, debt and ownership in his own firm (z^i, b^i, θ_i^i), and the portfolio of shares in other firms $(\widetilde{\theta}_k^i)_{k \neq i}$ given

[9] In order to express the fact that the planner replaces "markets", he must not have to worry about prices or respecting agents' budget constraints and thus has to be able to choose the date 0 consumption x_0^i of agents directly, subject only to the aggregate feasibility constraint (11).

by

$$\widetilde{\theta}^i_k f^k(\bar{z}^k, \bar{e}^k) = \theta^i_k f^k(z^k, e^k).$$

Given the outside income $\boldsymbol{m}^i$ derived from debt and other firms' securities, by constrained optimality, his choice of effort $e^i = \widetilde{e}^i(\boldsymbol{m}^i, z^i, \theta^i_i)$ would then have been optimal. Since $(\boldsymbol{x}^i, e^i)$ is preferred or indifferent for all agents and strictly preferred by at least one agent, the date 0 consumption must be at least as expensive, and strictly more for some agent: thus

$$x^i_0 \geq w^i_0 + \bar{q}_0 b^i - \sum_{k \neq i} \bar{Q}_k \widetilde{\theta}^i_k + \widetilde{Q}_i(z^i, b^i, (\widetilde{\theta}^i_k)_{k \neq i}, \theta^i_i)(1 - \theta^i_i) - z^i, i = 1, \ldots, I \tag{16}$$

with strict inequality for some i. Note that by (9), and (ii) in Definition 1,

$$\bar{Q}_k \widetilde{\theta}^i_k = \bar{q}_k f^k(\bar{z}^k \bar{e}^k) \widetilde{\theta}^i_k = \bar{q}_k f^k(z^k, e^k) \theta^i_k, \tag{17}$$

$$\widetilde{Q}_i(z^i, b^i, (\widetilde{\theta}^i_k)_{k \neq i}, \theta^i_i) = \bar{q}_i f^i(z^i, e^i). \tag{18}$$

Summing (16) over i, using (17) and (18) gives

$$\sum_{i=1}^{I} x^i_0 > \sum_{i=1}^{I} w^i_0 + \bar{q}_0 \sum_{i=1}^{I} b^i - \sum_{i=1}^{I} \bar{q}_i f^i(z^i, e^i) \left(\sum_{k=1}^{I} \theta^k_i - 1 \right) - \sum_{i=1}^{I} z^i. \tag{19}$$

By feasibility $\sum_{i=1}^{I} b^i = 0$ and $\sum_{k=1}^{I} \theta^k_i = 1$, $i = 1, \ldots, I$. But then (19) implies $\sum_{i=1}^{I} x^i_0 > \sum_{i=1}^{I} w^i_0 - \sum_{i=1}^{I} z^i$, contradicting the constrained feasibility of $(\boldsymbol{x}, \boldsymbol{e}, \boldsymbol{z}, \boldsymbol{b}, \boldsymbol{\theta})$. □

The standard framework for studying the optimal trade-off between risk sharing and incentives is the setting of a principal-agent problem. It is thus of some interest to note that the planner's problem of finding a CPO can be expressed as a generalized principal-agent problem. A principal (the planner), who can be thought of as owning all the resources, looks for a way of rewarding agents in the economy through the choice of consumption, investment and portfolio variables, so as to maximize a weighted sum of the agents' utilities under constraints which limit the risk-sharing possibilities at date 1 (constraints (14)), the incentive constraints (15), and subject to a reservation level of utility for himself equal to zero. This latter constraint can be expressed as the fact that the principal appropriates no resources of the economy for himself, and is thus equivalent to the resource availability constraints (11)-(13). If the principal wanted to decentralize the solution to his social welfare problem by providing agents with incentive contracts, then he would have to solve the following *contract design problem*: find functions $\phi^i : \mathcal{R}_+ \times \mathcal{R} \times \mathcal{R}^I_+ \longrightarrow \mathcal{R}$ such that a Nash equilibrium of the game with strategies $(z^i, b^i, \boldsymbol{\theta}^i, e^i)$ for the agents $(i = 1, \ldots, I)$ and payoffs $V^i(\boldsymbol{z}, \boldsymbol{b}, \boldsymbol{\theta}, \boldsymbol{e})$ where

$$V^i(\boldsymbol{z}, \boldsymbol{b}, \boldsymbol{\theta}, \boldsymbol{e}) = u^i_0(\phi^i(z^i, b^i, \boldsymbol{\theta}^i)) + u^i_1 \left(b^i \mathbf{1} + \sum_{i=1}^{I} \theta^i_k f^k(z^k, e^i) \boldsymbol{\eta}^k \right) - c^i(e^i)$$

is a CPO allocation. Proposition 2 asserts that the market provides a solution to this contract design problem given by

$$\phi^i(z^i, b^i, \boldsymbol{\theta}^i) = \omega^i_0 + \bar{q}_0 b^i - \sum_{k \neq i} \bar{Q}_k \theta^i_k + \widetilde{Q}^i(z^i, b^i, \boldsymbol{\theta}^i)(1 - \theta^i_i) - z^i$$

where $(\bar{q}_0, \bar{\boldsymbol{Q}}, \widetilde{\boldsymbol{Q}})$ are the prices and price perceptions of an RCPP equilibrium of the economy $\mathcal{E}(\boldsymbol{U}, \boldsymbol{\omega}_0, \boldsymbol{F})$. Note that the contracts ϕ^i are linear for investors and nonlinear for entrepreneurs.

