ELSEVIER

Journal of Mathematical Economics 38 (2002) 149–190

JOURNAL OF Mathematical ECONOMICS

www.elsevier.com/locate/jmateco

Capital market equilibrium with moral hazard

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Received 8 May 1999; received in revised form 5 May 2000; accepted 7 June 2002

Abstract

This paper studies a general equilibrium model of an economy with production under uncertainty in which firms' capital (ownership) structures creates a moral hazard problem for their managers. The concept of an equilibrium with rational, competitive price perceptions (RCPP) is introduced, in which investors correctly anticipate the optimal effort of entrepreneurs by observing their financial decisions, and entrepreneurs are aware that investors use their financial decisions as signals. The competitive element in the equilibrium valuation of firms comes from the fact that entrepreneurs cannot affect the market price of risks. It is shown that under appropriate spanning assumptions an RCPP is constrained Pareto optimal. Furthermore, if sufficiently many options are traded, then full optimality can be obtained despite the moral hazard problem: options serve both to increase the span of the market and to provide incentives for entrepreneurs.

Keywords: Agency; Incentives; Capital market equilibrium; Equity; Options; Constrained optimality; Spanning-overlap condition; Welfare theorems

1. Introduction

Economists have long held two opposing views on the merits of the stock market and the associated corporate form of organization. On the one hand, the stock market permits the substantial production risks of society to be diversified among many investors: this view underlies the capital asset pricing model (CAPM) which forms the basis for much of the modern theory of finance. On the other hand, the traditional view of classical economists, revived in modern times by Berle and Means (1932), Jensen and Meckling (1976) and the ensuing agency-cost literature, has emphasized the negative effect on incentives of the separation of ownership and control implied by the corporate form of organization. The object

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0304-4068/02/$ – see front matter
PII: S0304-4068(02)00068-X

of this paper is to provide a theoretical framework for reconciling these two perspectives, by showing circumstances under which the stock market can provide an optimal trade-off between the beneficial effect of risk sharing and the distortive effect on incentives. We argue furthermore that, when capital markets have become sufficiently developed by the introduction of a rich array of associated options markets, incentive structures can be created using these markets which compensate for the reduced ownership shares of top executives, so that agency costs can be eliminated, permitting a Pareto optimum to be achieved by the combined trading of equity and options.

To capture the dual role of financial markets in controlling risk sharing and incentives, we extend the classical model of capital market equilibrium (the general equilibrium model of a finance economy with production) to incorporate the effect of moral hazard. The new element is introduced in the spirit of Knight (1921) by modeling a firm as an entity arising from the organizational ability, foresight, and initiative of an *entrepreneur*. The activity of a firm consists of combining entrepreneurial effort and physical input (the value of capital and non-managerial labor) at an initial date: this gives rise to a random profit stream at the next date. In addition to entrepreneurs, there is another class of agents which we call *investors*: they have initial wealth at date 0 but no productive opportunities. The analysis is based on two key hypotheses: first that the effort of entrepreneurs is not observable, and second that the primitive states of nature, which model the risks to which firms are exposed, are too complex to make the writing and enforcement of contracts contingent on states feasible—in short, that states of nature not being verifiable, are not contractible. Under these conditions, there is no market for entrepreneurial effort and the markets for channeling capital from investors to firms must be either non-contingent or based on the realized outputs of the firms. This class of financial markets includes the bond and equity markets, which have a long tradition, and the much more recently introduced derivative markets, consisting of options on equity and portfolios of equity contracts.[1]

Entrepreneurs are taken to be the initial owners of their firms. They use the security markets to finance their investments and to diversify their risks. If in doing so they sell shares of their firms, then they must share the profit with outside shareholders, which decreases their incentives to invest effort in their firms. More generally, the incentives of an entrepreneur to invest effort depend directly on the positions he takes on his *inside* securities—those whose payoffs depend on his effort (his equity, options on his equity and indices which involve his firm)—and indirectly on the positions he takes on *outside* securities—those whose payoffs are independent of his effort (the bond, equity or options on equity of other firms and indices which do not involve his firm). Since we are interested in understanding the success or failure of financial markets in creating incentives, we assume that the income received or spent at date 0, or received at date 1 from the traded securities, is the only income received by the entrepreneur: no separate compensation package is arranged between the entrepreneur and the shareholders of his firm. Thus, the portfolio of securities chosen by each entrepreneur determines his incentives. The third key hypothesis of the paper is that the portfolio of securities chosen by each entrepreneur is publicly observable. Rational investors will use this knowledge to anticipate the effort that entrepreneurs will

[1] While the New York Stock Exchange was established in 1772, the first organized market for trading options is of very recent origin—the Chicago Board Options Exchange was opened in 1973.

invest in their firms. Entrepreneurs, being rational, in choosing their portfolios, will take into account that their financial trades signal to the market their incentives to make effort in their firms.

These assumptions on the way the capital market functions are formalized in a concept of equilibrium which we call a *capital market equilibrium with rational, competitive price perceptions* (RCPP). To decide whether an investment-financing plan is optimal, an entrepreneur needs to evaluate what would happen if he were to change this plan: his price perceptions describe how he perceives that the price of his equity (and associated inside securities) would react to any such change of plan. The price perceptions are assumed to be *rational* (i.e. each entrepreneur thinks that investors will correctly deduce from his investment-financing decision what his effort and the associated output of the firm will be) and *competitive* (an entrepreneur cannot affect the way the market prices risks, i.e. the state prices implicit in the equilibrium prices of the securities). To be consistent, the assumption of competitive price perceptions requires that an entrepreneur cannot alter the span of the financial markets by altering his production plan, i.e. that the assumption of *partial spanning* (PS) holds.

The price perceptions in essence act as a disciplining device for the entrepreneurs, forcing them to choose financial policies which create credible incentives for them to invest effort in their firms, and which thereby justify a high market valuation of the securities (in particular the equity) based on their firms. Sections 3–5 explore how effective this "market disciplining" is at yielding an efficient allocation of capital investment, risk sharing, and incentives. The study is divided into two parts: the first part takes the security structure as given and studies the constrained efficiency of the equilibrium; the second part derives a condition under which a weakened version of an RCPP equilibrium satisfies the First and Second Welfare Theorems. Section 5 shows that this condition can be satisfied by a security structure composed of standard capital market securities—bonds, equity, market indices and options—if the structure of the exogenous shocks is such that the outcome of production distinguishes among states, i.e. for fixed capital-effort inputs, the resulting vector of outputs of all firms is different in different states.

Studying the constrained optimality of equilibrium with a fixed security structure is a natural way of testing if rational and competitive price perceptions permit (possibly incomplete) financial markets to function"at their best". In an economy with only one firm the answer is positive: however, when there are several entrepreneurs the effort choice of each entrepreneur may depend on the effort of the other entrepreneurs through the payoffs of the securities based on their firms. This Nash equilibrium aspect of an RCPP equilibrium can lead to co-ordination failure in a market equilibrium which a planner, in choosing a constrained optimal allocation, could avoid. To establish constrained efficiency of an RCPP equilibrium, conditions need to be imposed which ensure that this interdependence between effort choices of entrepreneurs is sufficiently weak. In the standard one-good model of capital market equilibrium with production and no unobservable effort, in which each security is based on the output of a single firm, *partial spanning* (PS) is sufficient for an equilibrium to be constrained efficient. In the model with moral hazard (unobservable effort) this condition needs to be strengthened to *strong partial spanning* (SPS)—the requirement that the span of the securities generated by $I - 1$ firms is, for all capital-effort choices of these firms, contained in the subspace which they span at equilibrium (Proposition 1).

The second part examines the relation between security structures and Pareto optimality, under the assumption that the payoffs of the securities depend only on the observable outputs of the firms. If the security structure can be adapted to the characteristics of the economy, can a security structure be found which permits a Pareto optimal allocation to be decentralized as an RCPP equilibrium? Answering this question amounts to understanding how the condition of complete markets—which is the condition which must be satisfied in the absence of moral hazard—needs to be strengthened if the security structure is to permit agents to simultaneously control risk sharing and incentives.

We cannot completely answer this question and need to weaken the concept of equilibrium from an RCPP equilibrium to a *weak-RCPP equilibrium* in which the requirement that the effort of each entrepreneur is chosen optimally at date 1 is weakened to the requirement that each entrepreneur's effort satisfy the first-order condition for optimal choice of effort. We derive the condition that a market structure must satisfy to obtain the First and Second Welfare Theorems for a weak-RCPP equilibrium (Propositions 3 and 4). This condition—which we call the *spanning–overlap condition*—has a natural economic interpretation. For it requires that in addition to complete markets (spanning), for each entrepreneur there must be an "overlap" (i.e. an intersection of dimension greater than 1) between the subspace generated by his outside securities (those whose payoffs are independent of his effort) and the subspace generated by his inside securities (those whose payoffs depend on his effort): using an income stream which lies in the intersection, the magnitude of the incentive effect can be adjusted to any appropriate level (by the inside securities), while at the same time leaving the risk profile of the income stream unchanged (compensating by outside securities). Thus, the spanning–overlap condition ensures that incentives and risk sharing can be completely controlled.

In Section 5, we show that for any economy for which the state space is *technological* (the vector of firms' outputs distinguishes states), there is a security structure consisting of the riskless bond, the equity of each firm, an index of equity contracts and an appropriately chosen family of options such that the spanning–overlap condition is satisfied. In this case the riskless income stream satisfies the overlap condition since it lies in the outside subspace of each entrepreneur, and can also be generated by a portfolio of the firm's equity and options.

The idea that financial decisions of agents transmit information about their characteristics or actions which are not directly observable or knowable by the market, has been extensively explored in the finance literature. Concepts of equilibrium based on this idea and the idea of rational expectations have been used in many partial equilibrium models: for *adverse selection* in the signaling models of Ross (1977), and Leland and Pyle (1977), and the subsequent literature (see Harris and Raviv, 1992, for a survey); for problems of *moral hazard* by Jensen and Meckling (1976), Grossman and Hart (1982), Beck and Zorn (1982), Brander and Spencer (1989), Kihlstrom and Matthews (1990). The modeling of this paper is especially close to that of Kihlstrom–Matthews (KM) which carries out a partial equilibrium analysis of the efficiency of competitive and rational price perceptions when the only traded security is the equity of one entrepreneur. Apart from being a partial rather than a general equilibrium model the main difference between the KM model and the model in this paper lies in the timing. In KM all decisions and payments are made simultaneously, while we assume that the financial trades take place at an initial date, and the effort decision and production take place at date 1. This sequential structure, together with the assumption of separability between date 0 and date 1 utilities, leads to a more transparent model in which the concept of equilibrium is simpler to describe and analytically more tractable. This simplification in the structure of the model permits a general equilibrium analysis to be carried out with a more general security structure than a single equity contract, making it possible to identify at one end when a security structure leads to co-ordination failure at equilibrium and, at the other end, when a sufficiently rich security structure permits the first best to be attained, thereby avoiding the conflict between risk sharing, and incentives.

Since the original contributions of Helpman and Laffont (1975) and Rothschild and Stiglitz (1976), there have been repeated efforts to incorporate moral hazard and adverse selection into a general equilibrium framework (Prescott and Townsend, 1984a,b), with a recent renewal of interest (Dubey et al., 2001; Geanakoplos and Zame, 1995; Kocherlakota, 1998; Lisboa, 2001; Bisin and Gottardi, 1999). These papers differ by the nature of risks they consider (individual and/or aggregate), by the modeling of uncertainty (fixed outcomes with variable probabilities, or primitive causes (state of nature) with fixed probabilities), by the informational assumptions (observability/non-observability of trades) by the market structure (intermediaries versus financial markets) and by their analytical focus (definition and/or existence of equilibrium versus normative properties). We do not attempt a classification of the literature, since at this point it seems premature.

At this stage, however, an interesting connection should be pointed out between the analysis of Kocherlakota (1998) and our paper, which is likely to be worthwhile to exploit in future research. Kocherlakota considers a simple model in which each agent's endowment can have two outcomes, and the agent's effort affects the probability of the outcome. As in most models with fixed outcomes he assumes that the allocation of consumption is performed by an intermediary (which he calls the Monitoring Agency): the intermediary charges agents a price for their contingent consumption which reflects the effort which it is in their best interest to make, given the consumption that they demand, and agents take this into account in expressing their demand—a mechanism similar to the rational, competitive price perceptions of our model. As a result an agent's budget set consists of an Arrow–Debreu (present value) budget constraint combined with an incentive compatibility constraint. In Section 4, we show that the budget set of an entrepreneur in an RCPP equilibrium can be equivalently expressed in terms of state prices using an Arrow–Debreu budget constraint, combined with a spanning constraint (since the allocation takes place through financial markets), and an incentive compatibility constraint. Thus, our analysis and the paper of Kocherlakota are making progress towards identifying explicit mechanisms (intermediation or markets with price perceptions) which lead to abstract budget sets (an Arrow–Debreu budget constraint plus an incentive compatibility constraint) of the form postulated by Prescott and Townsend (1984a,b) to obtain abstract equilibria with good normative properties.

2. Equilibrium with rational competitive price perceptions

2.1. Characteristics of the economy

We consider a simple two-period one-good economy with production. There are two types of agents, *entrepreneurs* and *investors*. $\mathcal{I}_1 \neq \emptyset$ is the set of entrepreneurs, $\mathcal{I}_2 \neq \emptyset$ the set of investors, and the set $\mathcal{I} = \mathcal{I}_1 \cup \mathcal{I}_2$ of all agents is finite.[2] Every agent $i \in \mathcal{I}$ has an initial wealth ω_0^i at date 0. An agent i who is an entrepreneur has the opportunity to create a productive venture by investing an amount of capital $\kappa^i \in \mathbb{R}_+$ at date 0: the capital is carried over to date 1, at which time it becomes operational, the output $F_s^i(\kappa^i, e^i)$ depending on the date 1 effort e^i invested by the entrepreneur and on the realized shock (state of nature) s to which the firm is subjected. We assume that there is a finite set $\mathcal{S}$ of states of nature describing the shocks to which firms can be subjected, and that effort has to be made before the state of nature is realized. Let

$$\boldsymbol{y}^i = \boldsymbol{F}^i(\kappa^i, e^i) = (F_1^i(\kappa^i, e^i), \dots, F_S^i(\kappa^i, e^i))$$

denote firm i's resulting random output at date 1. Investors are agents who do not undertake productive ventures, so if $i \in \mathcal{I}_2$, then $\boldsymbol{F}^i(\kappa^i, e^i) \equiv \boldsymbol{0}$. Let $\mathbb{E}^i$ denote the set in which agent i chooses the effort e^i: if i is an investor then by convention $\mathbb{E}^i = \{0\}$.

Since capital must be invested at date 0, but the payoff (output) from productive activity is only obtained at date 1, entrepreneurs will typically need funds to finance their productive ventures. Furthermore, since the outcome of productive activity is uncertain, some form of risk sharing between entrepreneurs and investors is needed. Finally, entrepreneurs must be provided with appropriate incentives to invest effort in their firms. Our objective is to formulate a model in which we can pose the following question: under what conditions can markets—in particular financial markets of the type observed in a modern economy—be expected to satisfactorily solve the society's problem of financing, risk sharing, and incentives?

Two crucial assumptions will be made to capture in a stylized way in the model, the markets that *do* and *do not* exist in the real world. First, we assume that there is no explicit market on which entrepreneurial effort e^i is bought and sold. The standard explanation for the absence of such markets is based on the observation that entrepreneurial effort is a complex input which is not readily observable, measurable or monitored, at least not with the precision that would be needed to enforce contracts contingent on effort. Second, we assume that there are no markets for contracts contingent on the realization of the primitive states of nature that are the exogenous shocks to firms' outputs. This is a strong assumption, for there are some standard insurable risks to which firms are exposed—such as a fire in a firm's warehouse—for which there are insurance markets. However, the focus of this paper is on the more pervasive and complex "business risks"—shocks of varying magnitude to demand, to technology, to the competitive environment, and to input availability (as described in approximate terms in corporate quarterly and annual reports to shareholders)—which influence firms' profits and for which insurance contracts are not available. Given the difficulty of describing precisely ex-ante and verifying accurately ex-post the precise nature of these primitive shocks which affect firms' outputs, most financial contracts that are used for financing firms and sharing productive risks are either non-contingent (bonds) or based directly on the realized outputs of firms (equity and derivative securities, like options on equity). Thus, in the model we consider only financial contracts based on the realized outputs of firms. However, to make the model coherent, we must assume that investors and entrepreneurs *understand* the nature of the uncertainty (the shocks) to which firms

[2] Sets are denoted by calligraphic letters: the same letter in roman denotes the cardinality of the set, e.g. $\mathcal{I} = \{1, \dots, I\}$. Vectors, matrices and vector-valued functions are written in boldface.

are subjected. But since the transactions costs of itemizing the contingencies ex-ante and verifying them ex-post are taken to be very large, trading contingent contracts is unfeasible.