To obtain an intuitive understanding for the way in which the market solves the contract design problem, it is useful to compare the first-order conditions for constrained optimality with the first-order conditions in an RCPP equilibrium.

First-Order Conditions for CPO. In view of the boundary assumptions on the utility functions and assumption MCMP, at a CPO all the variables $\boldsymbol{x}^i$ are positive and, for entrepreneurs, the variables (z^i, e^i) are also positive. The only non-negativity constraints which need to be taken into account in deriving the first-order conditions (FOC) are the no-short-sales constraints $\theta^i_k \geq 0$. The FOC are more convenient to derive if the variables $(\boldsymbol{x}, \boldsymbol{e}; \boldsymbol{z}, \boldsymbol{b}, \boldsymbol{\theta})$ are replaced by the variables $(\boldsymbol{x}, \boldsymbol{e}; \boldsymbol{z}, \boldsymbol{b}, ((\mu^i_k)_{k \neq i}, \theta^i_i)^I_{i=1})$ where the relation between the two sets of variables is given by

$$\mu^i_k = \theta^i_k f^k(z^k, e^k), \quad i \neq k.$$

The new variables $(\mu^i_k)_{k \neq i}$ reflect the fact that the production of firm k affects agent i only in so far as it affects his outside income $\boldsymbol{m}^i$. In these new variables an allocation $(\boldsymbol{x}, \boldsymbol{e})$ is constrained feasible if there exist $(\boldsymbol{z}, \boldsymbol{b}, ((\mu^i_k)_{k \neq i}, \theta^i_i)^I_{i=1}) \in \mathcal{R}^I_+ \times \mathcal{R}^I \times \mathcal{R}^{II}_+$ such that

$$\sum_{i=1}^{I} x^i_0 = \sum_{i=1}^{I} w^i_0 - \sum_{i=1}^{I} z^i, \tag{20}$$

$$\sum_{i=1}^{I} b^i = 0, \tag{21}$$

$$\sum_{k \neq i} \mu^k_i \leq (1 - \theta^i_i) f^i(z^i, e^i), \quad i = 1, \ldots, I, \tag{22}$$

and for each agent $i = 1, \ldots, I$

$$\boldsymbol{x}^i_1 = -b^i \mathbf{1} + \sum_{k \neq i} \mu^i_k \boldsymbol{\eta}^k + \theta^i_i f^i(z^i, e^i) \boldsymbol{\eta}^i, \tag{23}$$

$$e^i = \tilde{e}^i(-b^i\mathbf{1} + \sum_{k\neq i}\mu^i_k\boldsymbol{\eta}^k, z^i, \theta^i_i). \tag{24}$$

A constrained Pareto optimal allocation is a solution of the problem

$$\max \sum_{i=1}^{I} \nu^i \left(u^i_0(x^i_0) + u^i_1(\boldsymbol{x}^i_1) - c^i(e^i)\right),$$

subject to the constraints (20)-(24), where ν_i is the relative weight attached to the utility of agent i. To express the cost of each constraint in units of date 0 consumption, we divide all the multipliers by the multiplier λ_0 induced by the date 0 constraint (20). This gives a set of normalized multipliers $(1, q_0, (q^i, \pi^i, \epsilon^i)^I_{i=1})$ associated respectively with each of the constraints (20)-(24), where $\pi^i = (\pi^i_1, \ldots, \pi^i_S)$. The first-order conditions with respect to the variables $(\boldsymbol{x}^i, e^i, z^i, b^i, \mu^i_k, \theta^i_i)$ of an entrepreneur i are

$$\frac{\partial u^i_1/\partial x^i_s}{u^{i\prime}_0} = \pi^i_s,\ s = 1, \ldots, S; \tag{25}$$

$$\frac{c^{i\prime}}{u^{i\prime}_0} = \left((1-\theta^i_i)\, q^i + \theta^i_i \boldsymbol{\pi}^i \cdot \boldsymbol{\eta}^i\right) \frac{\partial f^i}{\partial e^i} - \epsilon^i; \tag{26}$$

$$1 = \left((1-\theta^i_i)\, q^i + \theta^i_i \boldsymbol{\pi}^i \cdot \boldsymbol{\eta}^i\right) \frac{\partial f^i}{\partial z^i} + \epsilon^i \frac{\partial \tilde{e}^i}{\partial z^i}; \tag{27}$$

$$q_0 = \boldsymbol{\pi}^i \cdot \mathbf{1} + \epsilon^i \nabla_{m^i}\tilde{e}^i \cdot \mathbf{1}; \tag{28}$$

$$q^k \geq \boldsymbol{\pi}^i \cdot \boldsymbol{\eta}^k + \epsilon^i \nabla_{m^i}\tilde{e}^i \cdot \boldsymbol{\eta}^k, \quad k \neq i; \tag{29}$$

$$q^i = \boldsymbol{\pi}^i \cdot \boldsymbol{\eta}^i + \epsilon^i \frac{1}{f^i}\frac{\partial \tilde{e}^i}{\partial \theta^i_i}; \tag{30}$$

where $\nabla_{m^i}\tilde{e}^i$ is the vector of partial derivatives (the gradient) of the effort function $\tilde{e}^i(\boldsymbol{m}^i, z^i, \theta^i_i)$ with respect to $\boldsymbol{m}^i = (m^i_1, \ldots, m^i_S)$ and where (29) holds with equality if $\mu^i_k > 0$. To these equations should be added the FOC for the choice of optimal effort by entrepreneur i

$$c^{i\prime}(e^i) = \theta^i_i \nabla u^i_1 \left(\boldsymbol{m}^i + \theta^i_i f^i(z^i, e^i)\boldsymbol{\eta}^i\right) \cdot \boldsymbol{\eta}^i \frac{\partial f^i}{\partial e^i}(z^i, e^i)$$

This is just the marginal way of expressing the incentive constraint $e^i = \tilde{e}^i(\cdot)$ in (24). Dividing this equation by $u^{i\prime}_0$ to make it comparable with (25) - (30) gives

$$\frac{c^{i\prime}}{u^{i\prime}_0} = \theta^i_i \boldsymbol{\pi}^i \cdot \boldsymbol{\eta}^i \frac{\partial f^i}{\partial e^i}. \tag{31}$$

The first-order conditions with respect to the variables $(\boldsymbol{x}^i, \mu^i_k)$ of an investor are (25) and

$$q_0 = \boldsymbol{\pi}^i \cdot \mathbf{1} \tag{28'}$$

$$q^k \geq \boldsymbol{\pi}^i \cdot \boldsymbol{\eta}^k \quad (= \text{if } \mu^i_k > 0), \quad k \neq i. \tag{29'}$$

Economic Interpretation of FOC. Equation (25) defines the present-value vector $\boldsymbol{\pi}^i = (\pi_i, \ldots, \pi^i_S)$ of agent i: for any date 1 income stream $\boldsymbol{v} = (v_1, \ldots, v_S)$, $\boldsymbol{\pi}^i \cdot \boldsymbol{v}$ is the *present value* to agent i of the income stream $\boldsymbol{v}$. The variables $(q_0, q^1, \ldots, q^I)$ are the social values (*shadow prices*) of the income streams (securities) $(\mathbf{1}, \boldsymbol{\eta}^1, \ldots, \boldsymbol{\eta}^i)$. ϵ^i, which is the social cost of the incentive constraint (24), is the *social value of (one unit of) effort by agent i*. The equations (28) - (30) and (28′) - (29′), i.e., the first-order conditions with respect to $(b^i, \mu^i_k, \theta^i_i)$, express the limited sense in which there must be equalization of marginal rates of substitution to achieve a CPO allocation, full equalization being prevented by the fact that income can only be distributed indirectly using securities, and that the incentive constraints of the agents must be satisfied.

For each security, $\mathbf{1}$ or $\boldsymbol{\eta}^k$, the private benefit to agent i of an additional (marginal) unit of the security is $\boldsymbol{\pi}^i \cdot \mathbf{1}$ or $\boldsymbol{\pi}^i \cdot \boldsymbol{\eta}^k$. If agent i is an investor, then the private benefit coincides with the social benefit and (28′) and (29′) express the equalization of social (marginal) benefit and social (marginal) cost — these are the standard FOC for an optimal portfolio problem. Suppose now that agent i is an entrepreneur and $i \neq k$. An additional unit of security $\mathbf{1}$ or $\boldsymbol{\eta}^k$ creates more than just a *direct* marginal benefit: since the agent is an entrepreneur, an increase in his outside income has an *indirect* effect — for it changes his effort by $\Delta e^i = \nabla_{m^i}\tilde{e}^i \cdot \mathbf{1}$ or $\nabla_{m^i}\tilde{e}^i \cdot \boldsymbol{\eta}^k$, and since this effort has a social value ϵ^i, the social value of this indirect effect is $\epsilon^i \Delta e^i$. If $i \neq k$, in order for agent i to receive an additional unit of the security of his own firm, his holding $\theta^i_i f^i(z^i, e^i)$ must increase by one unit: this is equivalent to increasing θ^i_i by $\frac{1}{f^i}$. This increase in the shareholding of his own firm increases[10] his effort by $(1/f^i)\left(\partial\tilde{e}^i/\partial\theta^i_i\right)$, the social value of which is $\epsilon^i\,(1/f^i)\left(\partial\tilde{e}^i/\partial\theta^i_i\right)$. Thus (28)-(30) express equalization at the margin of the social cost and the social benefit of allocating an additional unit of $\mathbf{1}, \boldsymbol{\eta}^k$ or $\boldsymbol{\eta}^i$ to entrepreneur i where the social benefit is equal to the private benefit to the entrepreneur minus the indirect social cost of his changed effort.