We assume that J contracts are traded, contract j being characterized by a state independent function $V^j : \mathbb{R}^I \to \mathbb{R}$ describing the way the payoff of contract j depends on the realized output of the I firms in the economy.[3] Let $\boldsymbol{y} = (\boldsymbol{y}^1, \dots, \boldsymbol{y}^I)$ denote the random outputs of the firms and let $\boldsymbol{y}_s = (y_s^1, \dots, y_s^I)$ denote the realized outputs in state s. The payoff of security j in state s is then $V^j(\boldsymbol{y}_s)$. We let $V^j(\boldsymbol{y})$ denote the vector $(V^j(\boldsymbol{y}_s))_{s\in\mathcal{S}}$ and $\boldsymbol{V}(\boldsymbol{y}) = [V^1(\boldsymbol{y}), \dots, V^J(\boldsymbol{y})]$ denote the matrix of payoffs of the J securities. Although some of the analysis of the paper does not depend on the exact specification of the functions V^j, the securities that we have in mind and that underlie the theoretical framework are the securities traded on actual capital markets:

(i) *Bond*: To simplify the analysis we assume that the penalty for default for individual agents and for bankruptcy by firms is infinite, so that the personal debts of the agents and the debts incurred by firms are default-free. Thus, if the security structure includes a bond, it is the default-free bond, with the non-contingent payoff $\mathbf{1} = (1, \dots, 1)$.

(ii) *Equity*: We assume that the security structure always includes the equity of the I firms in the economy. It is convenient to number the securities so that the first I securities are the equity contracts of the firms. Also we assume that if firm i is partly financed by debt, the debt is taken personally by the entrepreneur: this simplifies the notation and, since bankruptcy for firms and default for agents is prohibited, does not change the properties of the model. Thus, the payoff of security i $(i = 1, \dots, I)$ is $V^i(\boldsymbol{y}^i) = \boldsymbol{y}^i$.

(iii) *Simple options*: If security j is a call (put) option on the equity of firm i with an exercise price τ (a simple option in the terminology of Ross, 1976), then $V^j(\boldsymbol{y}) = (\max\{y_s^i - \tau, 0\})_{s\in\mathcal{S}}$ $(V^j(\boldsymbol{y}) = (\min\{\tau - y_s^i, 0\})_{s\in\mathcal{S}})$.

(iv) *Indices and complex options*: If the payoff of security j is a weighted sum of the payoffs of some or all of the firms' equities, then we call security j a (market or sectoral) *index*. Such a security is characterized by the weights $\alpha^1, \dots, \alpha^I$ of the different firms in the index and $V^j(\boldsymbol{y}) = \sum_{i=1}^I \alpha^i \boldsymbol{y}^i$. Call and put options on an index are called *complex options*.

If agent i $(i \in \mathcal{I}_2)$ is an investor then at date 0 he chooses a portfolio of the J securities $z = (z_1^i, \dots, z_J^i) \in \mathbb{R}^J$ to distribute consumption between date 0 and date 1 and to choose the risk to which his date 1 consumption is exposed. When agent i $(i \in \mathcal{I}_1)$ is an entrepreneur he decides the amount of capital κ^i to invest in his firm, and chooses a portfolio $z^i \in \mathbb{R}^J$ of securities for financing this capital investment, diversifying his risks and creating incentives. We assume that the entrepreneur has the full initial property rights to his firm (he has the knowledge and skill to implement the technology $\boldsymbol{F}^i$). For incentive purposes, it is useful to distinguish between securities which depend on the output of his own firm (and hence his choice of effort) and those which depend on the outputs of other firms. To this end, let $\boldsymbol{y} = (\boldsymbol{y}^i, \boldsymbol{y}^{-i})$ where $\boldsymbol{y}^{-i} = (\boldsymbol{y}^k)_{k\neq i}$. For each i, the set of securities $\mathcal{J}$ can be partitioned into

$$\mathcal{J} = \mathcal{J}^i \cup \mathcal{J}^i_{-i} \cup \mathcal{J}_{-i}$$

[3] For simplicity of notation we treat entrepreneurs and investors symmetrically: for each $i \in \mathcal{I}_2$ the component $\boldsymbol{y}^i$ of $\boldsymbol{y}$ is a “dummy” zero vector.

where $\mathcal{J}^i$ is the set of securities whose payoffs depend exclusively on the output of firm i (equity of firm i, options on equity), $\mathcal{J}^i_{-i}$ is the set of securities whose payoffs depend on the output of firm i and the output of at least one other firm (indices with $\alpha^i > 0$ and derivatives on these indices) and $\mathcal{J}_{-i}$ is the set of securities whose payoffs do not depend on firm i's output (bond, equity of other firms, etc.). Let $\boldsymbol{q} = (q_1, \dots, q_J)$ denote the vector of prices of the securities.

A choice of the financial variables $(\kappa^i, \boldsymbol{z}^i)$ by entrepreneur i, in conjunction with a choice of effort e^i leads to a vector of consumption $\boldsymbol{x}^i = (x^i_0, x^i_{\mathbf{1}}) = (x^i_0, x^i_1, \dots, x^i_S)$ given by

$$x^i_0 = \omega^i_0 + q_i - \boldsymbol{q}\boldsymbol{z}^i - \kappa^i \tag{1}$$

$$x^i_{\mathbf{1}} = \boldsymbol{V}(\boldsymbol{F}^i(\kappa^i, e^i), \boldsymbol{y}^{-i})\boldsymbol{z}^i \tag{2}$$

where $\boldsymbol{y}^{-i}$ is the anticipated output of the other firms. The term q_i in the date 0 budget constraint comes from the assumption that entrepreneur i has full initial ownership of his firm, and from the convention that the first I securities are the firms' equities. If entrepreneur i chooses to keep a share z^i_i of his firm, then he obtains $q_i(1-z^i_i)$ from the sale of his equity. If agent i is an investor, the budget equations are the same with $\boldsymbol{F}^i(\kappa^i, e^i) \equiv \mathbf{0}$, $q_i = 0$, so that the terms related to his own "firm" are just dummy variables.[4]

It is clear from Eq. (2) that, since there are securities whose payoffs depend on firm i's realized output (we have assumed that firms' equity contracts are traded) the date 1 reward of an entrepreneur for his effort depends on his choice of financial variables. This captures the idea that the capital structure of a firm (in particular the inside equity and options held by the manager, and the firm's debt) affects the performance of its management. Since financing arrangements must be in place before a firm can become operational, we assume that the choice of effort e^i by an entrepreneur is made at date 1, after the financial decisions have been determined, but before the state of nature is realized.[5]

Each agent has a utility function $U^i\colon \mathbb{R}^{S+1}_+ \times \mathbb{R}_+ \to \mathbb{R}$, where $U^i(\boldsymbol{x}^i, e^i)$ is the utility associated with the consumption stream $\boldsymbol{x}^i = (x^i_0, x^i_1, \dots, x^i_S)$ and the effort level e^i. The utility function is assumed to be separable between date 0 and date 1, i.e. there exist functions u^i_0 and u^i_1, increasing in the consumption variables, such that

$$U^i(\boldsymbol{x}^i, e^i) = u^i_0(x^i_0) + u^i_1(\boldsymbol{x}^i_{\mathbf{1}}, e^i)$$

The date 1 utility function captures the trade-off between effort and consumption facing an entrepreneur. Although not indispensible the assumption of separability between date 0 and date 1 utility is made so as to simplify the definition and analysis of an equilibrium.

We let $\mathcal{E}(\boldsymbol{U}, \boldsymbol{\omega}_0, \boldsymbol{F}, \boldsymbol{V})$ denote an economy in which the utility functions of the I agents are $\boldsymbol{U} = (U^1, \dots, U^I)$, their date 0 endowments are $\boldsymbol{\omega}_0 = (\omega^1_0, \dots, \omega^I_0)$, the production functions are $\boldsymbol{F} = (\boldsymbol{F}^1, \dots, \boldsymbol{F}^I)$ and the security structure is characterized by the payoff functions $\boldsymbol{V} = (V^1, \dots, V^J)$.

[4] To simplify the expression of the market clearing conditions, we adopt the convention that if $i \in \mathcal{I}_2$, $z^i_i = 1$, $z^k_i = 0$ for $k \neq i$.

[5] The exact timing of effort is not important provided that the choice of effort does not precede the portfolio choice and strictly precedes the resolution of the uncertainty.

2.2. Optimal effort

After entrepreneur i has chosen his financial variables $(\kappa^i, \boldsymbol{z}^i)$ (in a way that we study later), he chooses the effort level e^i which maximizes $u_1^i(\boldsymbol{x}_1^i, e^i)$, where $\boldsymbol{x}_1^i$ is the date 1 consumption stream given by (2). Define the *effort correspondence* of entrepreneur i

$$\tilde{e}^i(\kappa^i, \boldsymbol{z}^i; \boldsymbol{y}^{-i}) = \arg\max_{e^i \in \mathbb{E}^i} \{u_1^i(\boldsymbol{x}_1^i, e^i) | x_1^i = \boldsymbol{V}(\boldsymbol{F}^i(\kappa^i, e^i), \boldsymbol{y}^{-i})\boldsymbol{z}^i\} \tag{E}$$

We assume that the problem (E) has a maximum, so that the correspondence $\tilde{e}^i$ is well defined on the domain $\mathcal{D}^i \subset \mathbb{R}_+ \times \mathbb{R}^J \times \mathbb{R}^{S(I-1)}$ consisting of the variables $(\kappa^i, \boldsymbol{z}^i; \boldsymbol{y}^{-i})$ such that $\boldsymbol{V}(\boldsymbol{F}^i(\kappa^i, e^i), \boldsymbol{y}^{-i})\boldsymbol{z}^i \in \mathbb{R}_+^S$ for some $e^i \in \mathbb{E}^i$. A variety of assumptions on the primitives of the model imply that the maximum of problem (E) is finite: we can either assume that $\mathbb{E}^i$ is a compact set, or if $\mathbb{E}^i$ is an unbounded subset of $\mathbb{R}_+$ that the marginal product $(\partial F_s^i/\partial e^i(\kappa^i, e^i))$ of effort tends to zero uniformly in κ^i, while the marginal cost $(-\partial u_1^i/\partial e^i(\boldsymbol{x}_1^i, e^i))$ tends to infinity uniformly in $\boldsymbol{x}_1^i$ when e^i tends to infinity.

2.3. RCPP equilibrium

Consider an investor[6] who is thinking of buying either the equity or options of firm i. To anticipate what the firm's profit will be, the investor needs to anticipate the entrepreneur's inputs (κ^i, e^i). In this model, we assume that the capital input κ^i is observable, while the effort e^i is not. However, as we have seen, e^i can be deduced if the entrepreneur's characteristics $(u_1^i, \boldsymbol{F}^i)$ and his financial variables are known: in the analysis that follows we assume that investors do indeed have access to this information and, hence, can deduce the effort e^i that the entrepreneur will invest in his firm.

In practice, there is an important distinction between accessibility of information regarding the inside financial variables $(\kappa^i, z_j^i, j \in \mathcal{J}^i)$ and information regarding the portfolio of other securities $(z_j^i, j \in \mathcal{J}_{-i}^i \cup \mathcal{J}_{-i})$ and the characteristics $(u_1^i, \boldsymbol{F}^i)$ of a firm's manager. Disclosure rules of the Securities and Exchange Commission require that proxy statements of publicly traded firms contain information regarding capital projects of the firm, as well as the equity and options holdings of the top management. Thus, the assumption that inside variables are known by investors conforms with the regulations of capital markets in the US.

More detailed information regarding the characteristics of the firm and its manager are less directly accessible, and it is essentially the job of security analysts to gain access to this type of information. While this information may not be available with the precision required by the model, analysts will, however, in the course of scrutinizing the earnings prospects of the firms they follow, acquire a good knowledge of the characteristics of the firms and their top management. Analysts who have followed the careers of top executives are likely to have a good estimate of the magnitude of their personal wealth and, hence, can impute at least the orders of magnitude of their outside incomes. Past performance gives information on their ability— which in the model is included in the function $\boldsymbol{F}^i$—and their motivation and ability to take risks—in the model, the functions u_1^i. The information collected by analysts

[6] In the discussion that follows we use the term "investor" in an extended sense: it refers not only to agents in $\mathcal{I}_2$ but also to any agent who buys securities for which he is not in an insider position. Thus, for example entrepreneur k buying shares of firm i with $k \neq i$ is considered as an investor on firm i's equity market.

spreads to investors through advisory services and the recommendations given by large brokerage companies. The assumption that the characteristics and financial trades of the entrepreneurs are known by all agents is, thus, the theoretical limit of a situation in which both the rules of disclosure and the activity of professionals in financial services result in a large amount of information being available to investors in the market.

If entrepreneurs' financial trades are known to investors, if investors make optimal use of this information to anticipate the outputs of firms, and in this way come to decide on the prices they are prepared to pay for the equity and options of the firms, then it seems reasonable to suppose that entrepreneurs will come to understand this. Hence, our second assumption regarding anticipations: entrepreneurs are aware that investors will use their financial decisions as "signals" of the effort that they will exert in their firms. The next step is to incorporate these two assumptions—namely that (1) investors use the available information (the financial variables) to correctly anticipate the firms' outputs, and (2) that entrepreneurs understand this—into a concept of equilibrium.

The description of an equilibrium thus consists of two parts. The first is the standard part which enumerates the *actions* of the agents and the *prices* of the securities; the second part describes the entrepreneurs' *perceptions* of the way their financial decisions affect the price that the "market" will pay for the securities—equity and options—based on the profits of their firms. To keep the notation symmetric we define a price perception for each agent and each security: the price perception of agent i depends on his observable actions $(\kappa^i, \boldsymbol{z}^i)$—the signal given to the market—and on the anticipations of other firms' outputs $\boldsymbol{y}^{-i}$. Let $\tilde{\boldsymbol{Q}}^i = (\tilde{Q}^i_j,)_{j\in\mathcal{J}})$ denote the price perception of agent i where

$$\tilde{Q}^i_j : \mathbb{R}_+ \times \mathbb{R}^J \times \mathbb{R}^{S(I-1)} \to \mathbb{R}_+$$

and $\tilde{Q}^i_j(\kappa^i, \boldsymbol{z}^i; \boldsymbol{y}^{-i})$ is the price that agent i expects for security j. Let $\tilde{\boldsymbol{Q}} = (\tilde{\boldsymbol{Q}}^1, \ldots, \tilde{\boldsymbol{Q}}^I)$ denote the price perceptions of all agents.

Definition 1. A *financial market equilibrium with price perceptions* $\tilde{\boldsymbol{Q}}$ for the economy $\mathcal{E}(\boldsymbol{U}, \boldsymbol{\omega}_0, \boldsymbol{F}, \boldsymbol{V})$ is a triple

$$((\bar{\boldsymbol{x}}, \bar{\boldsymbol{y}}, \bar{\boldsymbol{e}}, \bar{\boldsymbol{\kappa}}, \bar{z}), \bar{\boldsymbol{q}}; \tilde{\boldsymbol{Q}})$$

consisting of actions, prices, and price perceptions such that

(i) for each agent $i \in \mathcal{I}$ the action $(\bar{\boldsymbol{x}}^i, \bar{e}^i, \bar{\kappa}^i, \bar{z}^i)$ maximizes $U^i(\boldsymbol{x}^i, e^i)$ among consumption-effort streams such that[7]

$$x^i_0 = \omega^i_0 + \tilde{\boldsymbol{Q}}^i_i(\kappa^i, \boldsymbol{z}^i; \bar{\boldsymbol{y}}^{-i}) - \tilde{\boldsymbol{Q}}^i(\kappa^i, \boldsymbol{z}^i; \bar{\boldsymbol{y}}^{-i})z^i - \kappa^i$$
$$\boldsymbol{x}^i_{\boldsymbol{1}} = \boldsymbol{V}(\boldsymbol{F}^i(\kappa^i, e^i), \bar{\boldsymbol{y}}^{-i})\boldsymbol{z}^i.$$

[7] Without the assumption of separability between date 0 and date 1 utilities, the effort of an entrepreneur, and thus, the way the market interprets his observable actions, would depend on the date 0 consumption x^i_0. The consumption x^i_0 would then be the solution of a fixed point of the date 0 constraint

$$x^i_0 = \omega^i_0 + \tilde{\boldsymbol{Q}}^i_i(x^i_0, \kappa^i, \boldsymbol{z}^i; \boldsymbol{y}^{-i}) - \tilde{\boldsymbol{Q}}^i(x^i_0, \kappa^i, \boldsymbol{z}^i; \boldsymbol{y}^{-i})z^i - \kappa^i$$

This fixed point property, which is necessarily present in the KM model which has no timing, substantially complicates the analysis of the concept of equilibrium.

(ii) $\bar{\boldsymbol{y}}^i = \boldsymbol{F}^i(\bar{\kappa}^i, \bar{e}^i)$, $i \in \mathcal{I}$.
(iii) $\bar{\boldsymbol{q}} = \tilde{\boldsymbol{Q}}^i(\bar{\kappa}^i, \bar{z}^i; \bar{\boldsymbol{y}}^{-i})$, $i \in \mathcal{I}$.
(iv) $\sum_{i\in\mathcal{I}} \bar{z}^i_j = 1$, $j = 1, \dots, I$, $\quad \sum_{i\in\mathcal{I}} \bar{z}^i_j = 0$, for $j = I + 1, \dots, J$.

In an equilibrium with price perceptions, each entrepreneur takes the production plans and the prices of the securities of other entrepreneurs' firms as given. He chooses his own actions, anticipating that those which are observable (his financial decisions) will influence the prices of his securities in the way indicated by the function $\tilde{\boldsymbol{Q}}^i(\kappa^i, \boldsymbol{z}^i; \bar{\boldsymbol{y}}^{-i})$. By (ii), for each firm, the output (profit) anticipated by outside investors is compatible with the entrepreneur's choice of effort. By (iii) the price perceptions of each agent are consistent with the observed equilibrium prices $\bar{\boldsymbol{q}}$, and by (iv) the security markets clear.

Without more precise assumptions on the price perceptions $\tilde{\boldsymbol{Q}}^i$, the definition of equilibrium given so far only incorporates the first assumption that we discussed above—namely, that investors have correct expectations—but it does not yet explicitly incorporate the second—namely, that entrepreneurs are fully aware of this fact. To form his anticipations $\tilde{\boldsymbol{Q}}^i$, entrepreneur i needs to predict:

(a) the *output* of his firm that investors expect if they observe $(\kappa^i, \boldsymbol{z}^i)$;
(b) how the market will *price* the securities whose payoff depends on the value of this output (equity, options on equity, market indices involving firm i).

For part (a), we use the assumption that entrepreneur i knows that investors will deduce from the observation of $(\kappa^i, \boldsymbol{z}^i)$ what his likely effort $e^i \in \tilde{e}^i$ will be, and hence, what the likely output $\boldsymbol{F}^i(\kappa^i, e^i)$ of his firm will be. For part (b) we assume that the entrepreneur is, like an investor, a price-taker in the market for risky income streams. This price-taking assumption for price perceptions can be formalized as follows. Given that there are investors in the financial markets ($\mathcal{I}_2 \neq \emptyset$) who have no restrictions on the trading positions they can take, the equilibrium prices of the securities $\bar{\boldsymbol{q}}$ must not offer any arbitrage opportunity. It is well-known that this implies that there exists a vector $\bar{\boldsymbol{\pi}}$ of state prices $\bar{\boldsymbol{\pi}} = (\bar{\pi}_1, \dots, \bar{\pi}_S)$ such that $\bar{\boldsymbol{q}} = \bar{\boldsymbol{\pi}} \boldsymbol{V}(\bar{\boldsymbol{y}})$. If markets are complete (rank $\boldsymbol{V}(\bar{\boldsymbol{y}}) = S$) then the vector $\bar{\boldsymbol{\pi}}$ is unique; if markets are incomplete then there is a subspace of such vectors. Let $\mathcal{V}(\bar{\boldsymbol{y}})$ denote the marketed subspace at equilibrium, i.e. the subspace spanned by the columns of the matrix $\boldsymbol{V}(\bar{\boldsymbol{y}})$, which contains all the income streams that can be obtained by trading in the financial markets. If $\boldsymbol{m} \in \mathbb{R}^S$ is any stream in $\mathcal{V}(\bar{\boldsymbol{y}})$, then its value is $v_{\bar{q}}(\boldsymbol{m}) = \sum_{s\in\mathcal{S}} \bar{\pi}_s m_s$, where $\bar{\boldsymbol{\pi}} \in \mathbb{R}^S_{++}$ is any vector of state prices satisfying $\bar{\boldsymbol{\pi}} \boldsymbol{V}(\bar{\boldsymbol{y}}) = \bar{\boldsymbol{q}}$. Our assumption is that, as long as the entrepreneur envisions alternative production plans leading to security payoffs lying in the marketed subspace $\mathcal{V}(\bar{\boldsymbol{y}})$, then he will use the state prices implicit in the equilibrium prices $\bar{\boldsymbol{q}}$ to evaluate the corresponding security prices. While the price-taking assumption leads to a well-defined valuation of income streams in the marketed subspace, it does not extend in any natural way to income streams outside the marketed subspace: for if $\boldsymbol{m} \notin \mathcal{V}(\bar{\boldsymbol{y}})$, the value $\sum_{s\in\mathcal{S}} \bar{\pi}_s m_s$ can change when the vector of state prices satisfying $\bar{\boldsymbol{\pi}} \boldsymbol{V}(\bar{\boldsymbol{y}}) = \bar{\boldsymbol{q}}$ is changed, so that the valuation of the stream $\boldsymbol{m}$ is no longer well-defined.[8]

[8] This problem has been extensively discussed in the literature on equilibrium in a production economy with incomplete markets (see Ekern and Wilson, 1974; Radner, 1974; Drèze, 1974; Grossman and Hart, 1979, or the exposition in Magill and Quinzii, 1996, Chapter 6).

To stay within a framework that permits the competitive assumption to be retained without raising conceptual difficulties, we introduce the assumption of partial spanning.

Definition 2. We say that there is *partial spanning* (PS) at $\bar{\boldsymbol{y}}$ if for all $i \in \mathcal{I}$, for all $(\kappa^i, e^i) \in \mathbb{R}^2_+$ and $\boldsymbol{y}^i = \boldsymbol{F}^i(\kappa^i, e^i)$, the subspace $\mathcal{V}(\boldsymbol{y})$ is contained in the marketed subspace at $\bar{\boldsymbol{y}}$, i.e. $\mathcal{V}(\boldsymbol{y}) \subset \mathcal{V}(\bar{\boldsymbol{y}})$.

The partial spanning assumption is classical in the literature (see references in footnote 8): it means that a firm cannot create a "new security", i.e. an income stream which is not in the existing marketed subspace $\mathcal{V}(\bar{\boldsymbol{y}})$, by changing its production plan. With partial spanning the market prices of the securities are sufficient signals to value any possible alternative production plan of any firm and its associated securities.

Definition 3. A *financial market equilibrium with rational competitive price perceptions* (RCPP) is an equilibrium $((\bar{\boldsymbol{x}}, \bar{\boldsymbol{y}}, \bar{\boldsymbol{e}}, \bar{\boldsymbol{\kappa}}, \bar{z}), \bar{\boldsymbol{q}};\ \tilde{\boldsymbol{Q}})$ with price perceptions such that:

(i) PS holds at $\bar{\boldsymbol{y}}$,
(ii) for each $i \in \mathcal{I}$ the price perceptions are given by

$$\tilde{\boldsymbol{Q}}^i(\kappa^i, \boldsymbol{z}^i; \bar{\boldsymbol{y}}^{-i}) = \bar{\boldsymbol{\pi}}\boldsymbol{V}(\boldsymbol{F}^i(\kappa^i, \hat{e}^i), \bar{\boldsymbol{y}}^{-i})$$

for any $\bar{\boldsymbol{\pi}} \in \mathbb{R}^S_{++}$ such that $\bar{\boldsymbol{\pi}}\boldsymbol{V}(\bar{\boldsymbol{y}}) = \bar{\boldsymbol{q}}$ and for an effort choice $\hat{e}^i \in \tilde{e}^i(\kappa^i, \boldsymbol{z}^i; \bar{\boldsymbol{y}}^{-i})$ which maximizes

$$\bar{\boldsymbol{\pi}}\boldsymbol{F}^i(\kappa^i, e^i) - \bar{\boldsymbol{\pi}}\boldsymbol{V}(\boldsymbol{F}^i(\kappa^i, e^i), \bar{\boldsymbol{y}}^{-i})\bar{z}^i.$$

To check if his equilibrium financial decisions $(\bar{\kappa}^i, \bar{z}^i)$ are optimal, entrepreneur i considers alternative decisions $(\kappa^i, \boldsymbol{z}^i)$, recognizing that investors are rational and will deduce from $(\kappa^i, \boldsymbol{z}^i)$ what his associated optimal effort will be—namely, the solution of the optimal effort problem (E) if it is unique, or if it is multivalued, the solution which yields the highest date 0 income for entrepreneur i (recall that $u^i_1(\boldsymbol{x}^i_1, e^i)$ has the same value for each of the solutions).[9] This is the "rational" part of his anticipations. To evaluate the prices $\tilde{\boldsymbol{Q}}^i(\kappa^i, \boldsymbol{z}^i; \bar{\boldsymbol{y}}^{-i})$ that he would then get for his equity or the price that he would pay for the options on his firm, he uses any state price vector $\bar{\boldsymbol{\pi}}$ compatible with the equilibrium vector of security prices $\bar{\boldsymbol{q}}$. This is the "competitive" part of his expectations, which requires that PS holds at equilibrium. Note that, if agent i is an investor, or if agent i is an entrepreneur evaluating the securities of other firms on which he trades as an investor, the price anticipation is simply $\tilde{Q}^i_j = \sum_{s=1}^S \bar{\pi}_s V^j(\bar{\boldsymbol{y}}_s)$, which is the relation which has to hold in an equilibrium with correct anticipations.

PS is automatically satisfied if the financial markets are complete at equilibrium (rank $\boldsymbol{V}(\bar{\boldsymbol{y}}) = S$), but it can also be satisfied when the markets are incomplete as shown by the following examples.

[9] We assume that competition among the investors will lead them to pay the maximum price compatible with rational expectations. In this we differ from KM who do not make this assumption.

Example 1. The financial markets consist solely of the bond and equity markets, so that $J = I+1$. The production function of each firm has a simple factor structure: $\boldsymbol{F}^i(\kappa^i, e^i) = f^i(\kappa^i, e^i)\mathbf{1} + g^i(\kappa^i, e^i)\boldsymbol{\eta}^i$ where $f^i, g^i : \mathbb{R}^2_+ \to \mathbb{R}$ are concave increasing functions and $\boldsymbol{\eta}^i \in \mathbb{R}^S_+$ is a fixed vector, characterizing the risk structure of the firm. Then PS is satisfied if $g^i(\bar{\kappa}^i, \bar{e}^i) > 0$ for all $i \in \mathcal{I}_1$. The case $g^i(\kappa^i, e^i) = 1$ is the case studied by Kihlstrom and Matthews (1990) in partial equilibrium. The case $f^i(\kappa^i, e^i) = 0$ (equivalent to $f^i(\kappa^i, e^i) = a^i g^i(\kappa^i, e^i)$ with $a^i = E(\boldsymbol{\eta}^i)$) is studied in detail in Magill and Quinzii (1999).

Example 2. The financial securities consist of the riskless bond, equity, and options on each firm. Suppose the uncertainty (shocks) affecting the production in the economy is decomposed into a product of $\mathcal{I}_1$ spaces

$$\mathcal{S} = \mathcal{S}^1 \times \cdots \times \mathcal{S}^{I_1} = \{1, \ldots, S^1\} \times \cdots \times \{1, \ldots, S^{I_1}\}$$

so that a state of nature is an I_1-triple $s = (s^1, \ldots, s^{I_1})$ where s^i is the shock experienced by firm i. Then for any pair of states $s = (s^1, \ldots, s^{I_1}) \in \mathcal{S}$, $\hat{s} = (\hat{s}^1, \ldots, \hat{s}^{I_1}) \in \mathcal{S}$ with $s^i = \hat{s}^i$, $F^i_s(\kappa^i, e^i) = F^i_{\hat{s}}(\kappa^i, e^i)$ for all $(\kappa^i, e^i) \in \mathbb{R}^2_+$. If the vector $\boldsymbol{F}^i(\bar{\kappa}^i, \bar{e}^i)$ takes on S^i different values for the S^i individual states of firm i, and if there are options with striking prices in between the S^i different values taken by the output of firm i, for each firm $i \in \mathcal{I}_1$, then PS is satisfied.

While our modeling of uncertainty, which links random output to a state of nature, is standard in the general equilibrium literature with financial markets (GEI), and in the corporate finance literature developing the ideas of Jensen–Meckling (Beck and Zorn, 1982; Hughes, 1988; Kihlstrom and Matthews, 1990), a large part of the principal agent and moral hazard literature in economics uses a different approach to modeling uncertainty in which states of nature are not specified: instead, an (exogenously given) set of possible *outcomes* is specified, and it is assumed that the unobservable action of an agent influences the probabilities of these fixed outcomes.[10] Marshall (1976) refers to the first approach as the model with “fixed probabilities”, and to the second approach as the model “with variable probabilities”. The latter approach has its origin in insurance models in which the set of outcomes (e.g. accident or no accident) is straighforward to specify, while the primitive causes are typically very difficult to pinpoint. From a formal mathematical point of view, the model with primitive states of nature includes the model with fixed outcomes as a special case: it suffices to assume that there is a finite number of possible values for each firm’s output, that there are more states of nature than possible values for each firm, and that the effort of an entrepreneur influences the mapping of states to outcomes. However, as the next example shows, the assumption of partial spanning, as defined in Definition 2, may not hold in this case, so that the definition of the competitive part of the price expectations may need to be modified to cover the case with fixed outcomes.

Example 3. Suppose $S = 3$, $\boldsymbol{F}^i_s$ does not depend on κ^i, $e^i \in \{e^i_h, e^i_l\}$, $y^i \in \{y^i_h, y^i_l\}$,

$$\boldsymbol{F}^i(e^i_h) = (y^i_h, y^i_h, y^i_l), \qquad \boldsymbol{F}^i(e^i_l) = (y^i_h, y^i_l, y^i_l)$$

[10] See for example Ross (1973), Helpman and Laffont (1975), Holmström (1979), and Grossman and Hart (1983).

Thus, if agent i makes a high effort, states 1 and 2 lead to a high outcome, and only state 3 leads to a low outcome, while, if he makes a low effort, only state 1 leads to a high outcome. If the equilibrium outcome is that he makes a high effort, the securities based on the output of his firm will generate a subspace included in $\{y \in \mathbb{R}^3 | y_1 = y_2\}$. If he considers making a low effort, the payoff of the equity of his firm will be such that $y_1 \neq y_2$ and will not be in the equilibrium market subspace, unless the output of some other firm is different in states 1 and 2.

It is possible to define a notion of competitive and rational price perceptions for the fixed outcome case, provided one works in the *outcome space* in which the spanning assumption is automatically satisfied, and that agents' date 1 utilities are expected utilities. Then the "price" of an outcome, implicit in the equilibrium prices of the securities, can be decomposed into the product of the probability of the outcome (which depends on effort) and a risk-aversion coefficient which can be assimilated to a social marginal utility of income in this outcome state. The competitive assumption for price expectations amounts to assuming that agents take the vector of social marginal utilities of income as fixed, while the assumption that investors correctly anticipate how effort affects probabilities and how the observable trades of the entrepreneurs affect their effort decisions is the rational part of the expectation.

Since we do not propose to cover two models which require different notation in the same paper, we restrict the study to the case where the partial spanning assumption, in the sense of the finance literature, is satisfied. This choice is not completely a matter of indifference: we believe that if the outcomes of firms' production processes were determined ex-ante in an unambiguous way, security markets would not have the form that we observe: it would be natural to use contracts contingent on the realization of well-specified outcomes to reward the owners of the capital and the workers of a firm rather than contracts like bonds, equity, and options which are (linear or piecewise linear) functions of an outcome that will occur but which is not specified ex-ante.

3. Constrained optimality of RCPP

The concept of an RCPP equilibrium is a natural way of describing market behavior in a production economy with moral hazard when the agents who trade on the financial markets are well informed. To get a feel for how natural this concept is we turn to a study of its normative properties. At the first stage of development of financial markets, when the contracts traded consist solely of bonds and equity of firms, there is a clear *trade-off* between incentives and risk sharing. Entrepreneurs who want to finance their investment without incurring a large debt (which would put them in an inordinately risky situation) can choose to finance some of their investment by issuing equity, thus, opening the way to risk sharing and diversification: but issuing equity means they no longer receive the full benefit of their effort, so that their incentives to exert effort are diminished. Do these markets induce however entrepreneurs to make the optimal trade-off between incentives and risk sharing in their choice of debt and equity?

At a more mature stage of development of the financial markets, derivative securities such as options on firms' equity are introduced. Such contracts not only augment the opportunities for risk sharing, but also permit the introduction of non-linear reward schedules for entrepreneurs: non-linear schedules incorporate "high powered" incentives which can help to solve the moral-hazard problem induced by the reduced equity shares of entrepreneurs. If the entrepreneur receives a larger share of output when the firm's realized output is high than when it is low, then he will (typically) be induced to increase effort, to increase the likelihood of a high realization of output. Such an incentive scheme can be obtained by adding options to his share of equity: but would an entrepreneur choose to buy options to increase his incentives in this way, given that the income stream received from his firm will tend to be more risky? In short, *do market-induced choices of bonds, equity, and options by entrepreneurs and investors lead to the best possible use of these instruments*?