The social value ϵ^i of an additional unit of effort by entrepreneur i is defined by equation (26) which can be written as

$$\epsilon^i = \left(\theta^i_i \boldsymbol{\pi}^i \cdot \boldsymbol{\eta}^i \frac{\partial f^i}{\partial e^i} + (1-\theta^i_i)\, q^i \frac{\partial f^i}{\partial e^i}\right) - \frac{c^{i\prime}}{u^{i\prime}_0}, \tag{26'}$$

ϵ^i is the difference between the social marginal benefit $\theta^i_i \boldsymbol{\pi}^i \cdot \boldsymbol{\eta}^i \frac{\partial f^i}{\partial e^i} + (1 - \theta^i_i)\, q^i \frac{\partial f^i}{\partial e^i}$, namely the benefit to entrepreneur i plus the benefit to "outside investors" who receive the share $(1 - \theta^i_i)$ of his output, and the social

[10] In the text we take the most intuitive case where $\partial\tilde{e}^i/\partial\theta^i_i > 0$ i.e., increased ownership leads to increased effort. It can happen, when b^i is sufficiently large, that income effects make this term negative (see Section 4).

marginal cost, which here coincides with the private cost $c^{i\prime}/u_0^{i\prime}$, since entrepreneur i is the only one to bear the cost of his effort. Since effort is chosen optimally by entrepreneur i, by the "envelope theorem", or more precisely by the FOC (31), the welfare effect on the entrepreneur of a marginal change in his effort is zero. Substituting (31) into (26′) gives

$$\epsilon^i = (1-\theta_i^i)\, q^i \frac{\partial f^i}{\partial e^i} \tag{32}$$

The social value of an additional unit of effort by entrepreneur i is the value to agents other than himself of the additional output that this effort would create[11]*: thus $\epsilon^i > 0$ $(=0)$ if and only if $\theta_i^i < 1$ $(=1)$.* When $\theta_i^i < 1$ the effort of entrepreneur i affects all those agents j who obtain a share of his profit stream: there is thus an *external effect.* The incentive constraint implies that this external effect is not taken into account when agent i makes his effort decision and this creates a cost ϵ^i, which is the cost of separating ownership and control. This cost is however explicitly taken into account by the planner when he chooses $(z^i, b^i, \boldsymbol{\theta}^i)$.

The logic underlying the FOC (27) for the socially optimal investment in firm i should now be clear: the social cost of one unit of investment at date 0 must equal the direct social benefit (the first term on RHS of (27)) plus the indirect social benefit $\left(\epsilon^i\, \partial\tilde{e}^i/\partial z^i\right)$ from the increased effort by agent i induced by this increment to the capital input of his firm.

How the FOC for CPO are Achieved at Equilibrium. Since a stock market equilibrium is constrained Pareto optimal, entrepreneurs must — just like the planner in a CPO problem — be induced to take into account the external effect of their effort on the welfare of others, namely the terms in ϵ^i in equations (26)-(30). In the standard model of competitive equilibrium, where prices are assumed to be independent of the quantities chosen, the price system cannot cope efficiently with externalities. However, in an RCPP equilibrium, there is a "non-competitive" part, namely the rational-anticipations component of the perception function $\widetilde{Q}$: while entrepreneurs take the prices $(q_i)_i^I$ of the factors $\boldsymbol{\eta}^i$ as given, they recognize that the price that the market will pay for their shares depends on investors' expectations of the effort that they will make. Since investors can deduce from the entrepreneurs' financial decisions what their effort will be, financial decisions end up playing the role of signals: in the process of choosing their "signals", entrepreneurs are led to internalize the externality.

The way in which the price perceptions force entrepreneurs to internalize the externality, can be clearly understood by matching the FOC at an equilibrium with the FOC for a CPO allocation. Consider the maximum

[11] Note that their benefit is evaluated using q^i, and not $\boldsymbol{\pi}^j \boldsymbol{\eta}^i$ for $j \neq i$, and thus incorporates the incentive cost of giving them a marginal increment in the income stream $\boldsymbol{\eta}^i$.

problem of an entrepreneur in a stock market equilibrium ((i) in Definition 1). Let $\bar{\boldsymbol{\lambda}}^i = (\bar{\lambda}_0^i, \bar{\lambda}_1^i, \ldots, \bar{\lambda}_S^i) \in \mathcal{R}_+^{S+1}$ denote the vector of multipliers induced by the $S+1$ budget constraints: the normalized vector

$$\bar{\boldsymbol{\pi}}^i = \frac{1}{\bar{\lambda}_0^i}(\bar{\lambda}_1^i, \ldots, \bar{\lambda}_S^i) = (\bar{\pi}_1^i, \ldots, \bar{\pi}_S^i)$$

is the present-value vector of agent i at the equilibrium. The first-order conditions are

$$\frac{\partial u_1^i/\partial x_s^i}{u_0^{i\prime}} = \bar{\pi}_s^i, \quad s = 1, \ldots, S; \tag{33}$$

$$\frac{c^{i\prime}}{u_0^{i\prime}} = \bar{\theta}_i^i\, \bar{\boldsymbol{\pi}}^i \cdot \boldsymbol{\eta}^i \frac{\partial f^i}{\partial e^i}; \tag{34}$$