To answer this question we consider another way of arriving at an allocation where a "planner"—rather than the agents—chooses the financial variables, and examine if the planner could obtain a better allocation (in the Pareto sense) than that achieved in an RCPP equilibrium. Such a comparison only makes sense if the planner faces the same problem of unobservability of effort of the entrepreneurs and is restricted to the same opportunities for risk sharing as those available to the agents with the system of financial markets. In particular the planner cannot dictate effort levels to entrepreneurs—rather, these effort levels are chosen optimally by the entrepreneurs who take the reward structure given by the debt-equity-option choice of the planner and the effort levels of other agents (and hence their outputs) as given.

Definition 4. An allocation $(\boldsymbol{x}, \boldsymbol{e}) \in \mathbb{R}_+^{(S+1)I} \times \mathbb{R}_+^I$ is *constrained feasible* if there exist inputs and portfolios $(\boldsymbol{\kappa}, \boldsymbol{z}) \in \mathbb{R}_+^I \times \mathbb{R}^{IJ}$ such that

(i) $\sum_{i\in\mathcal{I}} z_j^i = 1,\ j = 1, \dots, I, \quad \sum_{i\in\mathcal{I}} z_j^i = 0,\ j = I+1, \dots, J,$
(ii) $\sum_{i\in\mathcal{I}} x_0^i = \sum_{i\in\mathcal{I}} \omega_0^i - \sum_{i\in\mathcal{I}} \kappa^i,$
(iii) $\boldsymbol{y}^i = \boldsymbol{F}^i(\kappa^i, e^i), \quad i \in \mathcal{I},$
(iv) $\boldsymbol{x}_{\boldsymbol{1}}^i = \boldsymbol{V}(\boldsymbol{y})z^i, \quad i \in \mathcal{I},$
(v) $e^i \in \tilde{e}^i(\kappa^i, z^i; \boldsymbol{y}^{-i}),\ i \in \mathcal{I}.$

An allocation $(\boldsymbol{x}, \boldsymbol{e})$ is *constrained Pareto optimal* (CPO) if it is constrained-feasible, and if there does not exist any alternative constrained feasible allocation $(\hat{\boldsymbol{x}}, \hat{\boldsymbol{e}})$ such that $U^i(\hat{\boldsymbol{x}}^i, \hat{e}^i) \geq U^i(\boldsymbol{x}^i, e^i),\ i \in \mathcal{I}$, with strict inequality for at least one i.

Constraints (i) are the feasibility constraints for the planner's choice of portfolio z. Constraints (ii) indicates that the planner does not need to respect a system of prices for the securities and the associated date 0 budget constraint implied for each agent: it is in this sense that the planner replaces the "market". Constraints (iv) indicate that the planner's choice of date 1 consumption streams, and hence, risk sharing, for the agents respects the existing structure of the financial securities. Constraints (v) are the incentive constraints which reflect the fact that the choice of effort is made by entrepreneur i (and not the planner), and is one that is optimal given the financial variables attributed to him, and given the effort levels of other agents.

In the capital market equilibrium described in the previous section in which many entrepreneurs are simultaneously making financing and effort decisions, there are two potential sources of inefficiency. The first arises from the property that the equilibrium is a Nash equilibrium in which the effort decision of each entrepreneur depends on the decisions of all other entrepreneurs (at date 1 agent i maximizes $u_1^i(\boldsymbol{V}(\boldsymbol{F}(\kappa^i, e^i), \bar{\boldsymbol{y}}^{-i}), e^i)$. The second arises from the moral hazard problem: for the choice of effort of an entrepreneur affects the payoffs of all securities based on the output of his firm (the securities in $\mathcal{J}^i \cup \mathcal{J}^i_{-i}$) and, thus, has an external effect on any investor or entrepreneur who buys these securities.

In order that the Nash equilibrium aspect of an RCPP equilibrium does not lead to the possibility of co-ordination failure—which a planner, but not the markets, could avoid—we introduce two assumptions on the security structure which restrict the interdependence of the effort decisions of the entrepreneurs. The first is that there are no securities whose payoffs depend on the production decisions of several firms: $\mathcal{J}^i_{-i} = \emptyset$ for all $i \in I$.

Definition 5. We say that the economy has *no index-based securities* if $\mathcal{J}^i_{-i} = \emptyset$, for all $i \in \mathcal{I}$.

The presence of market indices, or options on indices introduces a strong dependence between the decisions of the firms which are part of the index—and such dependence can lead to co-ordination failure as shown by the next example.

Example 4. Consider an economy with two entrepreneurs (agents 1, 2) and one (representative) investor (agent 3). The production of firm i ($i = 1, 2$) does not require capital, just the effort e^i of entrepreneur i which can take two possible values: for $i = 1, 2$, $\mathbb{E}^i = \{e^i_l, e^i_h\}$ with $e^i_l = 1$, $e^i_h = 2$. There are two states of nature at date 1 ($\mathcal{S} = \{1, 2\}$) and the production functions of the two firms are given by

$$\boldsymbol{F}^1(e^1) = (2e^1, e^1), \qquad \boldsymbol{F}^2(e^2) = (e^2, 2e^2)$$

Agents 1 and 2 are only endowed with their firms ($\omega^1_0 = \omega^2_0 = 0$) and agent 3 has an endowment $\omega^3_0 \geq 4$ at date 0. The utility functions of the agents are defined for non-negative consumption streams and are given by

$$U^i(x^i, e^i) = x^i_0 + \tfrac{1}{2}\ln x^i_1 + \tfrac{1}{2}\ln x^i_2 - c(e^i), \quad i = 1, 2, \quad U^3(x^3) = x^3_0 + \tfrac{1}{2}x^3_1 + \tfrac{1}{2}x^3_2$$

with $c(e^i_l) = 0$, $c(e^i_h) = \ln 4$. The securities are the equity of the two firms (securities 1, 2) and a call option on the market index $y^1 + y^2$ with exercise price $\tau = 5$. We show that this economy has two equilibria: one low-output equilibrium in which the two entrepreneurs choose the low effort level, and a high-output equilibrium in which the two entrepreneurs choose the high effort level. The high-ouput equilibrium Pareto dominates the low-output equilibrium which is, thus, not constrained Pareto optimal.

(a) *Low-output equilibrium*: Given the preferences of agent 3, the state prices must be $\bar{\pi}_1 = \bar{\pi}_2 = 1/2$ in any equilibrium and all securities must be priced at their expected values. Let us show that $((\bar{\boldsymbol{x}}, \bar{\boldsymbol{y}}, \bar{\boldsymbol{e}}, \bar{z}), \bar{\boldsymbol{q}}; \tilde{\boldsymbol{Q}})$ with

$$\begin{aligned}
&\bar{\boldsymbol{x}}^1 = \bar{\boldsymbol{x}}^2 = (0.5, 1, 1), \quad \bar{\boldsymbol{x}}^3 = (\omega^3_0 - 1, 1, 1),\\
&\bar{\boldsymbol{y}}^1 = (2, 1), \quad \bar{\boldsymbol{y}}^2 = (1, 2), \quad \bar{e}^1 = \bar{e}^2 = 1\\
&\bar{z}^1 = \bar{z}^2 = \bar{z}^3 = \left(\tfrac{1}{3}, \tfrac{1}{3}, 0\right), \quad \bar{\boldsymbol{q}} = \left(\tfrac{3}{2}, \tfrac{3}{2}, 0\right)\\
&\tilde{\boldsymbol{Q}}^1(z^1) = \left(\tfrac{3}{2}\tilde{e}^1(z^1), \tfrac{3}{2}, 0\right), \quad \tilde{\boldsymbol{Q}}^2(z^2) = \left(\tfrac{3}{2}, \tfrac{3}{2}\tilde{e}^2(z^2), 0\right), \quad \tilde{\boldsymbol{Q}}^3(z^3) = \left(\tfrac{3}{2}, \tfrac{3}{2}, 0\right)
\end{aligned}$$

is an RCPP equilibrium. All we need to show is that the choice of effort by entrepreneur 1 is optimal given his price expectations: by symmetry the choice of entrepreneur 2 will be optimal. Since investor 3 is risk neutral, once the securities are priced at their expected values, any portfolio choice is optimal.

First note that since agent 1 takes the effort decision of agent 2 as given ($\bar{e}^2 = 1$), there is no possibility for agent 1 to put the option "in the money". It is easy to see that $\tilde{e}(\bar{z}^1) = 1$ since the effort level $e^1_h = 2$ would lead to a date 1 consumption stream $\boldsymbol{x}^1_1 = (5/3, 4/3)$ and $(1/2)\ln(5/3) + (1/2)\ln(4/3) - \ln 4 < 0$. Among the portfolios which lead to the choice of effort e^1_l, $\bar{\boldsymbol{z}}^1$ is optimal since it maximizes $(3/2)(1 - z^1_1) - (3/2)z^1_2 + (1/2)\ln(2z^1_1 + z^1_2) + (1/2)\ln(z^1_1 + 2z^1_2)$. It remains to show that the entrepreneur 1 cannot increase his utility by choosing a portfolio $\boldsymbol{z}^1$ such that $\tilde{e}(\boldsymbol{z}^1) = e^1_h$: the optimal portfolio with this property is the solution of the constrained maximum problem

$$\max_{z^1_1, z^1_2} \tfrac{6}{2}(1 - z^1_1) - \tfrac{3}{2}z^1_2 + \tfrac{1}{2}\ln(4z^1_1 + z^1_2) + \tfrac{1}{2}\ln(2z^1_1 + 2z^1_2)$$

subject to the constraints

$$\tfrac{1}{2}\ln(4z^1_1 + z^1_2) + \tfrac{1}{2}\ln(2z^1_1 + 2z^1_2) - \ln 4 \geq \tfrac{1}{2}\ln(2z^1_1 + z^1_2) + \tfrac{1}{2}\ln(z^1_1 + 2z^1_2) \tag{IC}$$

$$\tfrac{6}{2}(1 - z^1_1) - \tfrac{3}{2}z^1_2 \geq 0, \quad 4z^1_1 + z^1_2 \geq 0, \quad 2z^1_1 + 2z^1_2 \geq 0 \tag{NN}$$

It is easy to see that the incentive constraint (IC) must be binding (the unconstrained maximum does not satisfy (IC)). The solution of the constrained maximum problem is $(\hat{z}^1_1, \hat{z}^1_2) = (0.421, -0.176)$ which leads to the consumption stream $\hat{\boldsymbol{x}}^1 = (2, 1.5, 0.49)$ and the utility level $U^1(\hat{\boldsymbol{x}}^1, e^1_h) = 0.464 < 0.5 = U^1(\bar{\boldsymbol{x}}^1, e^1_l)$.

(b) *High-output equilibrium*: It is easy to check by similar reasoning that $(\bar{\bar{\boldsymbol{x}}}, \bar{\bar{\boldsymbol{y}}}, \bar{\bar{\boldsymbol{e}}}, \bar{\bar{\boldsymbol{z}}}, \bar{\bar{\boldsymbol{q}}}; \tilde{\boldsymbol{Q}})$ with

$$\begin{aligned}
&\bar{\bar{\boldsymbol{x}}}^1 = \bar{\bar{\boldsymbol{x}}}^2 = (2, 1, 1), \quad \bar{\bar{\boldsymbol{x}}}^3 = (\bar{\omega}^3_0 - 4, 4, 4), \quad \bar{\bar{\boldsymbol{y}}}^1 = (4, 2),\\
&\bar{\bar{\boldsymbol{y}}}^2 = (2, 4), \quad \bar{e}^1 = \bar{e}^2 = 2\\
&\bar{\bar{\boldsymbol{z}}}^1 = \bar{\bar{\boldsymbol{z}}}^2 = (0, 0, 1), \quad \bar{\bar{\boldsymbol{z}}}^3 = (1, 1, -2), \quad \bar{\bar{\boldsymbol{q}}} = (3, 3, 1)\\
&\tilde{\boldsymbol{Q}}^1(\boldsymbol{z}^1) = \left(\tfrac{3}{2}\tilde{e}^1(\boldsymbol{z}^1), 3, \tfrac{1}{2}\max\{2\tilde{e}^1(\boldsymbol{z}^1) + 2 - 5, 0\} + \tfrac{1}{2}\max\{\tilde{e}^1(\boldsymbol{z}^1) + 4 - 5, 0\}\right)\\
&\tilde{\boldsymbol{Q}}^2(\boldsymbol{z}^2) = \left(3, \tfrac{3}{2}\tilde{e}^2(\boldsymbol{z}^2), \tfrac{1}{2}\max\{4 + \tilde{e}^2(\boldsymbol{z}^2) - 5, 0\} + \tfrac{1}{2}\max\{2 + 2\tilde{e}^2(\boldsymbol{z}^2) - 5, 0\}\right)\\
&\tilde{\boldsymbol{Q}}^3(\boldsymbol{z}^3) = (3, 3, 1)
\end{aligned}$$

is an RCPP equilibrium which Pareto dominates the low-output equilibrium since

$$U^1(\bar{\bar{\boldsymbol{x}}}^1, \bar{\bar{e}}^1) = U^2(\bar{\bar{\boldsymbol{x}}}^2, \bar{\bar{e}}^2) = 2 - \ln 4 = 0.614 > 0.5 = U^1(\bar{\boldsymbol{x}}^1, \bar{e}^1) = U^2(\bar{\boldsymbol{x}}^2, \bar{e}^2)$$

and $U^3(\bar{\bar{\boldsymbol{x}}}^3) = U^3(\bar{\boldsymbol{x}}^3) = \omega^3_0$. In this high-output equilibrium both entrepreneurs sell all the equity of their firms and receive their date 1 consumption from their holdings of the option.[11] Since the option is out of the money if either of the entrepreneurs makes a low effort, both are induced to choose high-effort at date 1. It is easy to check that any portfolio which induces a low effort of entrepreneur i when entrepreneur $j \neq i$ makes a high effort yields utility less than or equal to 0.5 and will, thus, never be chosen over $\bar{\bar{z}}^i$.

Thus, if the security structure includes a complex option and if its striking price is set at an inappropriate level, then the markets can lead to co-ordination failure—i.e. an RCPP equilibrium can fail to be CPO. Note, however, that if an option with a lower striking price were introduced (e.g. an option with a striking price of 2.5 which each entrepreneur can put in the money even if the other makes a low effort) then the low-output equilibrium would disappear. This is a special case of a result shown in Section 4 (Proposition 4). We will return to this topic in the next section where we study security structures characteristic of well-developed financial markets. In this section, we assume that there are no index-based securities.

When this assumption is satisfied, the date 1 income stream of any entrepreneur i can be decomposed into two components

$$\boldsymbol{x}_{\mathbf{1}}^i = \boldsymbol{m}_{\mathbf{1}}^i + \sum_{j \in \mathcal{J}^i} V^j(\boldsymbol{F}^i(\kappa^i, e^i)) z_j^i, \quad \boldsymbol{m}_{\mathbf{1}}^i = \sum_{j \in \mathcal{J}_{-i}} V^j(\boldsymbol{y}^{-i}) z_j^i. \tag{3}$$

We call $\boldsymbol{m}_{\mathbf{1}}^i$ the *outside income* of entrepreneur i at date 1 since this income comes from securities whose payoffs do not depend on his effort: for this income agent i is in the position of an outside investor. The component $\sum_{j \in \mathcal{J}^i} V^j(\boldsymbol{F}^i(\kappa^i, e^i)) z_j^i$ is agent i's *inside income* since it comes from securities whose payoffs depend on the production of his own firm and is, thus, affected by his effort. Let

$$V_{-i}(\boldsymbol{y}^{-i}) = [V^j(\boldsymbol{y}^{-i})]_{j \in \mathcal{J}_{-i}} \quad \text{and} \quad V^i(\boldsymbol{y}^i) = [V^j(\boldsymbol{y}^i)]_{j \in \mathcal{J}^i}$$

denote the associated matrix of payoffs of the outside and inside securities (respectively) of entrepreneur i, and let $\mathcal{V}_{-i}(\boldsymbol{y}^{-i})$ and $\mathcal{V}^i(\boldsymbol{y}^i)$ denote the associated subspaces of $\mathbb{R}^S$ spanned by their columns. It is through the outside subspace $\mathcal{V}_{-i}(\boldsymbol{y}^{-i})$ that the choice of effort by other entrepreneurs in the economy affects agent i: the second assumption that we shall make limits the extent to which changes in the effort choice of other entrepreneurs can affect agent i's outside subspace.

Definition 6. We say that there is *strong partial spanning* (SPS) at $\bar{\boldsymbol{y}}$, if for all $(\boldsymbol{\kappa}, \boldsymbol{e}) \in \mathbb{R}^{2I}$ and $\boldsymbol{y} = (\boldsymbol{F}^i(\kappa^i, e^i)_{i \in \mathcal{I}})$, $\mathcal{V}_{-i}(\boldsymbol{y}) \subset \mathcal{V}_{-i}(\bar{\boldsymbol{y}})$ for all $i \in I$.

[11] Note that the portfolio holdings which lead to the equilibrium choice of actions $(\bar{\bar{\boldsymbol{x}}}, \bar{\bar{\boldsymbol{y}}}, \bar{\bar{\boldsymbol{e}}})$ are not unique: all portfolios

$$z^1 = \left(\frac{1-\xi^1}{6}, \frac{1-\xi^1}{6}, \xi^1\right), \qquad z^2 = \left(\frac{1-\xi^2}{6}, \frac{1-\xi^2}{6}, \xi^2\right),$$
$$z^3 = \left(\frac{4+\xi^1+\xi^2}{6}, \frac{4+\xi^1+\xi^2}{6}, -\xi^1-\xi^2\right)$$

with $\xi^1 > 1-(3/4\sqrt{5})$, $\xi^2 > 1-(3/4\sqrt{5})$ lead to the effort choices $(\bar{\bar{e}}^1, \bar{\bar{e}}^2)$.

SPS ensures that there is partial spanning for every subset of $I-1$ firms. Note that even if markets were complete, SPS would not automatically be satisfied. It holds if the securities based on the outputs of any subset of $I-1$ firms suffice to complete the markets, or if each firm spans its own subspace, as in Examples 1 and 2. Note also that SPS implies PS: if firm i cannot create an income stream which lies outside $\mathcal{V}^{-k}(\bar{\boldsymbol{y}})$ for $k \neq i$, it cannot create an income stream lying outside $\mathcal{V}(\bar{\boldsymbol{y}})$. The following example shows that without SPS an RCPP equilibrium can fail to be CPO.

Example 5. Consider an economy with two entrepreneurs and an investor as in Example 4. There are two types of effort, $\mathbb{E}^i = \{a, b\}$ and the entrepreneurs' production functions are given by

$$\boldsymbol{F}^1(e^1) = \begin{cases} (2,0), & \text{if } e^1 = a \\ (0,4), & \text{if } e^1 = b \end{cases}, \qquad \boldsymbol{F}^2(e^2) = \begin{cases} (0,2), & \text{if } e^2 = a \\ (4,0), & \text{if } e^2 = b \end{cases}$$

The initial resources are $\boldsymbol{\omega}_0 = (0, 0, \omega_0^3)$ as in Example 4 and the utility functions defined for non-negative consumption streams, are given by

$$U^1(\boldsymbol{x}^1, e^1) = \lambda_0^1 x_0^1 + x_1^1, \quad U^2(\boldsymbol{x}^2, e^2) = \lambda_0^2 x_0^2 + x_2^2, \quad U^3(\boldsymbol{x}^3) = x_0^3 + \tfrac{1}{2}x_1^3 + \tfrac{1}{2}x_2^3$$

with $\lambda_0^1 < (1/2)$, $\lambda_0^2 < (1/2)$. Thus, the entrepreneurs can choose between two qualitatively different types of effort (a and b), which are equivalent in terms of cost (both are costless), but determine in which state the entrepreneur produces. Entrepreneur 1 is relatively more efficient at producing in state 2, while he derives utility at date 1 only from consumption in state 1: the situation is the reverse for entrepreneur 2. Finally, the securities consist of the equity of the two firms.

We show that the economy has two equilibria: one with inefficient specialization and the other with efficient specialization. Since the latter Pareto dominates the former, the economy has an RCPP equilibrium which is not CPO.

(a) *Equilibrium with inefficient specialization*: Let us show that $(\bar{\boldsymbol{x}}, \bar{\boldsymbol{y}}, \bar{\boldsymbol{e}}, \bar{\boldsymbol{z}}, \bar{\boldsymbol{q}};\ \tilde{\boldsymbol{Q}})$ with

$$\bar{\boldsymbol{x}}^1 = (0,2,0), \quad \bar{\boldsymbol{x}}^2 = (0,0,2), \quad \bar{\boldsymbol{x}}^3 = (\omega_0^3, 0, 0), \quad \bar{\boldsymbol{y}}^1 = (2,0), \quad \bar{\boldsymbol{y}}^2 = (0,2), \quad \bar{e}^1 = \bar{e}^2 = a$$

$$\bar{z}^1 = (1,0), \quad \bar{z}^2 = (0,1), \quad \bar{z}^3 = (0,0), \quad \bar{\boldsymbol{q}} = (1,1)$$

$$\tilde{Q}_1^1(z^1) = \begin{cases} 1, & \text{if } \tilde{e}^1(z^1) = a \\ 2, & \text{if } \tilde{e}(z^1) = b \end{cases}, \quad \tilde{Q}_2^1(z^1) = 1, \quad \tilde{Q}_1^2(z^2) = 1, \quad \tilde{Q}_2^2(z^2) = \begin{cases} 1, & \text{if } \tilde{e}^2(z^2) = a \\ 2, & \text{if } \tilde{e}^2(z^2) = b \end{cases}$$

is an RCPP equilibrium. As in Example 4, it suffices to show that the portfolio $\bar{z}^1 = (1, 0)$ is optimal for entrepreneur 1 given that $\bar{\boldsymbol{y}}^2 = (0, 2)$. The choice of a portfolio $\boldsymbol{z}^1 = (z_1^1, z_2^1)$ gives the entrepreneur the date 1 consumption

$$x_1^1 = \begin{cases} 2z_1^1, & \text{if } e^1 = a \\ 0, & \text{if } e^1 = b \end{cases}, \qquad x_2^1 = \begin{cases} 2z_2^1, & \text{if } e^1 = a \\ 4z_1^1 + 2z_2^1, & \text{if } e^1 = b \end{cases}$$

The portfolios for which $\tilde{e}^1(\boldsymbol{z}^1) = a$ must satisfy $z_1^1 \geq 0$, $z_2^1 \geq 0$ and $x_0^1 = 1 - z_1^1 - z_2^1 \geq 0$ to ensure non-negative consumption: among these $\bar{z}^1$ is clearly optimal. The portfolios for which $\tilde{e}^1(\boldsymbol{z}^1) = b$ must satisfy $4z_1^1 + 2z_2^1 \geq 0$. Agent 1's utility is then $U^1(\boldsymbol{x}^1, b) = \lambda_0^1(2 - (2z_1^1 + z_2^1)) \leq 2\lambda_0^1 < 2 = U^1(\bar{\boldsymbol{x}}^1, \bar{e}^1)$, so that $\bar{z}^1$ is optimal.

(b) *Equilibrium with efficient specialization*: A similar argument shows that $(\bar{\bar{\boldsymbol{x}}}, \bar{\bar{\boldsymbol{y}}}, \bar{\bar{\boldsymbol{e}}}, \bar{\bar{\boldsymbol{z}}}, \bar{\bar{\boldsymbol{q}}}; \tilde{\boldsymbol{Q}})$ with

$$\bar{\bar{\boldsymbol{x}}}^1 = (0,4,0),\quad \bar{\bar{\boldsymbol{x}}}^2 = (0,0,4),\quad \bar{\bar{\boldsymbol{x}}}^3 = (\omega_0^3, 0, 0),\quad \bar{\bar{\boldsymbol{y}}}^1 = (0,4),\quad \bar{\bar{\boldsymbol{y}}}^2 = (4,0),\quad \bar{\bar{e}}^1 = \bar{\bar{e}}^2 = b$$

$$\bar{\bar{z}}^1 = (0,1),\quad \bar{\bar{z}}^2 = (1,0),\quad \bar{\bar{z}}^3 = (0,0),\quad \bar{\bar{\boldsymbol{q}}} = (2,2)$$

$$\tilde{Q}_1^1(z^1) = \begin{cases} 1, & \text{if } \tilde{e}^1(z^1) = a \\ 2, & \text{if } \tilde{e}^1(z^1) = b \end{cases},\quad \tilde{Q}_2^1(z^1) = 2,\qquad \tilde{Q}_1^2(z^2) = 2,\quad \tilde{Q}_2^2(z^2) = \begin{cases} 1, & \text{if } \tilde{e}^2(z^2) = a \\ 2, & \text{if } \tilde{e}^2(z^2) = b \end{cases}$$

is an RCPP equilibrium in which each entrepreneur produces in the state in which he is relatively efficient, and this equilibrium Pareto dominates that in (a).

The next proposition shows that when these possibilities of co-ordination failure are avoided by reducing the extent to which the effort choice of an entrepreneur can be directly influenced by the effort choices of the other entrepreneurs, then the basic moral hazard problem is resolved by the competitive and rational price perceptions. Even though a planner choosing the financial decisions of entrepreneurs in a CPO allocation directly takes into account the fact that the effort of entrepreneur i affects all holders of securities based on firm i, while in equilibrium entrepreneur i chooses his financial decisions in his own self-interest, a planner cannot improve on the equilibrium allocation.

Proposition 1 (Constrained Pareto Optimality). *Let $(\bar{\boldsymbol{x}}, \bar{\boldsymbol{y}}, \bar{\boldsymbol{e}}, \bar{\boldsymbol{\kappa}}, \bar{z}, \bar{\boldsymbol{q}}; \tilde{\boldsymbol{Q}})$ be an RCPP equilibrium of the economy $\mathcal{E}(\boldsymbol{U}, \boldsymbol{\omega}_0, \boldsymbol{F}, \boldsymbol{V})$. If the security structure $\boldsymbol{V}$ has no index-based securities, and satisfies Strong Partial Spanning at $\bar{\boldsymbol{y}}$, then $(\bar{\boldsymbol{x}}, \bar{\boldsymbol{e}})$ is constrained Pareto optimal.*

Proof. Suppose the equilibrium allocation $(\bar{\boldsymbol{x}}, \bar{\boldsymbol{y}}, \bar{\boldsymbol{e}}, \bar{\boldsymbol{\kappa}}, \bar{z})$ is not CPO, then there exists a constrained feasible allocation $(\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{e}, \boldsymbol{k}, z)$ satisfying (i)–(v) of Definition 4, such that $U^i(\boldsymbol{x}^i, e^i) \geq U^i(\bar{\boldsymbol{x}}^i, \bar{e}^i), i \in \mathcal{I}$, with strict inequality for some i. Since $\mathcal{J}_{-i}^i = \emptyset$, the feasible date 1 consumption streams for agent i can be decomposed into outside and inside income components as in (3). The optimal choice of effort of agent i depends on κ^i, the portfolio of inside securities $(z_j^i, j \in \mathcal{J}^i)$ and depends on the portfolio of outside securities $(z_j^i, j \in \mathcal{J}_{-i})$ and the production plans of other firms only through the associated outside income stream $\boldsymbol{m}^i$. Thus, if $(\hat{\kappa}^i, \hat{\boldsymbol{z}}^i; \hat{\boldsymbol{y}}^{-i})$ are such that $\hat{\kappa}^i = \kappa^i, \hat{z}_j^i = z_j^i$ for $j \in \mathcal{J}^i$ and $\boldsymbol{m}_i = \sum_{j \in \mathcal{J}_{-i}} V^j(\hat{\boldsymbol{y}}^{-i})\hat{z}_j^i = \sum_{j \in \mathcal{J}_{-i}} V^j(\boldsymbol{y}^{-i})z_j^i$ then $\tilde{e}^i(\hat{\kappa}^i, \hat{\boldsymbol{z}}^i; \hat{\boldsymbol{y}}^{-i}) = \tilde{e}^i(\kappa^i, \boldsymbol{z}^i; \boldsymbol{y}^{-i})$. Since SPS holds, agent i could in the equilibrium situation have chosen the outside variables $(\hat{z}_j^i, j \in \mathcal{J}_{-i})$ to obtain the same outside income $\boldsymbol{m}^i$ as that chosen by the planner

$$\boldsymbol{m}^i = \sum_{j \in \mathcal{J}_{-i}} V^j(\boldsymbol{y}^{-i})z_j^i = \sum_{j \in \mathcal{J}_{-i}} V^j(\bar{\boldsymbol{y}}^{-i})\hat{z}_j^i \tag{4}$$

At the equilibrium the agent could also have chosen the same inside variables as the planner $(\hat{\kappa}^i, (\hat{z}_j^i, j \in \mathcal{J}^i)) = (\kappa^i, (z_j^i, j \in \mathcal{J}^i))$. By the remark made above, $\tilde{e}^i(\hat{\kappa}^i, \hat{\boldsymbol{z}}^i; \bar{\boldsymbol{y}}^{-i}) =$

$\tilde{e}^i(\kappa^i, \boldsymbol{z}^i; \boldsymbol{y}^{-i})$ so that e^i would also have been an optimal choice of effort of agent i. The date 0 consumption $\hat{x}_0^i$ of agent i would then have been

$$\hat{x}_0^i = \omega_0^i + \tilde{Q}_i^i(\hat{\kappa}^i, \hat{z}^i; \bar{\boldsymbol{y}}^{-i}) - \tilde{Q}^i(\hat{\kappa}^i, \hat{z}^i; \bar{\boldsymbol{y}}^{-i})\hat{z}^i - \hat{\kappa}^i$$

By Definition 3, $\tilde{Q}^i(\hat{\kappa}^i, \hat{z}^i; \bar{\boldsymbol{y}}^{-i}) = \bar{\boldsymbol{\pi}}\boldsymbol{V}(\boldsymbol{F}^i(\hat{\kappa}^i, \hat{e}^i); \bar{\boldsymbol{y}}^{-i})$ where $\hat{e}^i$ is an element of $\tilde{e}^i(\hat{\kappa}^i, \hat{z}^i; \bar{\boldsymbol{y}}^{-i})$ which maximizes the date 0 consumption.[12] Thus,

$$\begin{aligned}\hat{x}_0^i &\geq \omega_0^i + \bar{\boldsymbol{\pi}}\boldsymbol{F}^i(\hat{\kappa}^i, e^i) - \bar{\boldsymbol{\pi}}\boldsymbol{V}(\boldsymbol{F}^i(\hat{\kappa}^i, e^i), \bar{\boldsymbol{y}}^{-i})\hat{z}^i - \hat{\kappa}^i \\ &= \omega_0^i + \bar{\boldsymbol{\pi}}\boldsymbol{F}^i(\hat{\kappa}^i, e^i) - \bar{\boldsymbol{\pi}}\textstyle\sum_{j\in\mathcal{J}_{-i}} V^j(\bar{\boldsymbol{y}}^{-i})\hat{z}_j^i - \bar{\pi}\sum_{j\in\mathcal{J}^i} \boldsymbol{V}^j(\boldsymbol{F}^i(\hat{\kappa}^i, e^i))\hat{z}_j^i - \hat{\kappa}^i\end{aligned}$$

(4) and $\hat{\kappa}^i = \kappa^i$, $\hat{z}_j^i = z_j^i$, $j \in \mathcal{J}^i$ imply

$$\hat{x}_0^i \geq \omega_0^i + \bar{\boldsymbol{\pi}}\boldsymbol{F}^i(\kappa^i, e^i) - \bar{\boldsymbol{\pi}}\boldsymbol{m}^i - \bar{\boldsymbol{\pi}}\sum_{j\in\mathcal{J}^i} V^j(\boldsymbol{F}^i(\kappa^i, e^i))z_j^i - \kappa^i$$

which is equivalent to

$$\hat{x}_0^i \geq \omega_0^i + \bar{\boldsymbol{\pi}}\boldsymbol{F}^i(\kappa^i, e^i) - \bar{\boldsymbol{\pi}}\boldsymbol{x}_{\mathbf{1}}^i - \kappa^i$$

Since $U^i(\boldsymbol{x}^i, e^i) \geq U^i(\bar{x}^i, \bar{e}^i)$, $\forall i \in \mathcal{I}$, with strict inequality for some i, if $x_0^i < \hat{x}_0^i$ for some agent indifferent between $(\boldsymbol{x}^i, e^i)$ and $(\bar{\boldsymbol{x}}^i, \bar{e}^i)$, or if $x_0^i \leq \hat{x}_0^i$ for any agent who strictly prefers $(\boldsymbol{x}^i, e^i)$ to $(\bar{\boldsymbol{x}}^i, \bar{e}^i)$, then the optimality of an agent's equilibrium consumption would be contradicted. Thus, $x_0^i \geq \hat{x}_0^i$ $\forall i \in \mathcal{I}$, with strict inequality for some i, so that

$$\sum_{i\in\mathcal{I}} x_0^i > \sum_{i\in\mathcal{I}} \omega_0^i + \bar{\boldsymbol{\pi}}\sum_{i\in\mathcal{I}} \boldsymbol{F}^i(\kappa^i, e^i) - \bar{\boldsymbol{\pi}}\sum_{i\in\mathcal{I}} \boldsymbol{x}_{\mathbf{1}}^i - \sum_{i\in\mathcal{I}} \kappa^i$$

which contradicts the feasibility, namely, $\sum_{i\in\mathcal{I}} x_0^i \leq \sum_{i\in\mathcal{I}} \omega_0^i - \sum_{i\in\mathcal{I}} \kappa^i$ and $\sum_{i\in\mathcal{I}} \boldsymbol{x}_{\mathbf{1}}^i = \sum_{i\in\mathcal{I}} \boldsymbol{F}^i(\kappa^i, e^i)$, of the allocation chosen by the planner. □

When effort is non-observable, each entrepreneur chooses his effort level in his own interest despite the fact that it affects other agents in the economy through the payoff of the securities based on the profit of his firm (in particular the outside shareholders of his firm). This is the basic moral hazard problem created by the separation of ownership and control. In a CPO, the planner is fully aware of this externality and chooses the financial variables $(\boldsymbol{\kappa}, \boldsymbol{z})$ so as to mitigate this problem as much as possible. The choice of the portfolios $\boldsymbol{z} = (\boldsymbol{z}^i, i \in \mathcal{I})$ creates a reward structure, or contract, for each entrepreneur linking his payoff to the performance of his firm and the rest of the economy. When there are no index-based securities, this contract has the form

$$\phi^i(\boldsymbol{y}) = \sum_{j\in\mathcal{J}^i} V^j(y^i)z_j^i + m^i(\boldsymbol{y}^{-i})$$

where $\boldsymbol{y} = (y^i, \boldsymbol{y}^{-i})$, y^i being the realized output of the firm and $\boldsymbol{y}^{-i}$ the realized output of all other firms.