$$1 = \bar{\theta}_i^i\, \bar{\boldsymbol{\pi}}^i \cdot \boldsymbol{\eta}^i \frac{\partial f^i}{\partial z^i} + (1-\bar{\theta}_i^i)\frac{\partial \widetilde{Q}_i}{\partial z^i}; \tag{35}$$

$$q_0 = \bar{\boldsymbol{\pi}}^i \cdot \mathbf{1} + (1-\bar{\theta}_i^i)\frac{\partial \widetilde{Q}_i}{\partial b^i}; \tag{36}$$

$$\bar{Q}_k \geq \bar{\boldsymbol{\pi}}^i \cdot \boldsymbol{\eta}^k f^k + (1-\bar{\theta}_i^i)\frac{\partial \widetilde{Q}_i}{\partial \theta_k^i}, \quad (= \text{if } \bar{\theta}_k^i > 0),\ k \neq i; \tag{37}$$

$$\bar{Q}_i = \bar{\boldsymbol{\pi}}^i \cdot \boldsymbol{\eta}^i f^i + (1-\bar{\theta}_i^i)\frac{\partial \widetilde{Q}_i}{\partial \theta_i^i}. \tag{38}$$

By paying attention to the way potential shareholders react to his financial decisions $(z^i, b^i, \boldsymbol{\theta}^i)$, through the partial derivatives $(\partial\widetilde{Q}_i/\partial z^i$, etc...), entrepreneur i is led to take their interests into account. With the rational, competitive price perceptions $\widetilde{Q}_i$ defined by (9), these partial derivatives are given by

$$\frac{\partial \widetilde{Q}_i}{\partial z^i} = \bar{q}_i \frac{\partial f^i}{\partial z^i} + \bar{q}_i \frac{\partial f^i}{\partial e^i}\frac{\partial \tilde{e}^i}{\partial z^i}; \tag{39}$$

$$\frac{\partial \widetilde{Q}_i}{\partial b^i} = \bar{q}_i \frac{\partial f^i}{\partial e^i} \nabla_{m^i}\tilde{e}^i \cdot \mathbf{1}; \tag{40}$$

$$\frac{\partial \widetilde{Q}_i}{\partial \theta_k^i} = \bar{q}_i \frac{\partial f^i}{\partial e^i} \nabla_{m^i}\tilde{e}^i \cdot \boldsymbol{\eta}^k f^k, \quad k \neq i; \tag{41}$$

$$\frac{\partial \widetilde{Q}_i}{\partial \theta_i^i} = \bar{q}_i \frac{\partial f^i}{\partial e^i}\frac{\partial \tilde{e}^i}{\partial \theta_i^i}. \tag{42}$$

Substituting (39) - (42) into (33) - (38), and setting $q^i = \bar{q}_i$, $\epsilon^i = (1-\bar{\theta}_i^i)\, \bar{q}_i \frac{\partial f^i}{\partial e^i}$ for $i = 1, \ldots, I$, gives the FOC (25)-(31) for a constrained Pareto optimal allocation.

In letting himself be guided by the price perceptions $\widetilde{Q}_i(z^i, b^i, \boldsymbol{\theta}^i)$, an entrepreneur understands, for example, that if he doubles the share $(1-\theta_i^i)$

of his firm that he sells, this will not double the income he receives: for shareholders know that when his ownership share falls, the effort that the entrepreneur will invest in his firm will fall, and this is reflected in the smaller price $\widetilde{Q}_i$ that shareholders will pay for the shares. He also knows that if he uses the proceeds of the sale for personal consumption or to buy shares in other firms, he will get less than if he uses the proceeds to finance capital expenditure for the firm.

There is an interesting connection between Proposition 2 and the conditions for constrained (second best) optimality in an insurance market with moral hazard (Hellwig (1983), Henriet-Rochet (1991), Lisboa (1996)). In the insurance models, nonlinear prices are needed to obtain constrained optimality, and in such models the insurance companies are the natural intermediaries for implementing such "second-best optimal" nonlinear pricing. In the stock market, price perceptions induce nonlinear prices: thus rational behavior and anticipation on the part of agents can act as an alternative mechanism for achieving constrained efficiency to having intermediaries that charge explicit nonlinear prices.[12]

4 Qualitative properties of stock market equilibria

In this section we examine how equilibria with incentives differ from the familiar financial market equilibria based on risk sharing. The results which are summarized in Tables 1-4 show two types of equilibria for economies with the following characteristics: there are three (types of) agents, two entrepreneurs (agents 1 and 2) and one investor (agent 3); there are three states of nature of equal probability, and agents have additively separable utility functions

$$U^i(x_0, x_1, x_2, x_3, e) = v^i(x_0) + \delta_i \sum_{s=1}^{3}(1/3)v^i(x_s) - c^i(e)$$

$$\begin{aligned} v^i(x) &= \sqrt{x - a_i},\ a_1 = a_2 = 0,\ a_3 = 50, \\ \delta_i &= 0.9,\ c^i(e) = \beta e^\gamma,\ \beta = 1.8,\ \gamma = 2 \end{aligned}$$

Thus the utility functions for date 1 consumption are expected discounted utility, with v^i taken from the LRT (linear risk tolerance) family.[13] All

[12] In practice the underwriters who undertake to float an issue of shares on behalf of a firm help to make clear to the company how the market is going to evaluate their issue of shares. From the perspective of our model, in addition to matching supply and demand, their role is to help "entrepreneurs" to form rational, competitive price perceptions.