[12] If in Definition 3 (ii) of $\tilde{\boldsymbol{Q}}^i$ we were not selecting an effort choice $\hat{e}^i \in \tilde{e}^i(\kappa^i, \boldsymbol{z}^i; \bar{\boldsymbol{y}}^{-i})$ which maximizes the value of date 0 income of the entrepreneur, then some RCPP equilibria could be constrained inefficient (see Kihlstrom and Matthews, 1990).

The CPO problem, which amounts to choosing optimally the investment, risk, and incentive structure for the economy, is a generalized *principal-agent problem*, in which the planner (the principal) chooses the investment in each firm and the (constrained) optimal contract for each entrepreneur and investor in the economy. Proposition 1 asserts that, under the strong spanning condition, *a system of markets is capable of solving the principal agent problem* if agents have the behavior postulated in an RCPP equilibrium. The basic driving force for this optimality property of an RCPP equilibrium is that the social effect of each entrepreneur's choice of capital and reward structure—in particular the effect on outside investors—is transmitted to the entrepreneur through the rational price perceptions.

In choosing his financial decision $(\kappa^i, \boldsymbol{z}^i)$ in equilibrium, entrepreneur i takes into account how this decision affects the value

$$\tilde{Q}_i^i(\kappa^i, \boldsymbol{z}^i; \bar{\boldsymbol{y}}^{-i})(1 - z_i^i) - \sum_{j \in \mathcal{J}^i, j \neq i} \tilde{Q}_j^i(\kappa^i, \boldsymbol{z}^i; \bar{\boldsymbol{y}}^{-i}) z_j^i \tag{5}$$

which can be interpreted as the income he obtains from the sale of his equity net of the cost of options on his own equity used to decrease his risk and/or bond his interest to those of his firm's shareholders. When the assumption of rational competitive price perceptions is combined with the market clearing condition $(1 - \bar{z}_i^i = \sum_{k \neq i} \bar{z}_i^k, -\bar{z}_j^i = \sum_{k \neq i} \bar{z}_j^k)$, then (5) is equal to

$$\sum_{j \in \mathcal{J}^i} \bar{\boldsymbol{\pi}} V^j(\boldsymbol{F}^i(\kappa^i, \boldsymbol{z}^i), \bar{\boldsymbol{y}}^{-i}) \sum_{k \neq i} \bar{z}_j^k$$

which is exactly the value to all outside investors of the securities based on his firm. Thus, the price-perception functions $\tilde{\boldsymbol{Q}}^i$ induce an entrepreneur acting purely in his own self interest to take into account the external effect of his actions on the outside investors of his firm. In this way capital markets with informed participants can act as a disciplining device which ensures that self-interested behavior leads to a (constrained) socially optimal outcome.

4. Pareto optimality and the spanning–overlap condition

In the previous section, we examined the constrained optimality properties of RCPP equilibria of economies for which the security structure could in principle be very incomplete. In this section, we derive the abstract condition that a security structure must satisfy if it is to support Pareto optimal RCPP equilibria—that is, if some form of the First and Second Welfare Theorems is to hold for RCPP equilibria. From the standard GEI model without moral hazard we know that for general specifications of characteristics of the economy (preferences, endowments, technology) optimal risk sharing is attainable only if markets are complete.[13] If in addition the security structure is to provide appropriate incentives, then we show that for each entrepreneur there must be sufficient overlap between the subspace spanned by the securities which are influenced by his effort (the holdings of which directly affect his incentives) and the subspace spanned by the securities which are independent of

[13] See for example Magill–Shafer (1991, Section 4).

his effort and which can be used to adjust the risk profile of his income stream (without directly affecting his incentives). We call this strengthened condition that must be satisfied if a security structure is to support Pareto optimal equilibria, the *spanning–overlap condition*. In the next section, we derive conditions on the characteristics of the economy which imply that standard securities based on the outputs of the firms (bonds, equity, indices, and options) can satisfy the spanning–overlap condition.

To derive these results we introduce three analytical devices. The first is an artificial concept of equilibrium for which equilibria are always Pareto optimal: we call this an *artificial sole-proprietorship equilibrium* (ASP). This concept is artificial because it violates the basic postulate on which the paper is based—namely, that writing contracts contingent on states is unfeasible. Second, we transform an RCPP equilibrium into a more abstract mathematical form which permits a direct comparison to be made between an RCPP and an ASP equilibrium: we call this concept (which is equivalent to an RCPP equilibrium) an *abstract-RCPP equilibrium*. Finally, to make the analysis of this concept of equilibrium more tractable, we introduce a simplifying device which consists in replacing the incentive constraint expressing the optimal effort of an entrepreneur by the first-order condition that optimal effort must satisfy: the resulting equilibrium is called a *weak-RCPP equilibrium*. To permit this first-order approach to be used we assume that the effort of each entrepreneur is a continuous variable, so that $\mathbb{E}^i$ is an interval; to ensure that ASP equilibria are well behaved we make assumptions of convexity on the agents' preferences and on the technology of the firms, and to permit a first-order approach to be used, we assume that the utility and production functions are smooth.

Assumption $\mathcal{C}$ (Characteristics).

(i) For each $i \in \mathcal{I}_1$, $\mathbb{E}^i = \mathbb{R}_+$.

(ii) For each $i \in \mathcal{I}_1$, the production function $\boldsymbol{F}^i: \mathbb{R}^2_+ \to \mathbb{R}^S_+$ is concave, smooth, nondecreasing in κ^i and satisfies $\boldsymbol{F}^i(\kappa^i, 0) = 0$.

(iii) For each $i \in \mathcal{I}$ the utility function U^i is of the form $U^i(\boldsymbol{x}^i, e^i) = u_0^i(x_0^i) + u_1^i(\boldsymbol{x}_\mathbf{1}^i, e^i)$ where $u_0^i: \mathbb{R}_+ \to \mathbb{R}$ (respectively, $u_1^i: \mathbb{R}^{S+1}_+ \to \mathbb{R}$) is concave, smooth, increasing in x_0^i (respectively, $\boldsymbol{x}_\mathbf{1}^i$) and u_1^i is decreasing in e^i.

(iv) For each $i \in \mathcal{I}_1$, $(\partial u_1^i/\partial e^i)(\boldsymbol{x}_\mathbf{1}^i, e^i) \to -\infty$ as $e^i \to \infty$, and $(\partial F_s^i/\partial e^i)(\kappa^i, e^i) < M$ for some $M > 0$.

(v) For all $i \in \mathcal{I}$ and all $s \in \mathcal{S}$, $(\partial F_s^i/\partial \kappa^i)(\kappa^i, e^i) \to \infty$ as $\kappa^i \to 0$, $\forall e^i > 0$, and $(\partial F_s^i/\partial e^i)(\kappa^i, e^i) \to \infty$ as $e^i \to 0$, $\forall \kappa^i > 0$.

Note that we do not require that $F_s^i(\kappa^i, e^i)$ be an increasing function of e^i in each state: this allows us to include cases where the entrepreneur's effort can be directed to altering the risk profile of the output stream, which may require increasing output in "bad" states but perhaps decreasing it in "good" states. Assumption (iv) ensures that the optimal choice of effort is bounded, while assumption (v) guarantees that in any equilibrium all entrepreneurs produce.[14]

Recall the following definition.

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[14] If the production function does not depend on capital then we assume $(\partial F_s^i/\partial e^i)(e^i) \to \infty$ as $e^i \to 0$.

Definition 7. An allocation $(\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{\kappa}, \boldsymbol{e}) \in \mathbb{R}_+^{I(S+1)} \times \mathbb{R}_+^{IS} \times \mathbb{R}_+^{I} \times \mathbb{R}_+^{I}$ with $\boldsymbol{y}^i = \boldsymbol{F}^i(\kappa^i, e^i)$, $i \in \mathcal{I}$, is *feasible* if

$$\sum_{i \in \mathcal{I}} x_0^i \leq \sum_{i \in \mathcal{I}} \omega_0^i - \sum_{i \in \mathcal{I}} \kappa^i, \qquad \sum_{i \in \mathcal{I}} \boldsymbol{x}_{\mathbf{1}}^i \leq \sum_{i \in \mathcal{I}} \boldsymbol{F}^i(\kappa^i, e^i)$$

An allocation $(\boldsymbol{x}^*, \boldsymbol{y}^*, \boldsymbol{\kappa}^*, \boldsymbol{e}^*)$ *is Pareto optimal* if it is feasible and if there does not exist another feasible allocation $(\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{\kappa}, \boldsymbol{e})$ such that $U^i(\boldsymbol{x}^i, e^i) \geq U^i(\boldsymbol{x}^{i*}, e^{i*})$, $\forall i \in \mathcal{I}$, with strict inequality for at least one i.

Consider the artificial setting where there would be no costs associated with describing or verifying states of nature, so that financial contracts could be written contingent on the states of nature. Suppose that there were a complete set of such contracts—a complete set of Arrow securities—with payoffs independent of the agents' actions. Then there would be a simple mechanism for obtaining a Pareto optimal allocation, despite the non-observability of effort. The mechanism consists in separately resolving the problem of creating incentives and providing opportunities for risk sharing: each entrepreneur remains the sole proprietor of his firm so that he has both the full marginal benefit and cost of his effort and there is no distortion of incentives; then to adjust the risk profile of his income stream, he trades the appropriate Arrow securities (whose payoffs are independent of his actions). To describe the equilibrium which would prevail in such a setting, let the price of income at date 0 be normalized to 1 and let π_s denote the price (at date 0) of the Arrow security for state s which delivers one unit of income in state $s \in \mathcal{S}$. The budget set of agent i with Arrow securities and sole proprietorship is given by

$$B(\boldsymbol{\pi}, \omega_0^i, \boldsymbol{F}^i) = \left\{ (\boldsymbol{x}^i, e^i) \in \mathbb{R}_+^{S+2} \;\middle|\; \begin{array}{l} x_0^i = \omega_0^i - \boldsymbol{\pi}\boldsymbol{\zeta}^i - \kappa^i \\ \boldsymbol{x}_{\mathbf{1}}^i = \boldsymbol{\zeta}^i + \boldsymbol{F}^i(\kappa^i, e^i) \\ (\kappa^i, \boldsymbol{\zeta}^i) \in \mathbb{R}_+ \times \mathbb{R}^S \end{array} \right\}$$

where $\boldsymbol{\pi} = (\pi_1, \ldots, \pi_S)$ is the vector of prices and $\boldsymbol{\zeta}^i = (\zeta_1^i, \ldots, \zeta_S^i)$ is agent i's portfolio of the Arrow securities. As usual, the $S+1$ budget constraints with Arrow securities can be reduced to a single budget constraint, i.e. the set $B(\boldsymbol{\pi}, \omega_0^i, \boldsymbol{F}^i)$ can be written as

$$B(\boldsymbol{\pi}, \omega_0^i, \boldsymbol{F}^i) = \{(\boldsymbol{x}^i, e^i) \in \mathbb{R}_+^{S+2} | x_0^i + \boldsymbol{\pi}\boldsymbol{x}_{\mathbf{1}}^i = \omega_0^i + \boldsymbol{\pi}\boldsymbol{F}^i(\kappa^i, e^i) - \kappa^i\} \tag{6}$$

This leads to the following concept of equilibrium.

Definition 8. $(\bar{\boldsymbol{x}}, \bar{\boldsymbol{y}}, \bar{\boldsymbol{\kappa}}, \bar{\boldsymbol{e}}; \bar{\boldsymbol{\pi}})$ is an *artificial sole-proprietorship* (ASP) equilibrium if

(i) $(\bar{\boldsymbol{x}}^i, \bar{e}^i, \bar{\kappa}^i) \in \arg\max \{U^i(\boldsymbol{x}^i, e^i) | (\boldsymbol{x}^i, e^i) \in B(\bar{\boldsymbol{\pi}}, \omega_0^i, \boldsymbol{F}^i)\}$ and $\bar{\boldsymbol{y}}^i = \boldsymbol{F}^i(\bar{\kappa}^i, \bar{e}^i)$, $i \in \mathcal{I}$,

(ii) $\sum_{i \in \mathcal{I}} \bar{x}_0^i = \sum_{i \in \mathcal{I}} \omega_0^i - \sum_{i \in \mathcal{I}} \bar{\kappa}^i, \quad \sum_{i \in \mathcal{I}} \bar{\boldsymbol{x}}_{\mathbf{1}}^i = \sum_{i \in \mathcal{I}} \boldsymbol{F}^i(\bar{\kappa}^i, \bar{e}^i)$.

An ASP is not precisely an Arrow–Debreu equilibrium, since there are $S+1+I_1$ "goods" in the economy—the $S+1$ incomes at dates 0 and 1, and the I_1 effort levels of the entrepreneurs—but there are only $S+1$ markets. Despite the absence of the I_1 markets for the effort levels of entrepreneurs, the First and Second Welfare Theorems—as well as the existence of equilibrium—are satisfied by ASP equilibria. This is due to the following two properties of "Robinson Crusoe" economies:

(i) An agent who is both a producer and a consumer in a convex economy can be split into two "personalities": an entrepreneur who maximizes profit, and a consumer who takes the profit as given and maximizes utility over his budget set (see Magill and Quinzii, 1996, for an account of this property in a general framework).
(ii) Agent i, as the entrepreneur running firm i, buys the input "effort for firm i" from only one agent, himself as a consumer. The market for effort e^i can, thus, be "internalized" in the joint consumer–producer maximum problem of agent i in an ASP: any other ownership structure of the firm would fail to lead to Pareto optimality in the absence of a market for effort.

Proposition 2 (Properties of ASP equilibrium).

(i) *For any* $\boldsymbol{\omega}_0 \in \mathbb{R}^I_+$, $\boldsymbol{\omega}_0 \neq 0$, *there exists an ASP equilibrium.*
(ii) *If* $(\bar{\boldsymbol{x}}, \bar{\boldsymbol{y}}, \bar{\boldsymbol{\kappa}}, \bar{\boldsymbol{e}}; \bar{\boldsymbol{\pi}})$ *is an ASP equilibrium, then the allocation* $(\bar{\boldsymbol{x}}, \bar{\boldsymbol{y}}, \bar{\boldsymbol{\kappa}}, \bar{\boldsymbol{e}})$ *is Pareto optimal.*
(iii) *For any Pareto optimal allocation* $(\bar{\boldsymbol{x}}, \bar{\boldsymbol{y}}, \bar{\boldsymbol{\kappa}}, \bar{\boldsymbol{e}})$ *there exist incomes* $\boldsymbol{\omega}_0 \in \mathbb{R}^I$ *and state prices* $\bar{\boldsymbol{\pi}} \in \mathbb{R}^S_{++}$ *such that* $(\bar{\boldsymbol{x}}, \bar{\boldsymbol{y}}, \bar{\boldsymbol{\kappa}}, \bar{\boldsymbol{e}}; \bar{\boldsymbol{\pi}})$ *is an ASP equilibrium of the economy* $\mathcal{E}(\boldsymbol{U}, \boldsymbol{\omega}_0, \boldsymbol{F})$.

Proof. The existence proof is standard. To prove the equivalence between PO allocations and ASP equilibria in the differentiable case if suffices to note that the FOC for Pareto optimality are the same as the FOC for the agents' maximum problems in an ASP equilibrium:

$$\frac{\partial u_1^i(\bar{\boldsymbol{x}}_1^i, \bar{e}^i)/\partial x_s^i}{u_0^{i'}(\bar{x}_0^i)} = \bar{\pi}_s, \qquad -\frac{\partial u_1^i(\bar{\boldsymbol{x}}_1^i, \bar{e}^i)/\partial e^i}{u_0^{i'}(\bar{x}_0^i)} = \sum_{s\in S} \bar{\pi}_s \frac{\partial F_s^i}{\partial e^i}(\bar{\kappa}^i, \bar{e}^i),$$

$$1 = \sum_{s\in S} \bar{\pi}_s \frac{\partial F_s^i}{\partial \kappa^i}(\bar{\kappa}^i, \bar{e}^i),\ i \in \mathcal{I}$$

In both cases the problems are convex so that the FOC are necessary and sufficient. Differentiability assumptions are not required for the results of Proposition 2. We leave it to the reader to adapt the standard arguments in the non-differentiable case. □

If an RCPP equilibrium is to lead to a Pareto optimum, then the equilibrium needs to mimic an ASP equilibrium: to this end it is useful to write the budget set of an RCPP equilibrium in a form that brings it closer to the ASP budget set. This can be done in two steps as follows. Incorporating Definition 3(ii) of price perceptions directly into agent i's date 0 budget equation (in Definition 1(i)), we can write the budget set in an RCPP equilibrium as

$$\mathbb{B}(\boldsymbol{\pi}, \omega_0^i, \boldsymbol{F}^i, \boldsymbol{y}) = \left\{ (\boldsymbol{x}^i, e^i) \in \mathbb{R}^{S+2}_+ \;\middle|\; \begin{array}{l} x_0^i = \omega_0^i + \boldsymbol{\pi}\boldsymbol{F}(\kappa^i, e^i) - \boldsymbol{\pi}\boldsymbol{V}(\boldsymbol{F}^i(\kappa^i, e^i), \boldsymbol{y}^{-i})\boldsymbol{z}^i - \kappa^i \\ \boldsymbol{x}_1^i = \boldsymbol{V}(\boldsymbol{F}^i(\kappa^i, e^i), \boldsymbol{y}^{-i})\boldsymbol{z}^i \\ e^i \in \tilde{e}^i(\kappa^i, \boldsymbol{z}^i; \boldsymbol{y}^{-i}) \\ (\kappa^i, \boldsymbol{z}^i) \in \mathbb{R}_+ \times \mathbb{R}^J \end{array} \right\}$$

where $\boldsymbol{\pi} \in \mathbb{R}^S_{++}$ satisfies the no-arbitrage condition $\boldsymbol{\pi} \boldsymbol{V}(\boldsymbol{y}) = \boldsymbol{q}$. Note that if we multiply the date 1 budget equation for state s by π_s and add, we obtain $\boldsymbol{\pi x}^i_{\mathbf{1}}$, namely the present value of agent i's date 1 consumption stream: adding the present value of the date 1 equations to the date 0 equation gives the equivalent budget set[15]

$$\mathcal{B}(\boldsymbol{\pi}, \omega^i_0, \boldsymbol{F}^i, \boldsymbol{y}) = \left\{ (\boldsymbol{x}^i, e^i) \in \mathbb{R}^{S+2}_{+} \;\middle|\; \begin{array}{l} x^i_0 + \boldsymbol{\pi x}^i_{\mathbf{1}} = \omega^i_0 + \boldsymbol{\pi F}^i(\kappa^i, e^i) - \kappa^i \\ \boldsymbol{x}^i_{\mathbf{1}} = \boldsymbol{V}(\boldsymbol{F}^i(\kappa^i, e^i), \boldsymbol{y}^{-i})\boldsymbol{z}^i \\ e^i \in \tilde{e}^i(\kappa^i, \boldsymbol{z}^i; \boldsymbol{y}^{-i}) \\ (\kappa^i, \boldsymbol{z}^i) \in \mathbb{R}_+ \times \mathbb{R}^J \end{array} \right\} \tag{7}$$

We are, thus, led to a concept of equilibrium in which the vector of state (present-value) prices $\boldsymbol{\pi}$ implicit in the vector of security prices $\boldsymbol{q}$ in an RCPP equilibrium become the explicit prices: the competitive part of the price perceptions lead to the date 0 present-value constraint, while the rational part is incorporated into the incentive constraint $e^i \in \tilde{e}^i(\kappa^i, \boldsymbol{z}^i; \boldsymbol{y}^{-i})$.

Definition 9. $(\bar{\boldsymbol{x}}, \bar{\boldsymbol{y}}, \bar{\boldsymbol{e}}, \bar{\boldsymbol{\kappa}}, \bar{z}; \bar{\boldsymbol{\pi}})$ is an *abstract RCPP equilibrium* of the economy $\mathcal{E}(\boldsymbol{U}, \boldsymbol{\omega}_0, \boldsymbol{F}, \boldsymbol{V})$ if

(i) for each agent $i \in \mathcal{I}$, $(\bar{\boldsymbol{x}}^i, \bar{e}^i, \bar{\kappa}^i, \bar{\boldsymbol{z}}^i)$ is the action which maximizes $U^i(\boldsymbol{x}^i, e^i)$ over the budget set $\mathcal{B}(\bar{\boldsymbol{\pi}}, \omega^i_0, \boldsymbol{F}^i, \bar{\boldsymbol{y}})$, and $(\bar{\kappa}^i, \bar{\boldsymbol{z}}^i)$ finances $\bar{\boldsymbol{x}}^i$,
(ii) $\bar{\boldsymbol{y}}^i = \boldsymbol{F}^i(\bar{\kappa}^i, \bar{e}^i), \quad i \in \mathcal{I}$,
(iii) $\sum_{i\in\mathcal{I}} \bar{z}^i_j = 1, \quad j = 1, \ldots, I, \quad \sum_{i\in\mathcal{I}} \bar{z}^i_j = 0, \quad j = I+1, \ldots, J.$

Remark. It is easy to deduce from the previous reasoning that if there is partial spanning at $\bar{\boldsymbol{y}}$, then $(\bar{\boldsymbol{x}}, \bar{\boldsymbol{y}}, \bar{\boldsymbol{e}}, \bar{\boldsymbol{\kappa}}, \bar{z}; \bar{\boldsymbol{\pi}})$ an abstract RCPP equilibrium if and only if $(\bar{\boldsymbol{x}}, \bar{\boldsymbol{y}}, \bar{\boldsymbol{e}}, \bar{\boldsymbol{\kappa}}, \bar{z}, \bar{\boldsymbol{q}}; \tilde{\boldsymbol{Q}})$ with $\bar{\boldsymbol{q}} = \boldsymbol{V}(\bar{\boldsymbol{y}})$ and $\tilde{\boldsymbol{Q}}$ given by Definition 3(ii) is an RCPP equilibrium.

The abstract form of an RCPP equilibrium makes clear that an RCPP equilibrium can be viewed as a "constrained" ASP: the additional constraints are *the spanning condition* that the date 1 consumption must be obtained through trading securities based on the observable outputs of the firms, and the *incentive constraint* that effort of each entrepreneur is optimal given his portfolio. Let $(\bar{\boldsymbol{x}}, \bar{\boldsymbol{y}}, \bar{\boldsymbol{\kappa}}, \bar{\boldsymbol{e}}; \bar{\boldsymbol{\pi}})$ be an ASP equilibrium: is it possible that for some sufficiently rich security structure $\boldsymbol{V}$ the same allocation is obtained as an abstract RCPP equilibrium of $\mathcal{E}(\boldsymbol{U}, \boldsymbol{\omega}_0, \boldsymbol{F}, \boldsymbol{V})$? For this to happen it is sufficient that the choice of each agent in the "big" budget set of the ASP equilibrium also satisfies the spanning and incentive constraints of the abstract RCPP, in which case such a security structure yields at least one RCPP which is Pareto optimal. To make finding such security structures a tractable problem we introduce our third and final device: we replace the incentive constraint $e^i \in \tilde{e}^i(\kappa^i, \boldsymbol{z}^i; \boldsymbol{y}^{-i})$ by the requirement that e^i satisfies the first-order condition for optimal effort.

15 A reader familiar with *Theory of Incomplete Markets* (Magill and Quinzii, 1996, Section 10) will note that the budget set $\mathcal{B}$ written in terms of the vector of state prices $\boldsymbol{\pi}$ is the equivalent for this model of the budget set of an agent in a *no-arbitrage equilibrium*. However, in a model with moral hazard the portfolio variables $z = (z^1, \ldots, z^I)$, which disappear in the standard no-arbitrage equilibrium, need to be retained since they condition the incentives of the entrepreneurs.

Since we want the class of security structures that we consider to include options, we assume that the payoff functions $V^j\colon \mathbb{R}^I_+ \to \mathbb{R}$ can have points of non-differentiability, but have well-defined left and right derivatives $(\partial V^j/\partial y^i_-)$ and $(\partial V^j/\partial y^i_+)$ everywhere on $\mathbb{R}^I_+$. To express the marginal conditions for the optimal choice of effort we introduce notation for the marginal cost and marginal benefit of an additional unit of effort by agent i. Let $\pi^i_s(\boldsymbol{x}^i, e^i) = (\partial u^i_1(\boldsymbol{x}^i_{\boldsymbol{1}}, e^i)/\partial x^i_s)/u^{i'}(x^i_0)$ denote the present value (to agent i) of an additional unit of income in state s. Let $b^i_j(\boldsymbol{x}^i, \kappa^i, e^i, \boldsymbol{y}^{-i})[b^i_{j-}(\,), b^i_{j+}(\,)]$ denote the derivative [respectively, left, right derivative] of the present value of security j's payoff with respect to agent i's effort e^i,

$$b^i_j(\boldsymbol{x}^i, \kappa^i, e^i, \boldsymbol{y}^{-i}) = \frac{\partial}{\partial e^i}\left(\sum_{s\in\mathcal{S}} \pi^i_s(\boldsymbol{x}^i, e^i) V^j(F^i_s(\kappa^i, e^i), \boldsymbol{y}^{-i}_s)\right)$$

$b^i_{j-}[b^i_{j+}]$ being defined with the left [right] derivative $(\partial/\partial e^i_-)$ [resp. $(\partial/\partial e^i_+)$]. If V^j is differentiable at $(F^i_s(\kappa^i, e^i), \boldsymbol{y}^{-i}_s)$ for all $s \in \mathcal{S}$, then $b^i_{j-}(\boldsymbol{x}^i, \kappa^i, e^i, \boldsymbol{y}^{-i}) = b^i_{j+}(\boldsymbol{x}^i, \kappa^i, e^i, \boldsymbol{y}^{-i}) = b^i_j(\boldsymbol{x}^i, \kappa^i, e^i, \boldsymbol{y}^{-i})$, and if $j \in \mathcal{J}_{-i}$, security j is independent of the effort of agent i, so that $b^i_j = b^i_{j+} = b^i_{j-} \equiv 0$. Define the vectors of derivatives

$$\boldsymbol{b}^i_- = (b^i_{1-}, \dots, b^i_{J-}), \qquad \boldsymbol{b}^i_+ = (b^i_{1+}, \dots, b^i_{J+}), \qquad \boldsymbol{b}^i = (b^i_1, \dots, b^i_J)$$

of the present values of the J securities with respect to e^i. Given that he holds the portfolio $\boldsymbol{z}^i$ of the securities, the marginal benefit to entrepreneur i from a small change in his effort is $b^i_-(\boldsymbol{x}^i, \kappa^i, e^i, \boldsymbol{y}^{-i})\boldsymbol{z}^i = \sum_{j=1}^J b^i_{j-}(\boldsymbol{x}^i, \kappa^i, e^i, \boldsymbol{y}^{-i})z^i_j$ for a decrease, $b^i_+(\boldsymbol{x}^i, \kappa^i, e^i, \boldsymbol{y}^{-i})\boldsymbol{z}^i$ for an increase, while the marginal cost of effort (measured in units of date 0 consumption) is

$$\gamma^i(\boldsymbol{x}^i, e^i) = -\frac{\partial u^i_1(\boldsymbol{x}^i_{\boldsymbol{1}}, e^i)/\partial e^i}{u^{i'}_0(x^i_0)}$$

Thus, replacing the incentive constraint $e^i \in \tilde{e}^i(\kappa^i, \boldsymbol{z}^i; \boldsymbol{y}^{-i})$ in the budget set (7) by the first-order condition for optimal effort, leads to the following "weakened" version of an (abstract) RCPP equilibrium.

Definition 10. $(\bar{\boldsymbol{x}}, \bar{\boldsymbol{y}}, \bar{\boldsymbol{e}}, \bar{\boldsymbol{\kappa}}, \bar{\boldsymbol{z}}; \bar{\boldsymbol{\pi}})$ is a *weak-RCPP equilibrium* if

(i) for each agent $i \in \mathcal{I}$, $(\bar{\boldsymbol{x}}^i, \bar{e}^i, \bar{\kappa}^i, \bar{\boldsymbol{z}}^i)$ is the action which maximizes $U^i(\boldsymbol{x}^i, e^i)$ over the budget set

$$\mathcal{B}'(\bar{\boldsymbol{\pi}}, \omega^i_0, \boldsymbol{F}^i, \bar{\boldsymbol{y}}) = \left\{ (\boldsymbol{x}^i, e^i) \in \mathbb{R}^{S+2}_+ \;\middle|\; \begin{array}{l} x^i_0 + \bar{\boldsymbol{\pi}}\boldsymbol{x}^i_{\boldsymbol{1}} = \omega^i_0 + \bar{\boldsymbol{\pi}}\boldsymbol{F}^i(\kappa^i, e^i) - \kappa^i \\ \boldsymbol{x}^i_{\boldsymbol{1}} = \boldsymbol{V}(\boldsymbol{F}^i(\kappa^i, e^i), \bar{\boldsymbol{y}}^{-i})\boldsymbol{z}^i \\ b^i_-(\boldsymbol{x}^i, \kappa^i, e^i, \bar{\boldsymbol{y}}^{-i})\boldsymbol{z}^i \ge \gamma^i(\boldsymbol{x}^i, e^i) \ge b^i_+(\boldsymbol{x}^i, \kappa^i, e^i, \bar{\boldsymbol{y}}^{-i})\boldsymbol{z}^i \\ (\kappa^i, \boldsymbol{z}^i) \in \mathbb{R}_+ \times \mathbb{R}^J \end{array} \right\} \tag{8}$$

and $(\bar{\kappa}^i, \bar{z}^i)$ finances $\bar{\boldsymbol{x}}^i$.

(ii) $\bar{\boldsymbol{y}}^i = \boldsymbol{F}^i(\bar{\kappa}^i, \bar{e}^i), \quad i \in \mathcal{I}$.

(iii) $\sum_{i\in\mathcal{I}} \bar{z}^i_j = 1, \quad j = 1, \dots, I, \qquad \sum_{i\in\mathcal{I}} \bar{z}^i_j = 0, \quad j = I+1, \dots, J$.

Consider an ASP equilibrium $(\bar{\boldsymbol{x}}, \bar{\boldsymbol{y}}, \bar{\boldsymbol{\kappa}}, \bar{\boldsymbol{e}}; \bar{\boldsymbol{\pi}})$ and a security structure $\boldsymbol{V}$ which is differentiable at $\bar{\boldsymbol{y}}$. To find a vector of portfolios $\bar{z} = (\bar{z}^1, \dots, \bar{z}^I)$ such that $(\bar{\boldsymbol{x}}, \bar{\boldsymbol{y}}, \bar{\boldsymbol{\kappa}}, \bar{\boldsymbol{e}}, \bar{z}; \bar{\boldsymbol{\pi}})$ is a weak-RCPP equilibrium for the security structure $\boldsymbol{V}$, two conditions must be satisfied: first, for each agent $i \in \mathcal{I}$ there must exist a portfolio $\bar{z}^i \in \mathbb{R}^J$ such that

$$\begin{bmatrix} \boldsymbol{V}(\bar{\boldsymbol{y}}) \\ b^i(\bar{\boldsymbol{x}}^i, \bar{\kappa}^i, \bar{e}^i, \bar{\boldsymbol{y}}^{-i}) \end{bmatrix} \bar{z}^i = \begin{bmatrix} \bar{\boldsymbol{x}}^i_{\mathbf{1}} \\ \gamma^i(\bar{\boldsymbol{x}}^i, \bar{e}^i) \end{bmatrix} \tag{SI}$$

and second, $\sum_{i\in\mathcal{I}} \bar{z}^i_j = 1$, $j = 1, \dots, I$, $\sum_{i\in\mathcal{I}} \bar{z}^i_j = 0$, $j = I+1, \dots, J$ so that the security markets clear. To simplify notation let $\bar{b}^i_j = b^i_j(\bar{\boldsymbol{x}}^i, \bar{\kappa}^i, \bar{e}^i, \bar{\boldsymbol{y}}^{-i})$.