[13] For an expected utility function $E(v(x))$, the *risk tolerance* is defined by $T(x) = -v'(x)/v''(x)$. The function v is in the LRT family if $T(x) = A + Bx$. A is the intercept and B is the coefficient of *marginal risk tolerance*. Here $A_1 = A_2 = 0$, $A_3 = -100$ and $B_i = 2$ for all agents.

agents have the same coefficient of *marginal risk tolerance* (equal to 2) and agent 3, with a negative intercept, is less risk tolerant than the others. The entrepreneurs' production possibilities are given by

$$\boldsymbol{F}^i(z,e) = (z)^{1/2}(e)^{3/4}\boldsymbol{\eta}^i \quad \text{with} \quad \boldsymbol{\eta}^1 = \begin{bmatrix} 22 \\ 15 \\ 8 \end{bmatrix},\ \boldsymbol{\eta}^2 = \begin{bmatrix} 30 \\ 10 \\ 14 \end{bmatrix}.$$

Thus activity 1 with mean $E(\boldsymbol{\eta}^1) = 15$ and standard deviation $\sigma(\boldsymbol{\eta}^1) = 5.7$ is less productive, but less risky, than activity 2, for which $E(\boldsymbol{\eta}^2) = 18$ and $\sigma(\boldsymbol{\eta}^2) = 8.6$. The two activities are positively correlated with correlation coefficient $\text{cor}(\boldsymbol{\eta}^1, \boldsymbol{\eta}^2) = 0.76$. The economy has a fixed date 0 wealth: $w_0^1 + w_0^2 + w_0^3 = 400$. We consider two distributions of initial wealth between entrepreneurs and investors given by

$$(80, 80, 240) \quad \text{and} \quad (20, 20, 360).$$

To show how the incentive effects change the predictions of the model with respect to risk sharing, security prices, and the use of debt versus equity, when compared with the standard CAPM-like model of finance, we compute two types of equilibria. First, the RCPP stock market equilibrium (Tables 1 and 3); second, the risk sharing equilibrium of the associated finance economy in which firms have the same physical investment and output (z^i, y^i) as in the RCPP equilibrium, but where the production plans are taken as fixed and independent of the consumption-portfolio choices of the agents. The consumption-portfolio choices and security prices of this latter equilibrium are those that would be predicted by an outside observer knowing the agents' risk-impatience characteristics and the firms' production plans, but who is not aware of the feedback between the entrepreneurs' financial decisions and their choices of effort. Since we have chosen utility functions in the LRT family and since there are well-known properties for the equilibria of a finance economy with such preferences, we call this latter type of equilibrium an LRT equilibrium (Tables 2 and 4).

Comparing RCPP and LRT Equilibria. The main difference between the two types of equilibria lies in the capital structure of the firms. An LRT equilibrium is a classical risk sharing equilibrium, and by a well-known result in the finance literature[14], in such an equilibrium agents have fully diversified portfolios, $\theta_1^i/\theta_2^i = 1$, $i = 1, 2, 3$ (see Tables 2 and 4). By contrast, in an RCPP equilibrium (Tables 1 and 3) because entrepreneurs know that retaining an increased ownership share implies an increased equity price,

[14] For a summary of the properties of LRT economies, see for example Magill and Quinzii (1996, Section 17).

and because increasing debt has the same effect, *the incentive effects induce entrepreneurs to retain a higher proportion of their firm than in an LRT equilibrium*: as a result, *entrepreneurs typically make more use of debt to finance their capital investment in an RCPP equilibrum than in an LRT equilibirum.*

The qualitative difference in capital structure in the two types of equilibria translates into a qualitative difference in the prices of the securities or equivalently their rates of return (as shown in the last row of Tables 1-4). If r denotes the *rate of interest* and if $r_i - r$ is the *risk premium* on the equity of firm i, where

$$1 + r = \frac{1}{q_0}, \quad 1 + r_i = \frac{E(y^i)}{Q_i}, \quad i = 1, 2.$$

and $\boldsymbol{y}^i = (y_1^i, \ldots, y_S^i)$ is the date 1 profit stream of firm i, then *the rate of interest is higher and the risk premia on securities are lower in an RCPP equilibrium than in an LRT equilibrium.* Entrepreneurs, by restricting the supply of their firms' shares that they offer for sale, drive up the prices of equity contracts, thus lowering their risk premia[15]. The entrepreneurs who need outside funds to finance their capital investment resort to increased borrowing, thereby increasing the rate of interest.

The difference between the incentive effects of equity and debt can be seen by comparing the RCPP equilibria in Tables 1 and 3. The reduced initial wealth of entrepreneurs in the latter equilibrium forces them to draw more extensively on outside sources of funds, their capital investment in the two equilibria being essentially unchanged: were they to raise funds exclusively by selling shares in their firms, the negative effect on incentives would lead to a fall in output and to a fall in the price of their shares. To avoid this decrease in the price of their equity, entrepreneurs increase their reliance on debt: incurring debt counterbalances the effect of selling equity, since increasing debt has a positive effect on incentives[16], leading to a higher output and higher equity prices. In the equilibrium of Table 3 the effect of increasing debt dominates the effect of selling equity, so that effort and output increase (by about 15%).

Qualitative Properties of Effort Function. The way in which owner-

[15] This result seems to make the "equity premium puzzle" even more of a puzzle. However the observed high return on equity comes from capital gains rather than a high dividend yield, and capital gains are not present in our two-period model. A multiperiod model would be needed to determine whether the incentive-based restriction of the supply of equity could be a factor contributing to large capital gains

[16] A one unit increase in debt leads to a one unit decrease in consumption in each state at date 1 and hence to an increase in the marginal utility of consumption in each state. This increased marginal benefit (payoff) of effort implies that more debt leads to more effort (see footnote 17). Thus, in this model, the market interprets an increase in debt as a "favorable signal".

Table 1: RCPP Stock Market Equilibrium

$w_0 = (80, 80, 240)$

	x_0	x_1	x_2	x_3	$E(x_1)$	$\sigma(x_1)$	e	z	b	θ_1 (%)	θ_2 (%)
agent 1	64	107	68	28	68	32	0.98	43	17	87	0
agent 2	83	201	29	64	98	74	1.24	85	57	0	80
agent 3	126	158	108	111	126	23	0	0	-74	13	20
aggregate[a] returns[b](%)	273	466	205	203	292	124			7.8	4.7	7.3

[a] aggregate consumption at each date in each state which for date 1 is equal to aggregate output

[b] the last row of the "b" column gives the interest rate r (percent); the last row of the θ_i column gives the risk premium $r_i - r$ (percent) for each firm $i = 1, 2$.

Table 2: LRT Equilibrium

$w_0 = (80, 80, 240)$

	x_0	x_1	x_2	x_3	$E(x_1)$	$\sigma(x_1)$	b	θ_1 (%)	θ_2 (%)
agent 1	64	119	44	44	69	35	14	29	29
agent 2	84	157	59	59	92	47	19	38	38
agent 3	125	190	102	102	131	42	-33	33	33
aggregate[a] returns[b](%)	273	466	205	203	292	124	6.4	8.1	11.8

[a,b] same definition as in Table 1

Table 3: RCPP Stock Market Equilibrium

$w_0 = (20, 20, 360)$

	x_0	x_1	x_2	x_3	$E(x_1)$	$\sigma(x_1)$	e	z	b	θ_1 (%)	θ_2 (%)
agent 1	32	82	44	6	44	31	1.16	46	37	71	0
agent 2	51	169	9	41	73	69	1.41	86	72	0	67
agent 3	186	274	180	181	212	44	0	0	-109	29	33
aggregate[a] returns[b](%)	269	525	233	228	329	139			18.5	4.9	7.2

[a,b] same definition as in Table 1

Table 4: LRT Equilibrium

$w_0 = (20, 20, 360)$

	x_0	x_1	x_2	x_3	$E(x_1)$	$\sigma(x_1)$	b	θ_1 (%)	θ_2 (%)
agent 1	33	71	28	27	42	21	7	15	15
agent 2	51	112	43	42	66	33	12	24	24
agent 3	185	342	162	159	221	85	-19	61	61
aggregate[a] returns[b](%)	269	525	233	228	329	139	15.8	8.7	12.5

[a,b] same definition as in Table 1

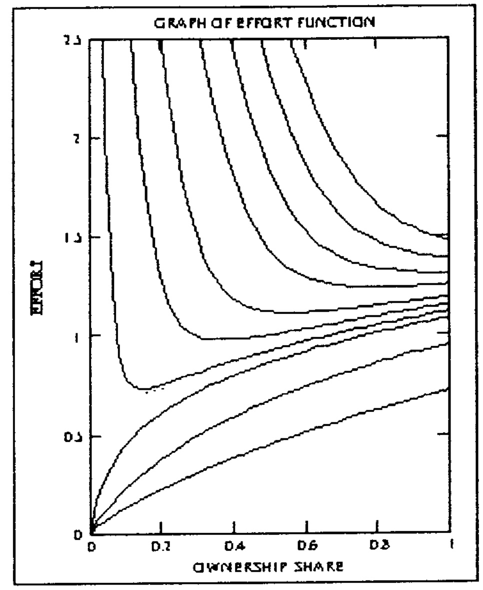

FIGURE 1. Graph of the effort function of entrepreneur 2 for different values of b^2 : starting with the top curve the values are b^2= 102, 87, 72, 57, 37, 22, 7, -8, -93, -393.

ship and debt jointly influence effort is shown in Figure 1. Entrepreneur 2's capital investment and his ownership share of firm 1 have been set at the equilibrium values in Table 1 ($\bar{z}^2 = 85, \bar{\theta}_1^2 = 0$) so that his optimal effort can be expressed as a function of his ownership share θ_2^2 and his debt b^2

$$e^2 = h^2(\theta_2^2, b^2) = \bar{e}^2(-b^2\mathbf{1}, \bar{z}^2, \theta_2^2).$$

Figure 1 shows the graph of the effort function $e^2 = h^2(\theta_2^2, b^2)$ viewed as a function of θ_2^2 for different fixed values of b^2. The graph of the effort function of entrepreneur 1 has the same general form.

Increasing debt always leads to an increase in effort.[17] The effect of changing the ownership share is more subtle. *When an entrepreneur has positive outside wealth ($b^2 < 0$) then an increase in θ_2^2 always increases effort. When the entrepreneur is endebted ($b^2 > 0$) then for any fixed level b^2 of debt, there is a critical level $\widehat{\theta}_2^2(b^2)$ such that for $\theta_2^2 < \widehat{\theta}_2^2$ effort is a decreasing function of θ_2^2 and for $\theta_2^2 > \widehat{\theta}_2^2$ effort is an increasing function.*[18]

[17] It is easy to see, by differentiating the first-order condition defining the optimal effort function $\bar{e}^i$ that the property $\partial e^i/\partial b^i > 0$ holds generally when u_1^i is an expected utility

[18] This behaviour of $\partial e^i/\partial \theta_i^i$ holds for LRT utility functions with a zero intercept and a

The negative slope of the optimal effort function for small values of the equity share is akin to the income effect dominating the substitution effect in a standard microeconomic choice problem (interpreting effort as labor and θ^i_i as a wage, since the reward for effort is proportional to $\theta^i_i f^i(\bar{z}^i, e^i)$). The equation determining entrepreneur i's optimal effort is

$$c^{i\prime}(e^i) = \theta^i_i \left(\sum_{s=1}^{S} \frac{\partial u^i_1(\boldsymbol{x}^i_1)}{\partial x^i_s} \eta^i_s \right) \frac{\partial f^i}{\partial e^i} \quad \text{with } \boldsymbol{x}^i_1 = -b^i \mathbf{1} + \theta^i_i f^i(z^i, e^i) \boldsymbol{\eta}^i.$$

Increasing the agents' ownership share θ^i_i has two effects: the direct (substitution) effect is to *increase* the marginal benefit from an additional unit of effort; the indirect (income) effect is to increase date 1 consumption x^i_1 and thus to decrease $\partial u^i / \partial x^i_s$ (assuming additive separability), thus *decreasing* the marginal benefit of effort. When $\boldsymbol{x}^i_1$ is small (small θ^i_i and large b^i), the marginal utility of consumption decreases fast and the negative indirect effect dominates, leading to the apparently paradoxical result that a reduced ownership share leads to increased effort. When $\boldsymbol{x}^i_1$ is large (large θ^i_i and small or negative b^i) marginal utility changes very little with an additional unit of consumption, and the direct positive effect dominates: hence the intuitive result that increased ownership leads to increased effort.

When $b^2 > 0$, the effort curves are asymptotic to the vertical axis, implying that effort must increase enormously when $\theta^2_2 \longrightarrow 0$: this is the *no-bankruptcy effect.* Since in this model the cost of bankruptcy is infinite, to be sure that the inequality $-b^i + \theta^i_i f^i(z^i, e^i)\eta^i_s \geq 0$ is satisfied for all states, the smaller θ^i_i, the greater the effort agent i must expend to stay out of bankruptcy. While shareholders of firm i would be happy to see entrepreneur i incurring a large debt and owning only a small share of his firm, the entrepreneur in choosing his financial variables (z^i, b^i, θ^i_i) will normally stay out of this region!

Note that for $\bar{z}^2$ fixed and $\bar{\theta}^2_1 = 0$, the perception function $\widetilde{Q}_2$ is a function of (b^2, θ^2_2)

$$\widetilde{Q}_2(b^2, \theta^2_2) = -q_2 f^2(\bar{z}^2, h^2(b^2, \theta^2_2))$$

so that up to a monotone transformation of the vertical axis, the same graph illustrates the perception function $\widetilde{Q}_2(b^2, \theta^2_2)$. Thus the general qualitative properties of the way effort responds to debt and ownership share translate into equivalent properties for the perception function $\widetilde{Q}_2(b^2, \theta^2_2)$. In particular selling equity can always be achieved without a drop in the price, provided debt is incurred at the same time.

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coefficient of marginal risk tolerance greater than one ($v^i(x^i_s) = (x^i_s)^\alpha$ with $0 < \alpha < 1$).

5 Conclusion

With the exception of the well-known papers of Prescott and Townsend (1984a,b), general equilibrium theory and the economics of asymmetric information are two branches of economic theory which have remained surprisingly separate. With some exaggeration general equilibrium studies circumstances under which markets "work", while the theory of asymmetric information reveals the circumstances which make markets "fail". Prescott and Townsend argue that in principle markets can resolve problems posed by asymmetry of information: however, to establish this result, they postulate the existence of an extensive array of markets for contracts (which rather like Arrow-Debreu contracts) are difficult to identify in the real world.

The approach of this paper is somewhat different: it seeks to formalize in a general equilibrium setting why the markets that we actually observe for debt and equity may perform rather well even in the presence of moral hazard. The main requirement, in addition to perfect competition, is that participants on these markets be rational, and that this rationality be common knowledge. This is formalized in the concept of rational, competitive price perceptions: it is the anticipatory aspect of perceptions which provides the disciplinary forces that induce agents to act in the appropriate way.

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