Proposition 3 (Second Welfare Theorem). *Let $(\bar{\boldsymbol{x}}, \bar{\boldsymbol{y}}, \bar{\boldsymbol{\kappa}}, \bar{\boldsymbol{e}}; \bar{\boldsymbol{\pi}})$ be an ASP equilibrium. If $\boldsymbol{V}$ is a security structure based on the observable outputs of the firms, which is differentiable at $\bar{\boldsymbol{y}}$, and satisfies*

(i) (SPANNING) *Rank $\boldsymbol{V}(\bar{\boldsymbol{y}}) = S$.*

(ii) (OVERLAP) *For each $i \in \mathcal{I}_1$, there exists an outside income stream $\boldsymbol{v}^i \in \mathcal{V}_{-i}(\bar{\boldsymbol{y}})$ and coefficients $(\lambda^i_j)_{j\in\mathcal{J}^i\cup\mathcal{J}^i_{-i}}$ such that*

$$(\alpha)\ \boldsymbol{v}^i = \sum_{j\in\mathcal{J}^i\cup\mathcal{J}^i_{-i}} \boldsymbol{V}^j(\bar{\boldsymbol{y}})\lambda^i_j,$$

$$(\beta)\ \sum_{j\in\mathcal{J}^i\cup\mathcal{J}^i_{-i}} \bar{b}^i_j\lambda^i_j \neq 0,$$

then there exist portfolios $\bar{z} = (\bar{z}^1, \dots, \bar{z}^I)$ such that $(\bar{\boldsymbol{x}}, \bar{\boldsymbol{y}}, \bar{\boldsymbol{\kappa}}, \bar{\boldsymbol{e}}, \bar{z}; \bar{\boldsymbol{\pi}})$ is a weak-RCPP equilibrium.

Proof. It is clear that for an entrepreneur $i \in \mathcal{I}$, the budget set (8) is contained in the ASP budget set (6), i.e. $\mathcal{B}'(\bar{\boldsymbol{\pi}}, \omega^i_0, \boldsymbol{F}^i, \bar{\boldsymbol{y}}) \subset B(\bar{\boldsymbol{\pi}}, \omega^i_0, \boldsymbol{F}^i)$, while for an investor $i \in \mathcal{I}_2$ the two budget sets coincide, since by (i) rank $\boldsymbol{V}(\bar{\boldsymbol{y}}) = S$. Suppose for the moment that we can show that the optimal choice $(\bar{\boldsymbol{x}}^i, \bar{\kappa}^i, \bar{e}^i)$ of each entrepreneur $i \in \mathcal{I}_1$ in the larger budget set B can also be obtained in $\mathcal{B}'$ with a portfolio $\bar{z}^i$: then clearly $(\bar{\boldsymbol{x}}^i, \bar{\kappa}^i, \bar{e}^i, \bar{z}^i)$ is optimal in $\mathcal{B}'$. The proof of Proposition 3 can then be completed by the following argument. Choose an investor and call him agent I: such an investor exists since $\mathcal{I}_2 \neq \emptyset$. For each of the other investors $i \in \mathcal{I}_2$, $i \neq I$, choose a portfolio $\bar{z}^i \in \mathbb{R}^J$ such that $\bar{\boldsymbol{x}}^i_{\mathbf{1}} = \boldsymbol{V}(\bar{\boldsymbol{y}})\bar{z}^i$: such a portfolio exists since the markets are complete. For agent I choose

$$\bar{z}^I_j = 1 - \sum_{i\in\mathcal{I}, i\neq I} \bar{z}^i_j, \quad j = 1, \dots, I, \qquad \bar{z}^I_j = -\sum_{i\in\mathcal{I}, i\neq I} \bar{z}^i_j, \quad j = I+1, \dots, J$$

This ensures that the market clearing equations hold for the securities, and this in turn implies that $\boldsymbol{V}(\bar{\boldsymbol{y}})\sum_{i\in\mathcal{I}}\bar{z}^i=\sum_{i\in\mathcal{I}_1}\bar{\boldsymbol{y}}^i$ (the first I securities are equity with $\bar{\boldsymbol{y}}^i=0$ if $i\in\mathcal{I}_2$, and the remaining securities are in zero net supply). Since for all $i\neq I$, $\bar{\boldsymbol{x}}^i_{\mathbf{1}}=\boldsymbol{V}(\bar{\boldsymbol{y}})\bar{z}^i$, and since markets clear at an ASP

$$\bar{\boldsymbol{x}}^I_{\mathbf{1}}=\sum_{i\in\mathcal{I}_1}\bar{\boldsymbol{y}}^i-\sum_{i\neq I}\boldsymbol{x}^i_{\mathbf{1}}=\boldsymbol{V}(\bar{\boldsymbol{y}})\bar{z}^I$$

Since the date 0 constraint in $\mathcal{B}'$ is the same as in the budget set B, for each investor i in $\mathcal{I}_2$, $\bar{\boldsymbol{x}}^i\in\mathcal{B}'$ and is optimal in this budget set.

It only remains to prove that for each entrepreneur $i\in\mathcal{I}_1$, $\bar{\boldsymbol{x}}^i_{\mathbf{1}}$ lies in the budget set $\mathcal{B}'$. We begin by choosing a portfolio $\tilde{z}^i\in\mathbb{R}^J$ such that $\bar{\boldsymbol{x}}^i_{\mathbf{1}}=\boldsymbol{V}(\bar{\boldsymbol{y}})\tilde{z}^i$: such a portfolio exists since rank $\boldsymbol{V}(\bar{\boldsymbol{y}})=S$. Let us decompose the stream $\bar{\boldsymbol{x}}^i_{\mathbf{1}}$ into components on the i-independent and i-dependent subspaces

$$\begin{aligned}\bar{\boldsymbol{x}}^i_{\mathbf{1}}&=\sum_{j\in\mathcal{J}_{-i}}V^j(\bar{\boldsymbol{y}})\tilde{z}^i_j+\sum_{j\in\mathcal{J}^i\cup\mathcal{J}^i_{-i}}V^j(\bar{\boldsymbol{y}})\tilde{z}^i_j\\&=\sum_{j\in\mathcal{J}_{-i}}V^j(\bar{\boldsymbol{y}})\tilde{z}^i_j-\rho\boldsymbol{v}^i+\sum_{j\in\mathcal{J}^i\cup\mathcal{J}^i_{-i}}V^j(\bar{\boldsymbol{y}})(\tilde{z}^i_j+\rho\lambda^i_j)\end{aligned}$$

for any $\rho\in\mathbb{R}$, where $\boldsymbol{v}^i$ and (λ^i_j) are given by (ii). Since $\boldsymbol{v}^i\in\mathcal{V}_{-i}(\bar{\boldsymbol{y}})$ there exist (μ^i_j) such that $\boldsymbol{v}^i=\sum_{j\in\mathcal{J}_{-i}}V^j(\bar{\boldsymbol{y}})\mu^i_j$. Thus, for all $\rho\in\mathbb{R}$, there exist a portfolio $\boldsymbol{z}^{i\rho}$ with

$$\boldsymbol{z}^{i\rho}=\begin{cases}\tilde{z}^i_j-\rho\mu^i_j, & \text{if } j\in\mathcal{J}_{-i}\\ \tilde{z}^i_j+\rho\lambda^i_j, & \text{if } j\in\mathcal{J}^i\cup\mathcal{J}^i_{-i}\end{cases}$$

which leads to the same consumption stream $\bar{\boldsymbol{x}}^i_{\mathbf{1}}$. Since by (ii) (β), $\sum_{j\in\mathcal{J}^i\cup\mathcal{J}^i_{-i}}\bar{b}^i_j\lambda^i_j\neq 0$, there exists $\bar{\rho}\in\mathbb{R}$ such that

$$\bar{\boldsymbol{b}}^i\boldsymbol{z}^{i\bar{\rho}}=\sum_{j\in\mathcal{J}^i\cup\mathcal{J}^i_{-i}}\bar{b}^i_j\tilde{z}^i_j+\bar{\rho}\left(\sum_{j\in\mathcal{J}^i\cup\mathcal{J}^i_{-i}}\bar{b}^i_j\lambda^i_j\right)=\gamma^i(\bar{\boldsymbol{x}}^i,\bar{e}^i).$$

We know that $(\bar{\boldsymbol{x}}^i,\bar{e}^i,\bar{\kappa}^i)$ satisfies the date 0 budget equation in $\mathcal{B}'$, since $(\bar{\boldsymbol{x}},\bar{\boldsymbol{y}},\bar{\boldsymbol{\kappa}},\bar{\boldsymbol{e}};\bar{\boldsymbol{\pi}})$ is an ASP equilibrium. If we let $\bar{z}^i=\boldsymbol{z}^{i\bar{\rho}}$, then the spanning and incentive (FOC for effort) equations (SI) are also satisfied, so that $(\bar{\boldsymbol{x}},\bar{e}^i,\bar{\kappa}^i,\bar{z}^i)$ is optimal in $\mathcal{B}'$. □

The pair of conditions ((i) and (ii)), which we refer to as the *spanning–overlap condition* make precise the sense in which the security structure must be sufficiently rich if an ASP equilibrium is to be realizable as an RCPP equilibrium. The first condition—complete markets—requires that each agent can attain any desired risk profile for his date 1 consumption stream. This is the usual condition that must be satisfied in a GEI equilibrium without moral hazard. The second property that the security structure must have—the overlap condition—requires that for each entrepreneur i there be an appropriate overlap between the subspace $\mathcal{V}_{-i}(\bar{\boldsymbol{y}})$ spanned by the i-independent securities ($j \in \mathcal{J}_{-i}$), and the subspace $\hat{\mathcal{V}}^i(\bar{\boldsymbol{y}}) = \mathcal{V}^i(\bar{\boldsymbol{y}}) + \mathcal{V}^i_{-i}(\bar{\boldsymbol{y}})$ spanned by the i-dependent securities ($j \in \mathcal{J}^i \cup \mathcal{J}^i_{-i}$) : more precisely there must be a non-zero income stream ($\boldsymbol{v}^i$) in the intersection of these two subspaces (condition (ii) (α)) and a portfolio (λ^i_j) of i-dependent securities which generates it, which has a non-trivial incentive effect for entrepreneur i (condition (ii) (β)). When these conditions are satisfied, by adjusting the component of this income stream on the i-dependent securities, the magnitude of the incentive effect can be adjusted to the appropriate level while leaving the risk profile of the entrepreneur's consumption stream unchanged. In this way both risk sharing and incentives can be completely controlled.

Let us now show that the spanning–overlap condition is sufficient to prove the First Welfare Theorem for weak-RCPP equilibria.

Proposition 4 (First Welfare Theorem). *If $(\bar{\boldsymbol{x}}, \bar{\boldsymbol{y}}, \bar{\boldsymbol{e}}, \bar{\boldsymbol{\kappa}}, \bar{z}; \bar{\boldsymbol{\pi}})$ is a weak-RCPP equilibrium of the economy $\mathcal{E}(\boldsymbol{U}, \boldsymbol{\omega_0}, \boldsymbol{F}, \boldsymbol{V})$ for which the security structure $\boldsymbol{V}$ at $\bar{\boldsymbol{y}}$ is differentiable and satisfies the spanning–overlap condition, then $(\bar{\boldsymbol{x}}, \bar{\boldsymbol{y}}, \bar{\boldsymbol{\kappa}}, \bar{\boldsymbol{e}})$ is Pareto optimal.*

Proof. For each investor $i \in \mathcal{I}_2$, $\bar{\boldsymbol{x}}^i$ maximizes $U^i(\boldsymbol{x}^i, 0)$ over the budget set

$$\mathcal{B}'(\bar{\boldsymbol{\pi}}, \omega_0^i, \bar{\boldsymbol{y}}) = \left\{ \boldsymbol{x}^i \in \mathbb{R}^{S+1}_+ \;\middle|\; \begin{array}{l} x_0^i + \bar{\boldsymbol{\pi}}\boldsymbol{x}^i_{\boldsymbol{1}} = \omega_0^i \\ \boldsymbol{x}^i_{\boldsymbol{1}} = \boldsymbol{V}(\bar{\boldsymbol{y}})z^i,\, z^i \in \mathbb{R}^J \end{array} \right\}$$

Since the markets are complete, the second constraint is automatically satisfied so that the investor's weak-RCPP budget set coincides with the ASP budget set

$$\mathcal{B}'(\bar{\boldsymbol{\pi}}, \omega_0^i, \bar{\boldsymbol{y}}) = B(\bar{\boldsymbol{\pi}}, \omega_0^i) = \{\boldsymbol{x}^i \in \mathbb{R}^{S+1}_+ | x_0^i + \bar{\boldsymbol{\pi}}\boldsymbol{x}^i_{\boldsymbol{1}} = \omega_0^i\}$$

For each entrepreneur $i \in \mathcal{I}_1$, $(\bar{\boldsymbol{x}}^i, \bar{e}^i, \bar{\kappa}^i, \bar{z}^i)$ maximizes $U^i(\bar{\boldsymbol{x}}^i, \bar{e}^i)$ over the weak-RCPP budget set

$$\mathcal{B}'(\bar{\boldsymbol{\pi}}, \omega_0^i, \boldsymbol{F}^i, \bar{\boldsymbol{y}}) = \left\{ (\boldsymbol{x}^i, e^i) \in \mathbb{R}^{S+2}_+ \;\middle|\; \begin{array}{l} x_0^i + \bar{\boldsymbol{\pi}}\boldsymbol{x}^i_{\boldsymbol{1}} = \omega_0^i + \bar{\boldsymbol{\pi}}\boldsymbol{F}^i(\kappa^i, e^i) - \kappa^i \\ \boldsymbol{x}^i_{\boldsymbol{1}} = \boldsymbol{V}(\boldsymbol{F}^i(\kappa^i, e^i), \bar{\boldsymbol{y}}^{-i})z^i \\ b^i(\boldsymbol{x}^i, \kappa^i, e^i, \bar{\boldsymbol{y}}^{-i})z^i = \gamma^i(\boldsymbol{x}^i, e^i) \\ (\kappa^i, z^i) \in \mathbb{R}_+ \times \mathbb{R}^J \end{array} \right\}$$

where the incentive constraint is written with equality since $\boldsymbol{V}$ is differentiable at $\bar{\boldsymbol{y}}$. If we show that the multipliers $\boldsymbol{\rho}^i = (\rho_1^i, \dots, \rho_S^i)$ associated with the date 1 spanning constraint ($\boldsymbol{x}^i_{\boldsymbol{1}} = \boldsymbol{V}z^i$), and the multiplier ϵ^i associated with the incentive constraint ($\gamma^i = \boldsymbol{b}^i z^i$) are zero, then the FOC for maximizing U^i over $\mathcal{B}'$ reduce to the FOC for maximizing U^i over the ASP budget set

$$B(\bar{\boldsymbol{\pi}}, \omega_0^i) = \{\boldsymbol{x}^i \in \mathbb{R}^{S+1}_+ | x_0^i + \bar{\boldsymbol{\pi}}\boldsymbol{x}^i_{\boldsymbol{1}} = \omega_0^i + \bar{\boldsymbol{\pi}}\boldsymbol{F}^i(\kappa^i, e^i) - \kappa^i\}$$

Then since $(U^i, \boldsymbol{F}^i)$ are concave, $(\bar{\boldsymbol{x}}^i, \bar{e}^i, \bar{\kappa}^i)$ maximizes $U^i(\boldsymbol{x}^i, e^i)$ over the ASP budget set $B(\bar{\boldsymbol{\pi}}, \omega_0^i)$.

To prove that the vector of multipliers $(\boldsymbol{\rho}^i, \epsilon^i)$ is zero, consider the FOC with respect to the portfolio $\boldsymbol{z}^i$ which must be satisfied at $(\bar{\boldsymbol{x}}, \bar{\boldsymbol{y}}, \bar{\boldsymbol{e}}, \bar{\boldsymbol{\kappa}}, \bar{\boldsymbol{z}}; \bar{\boldsymbol{\pi}})$ by each entrepreneur i

$$\sum_{s=1}^{S} \rho_s^i \boldsymbol{V}(F_s^i(\bar{\kappa}^i, \bar{e}^i), \bar{\boldsymbol{y}}_s^{-i}) + \epsilon^i \boldsymbol{b}^i(\bar{\boldsymbol{x}}^i, \bar{e}^i, \bar{\kappa}^i, \bar{\boldsymbol{y}}^{-i}) = \boldsymbol{0} \Leftrightarrow (\boldsymbol{\rho}^i, \epsilon^i) \begin{bmatrix} \boldsymbol{V}(\bar{\boldsymbol{y}}) \\ \bar{\boldsymbol{b}}^i \end{bmatrix} = \boldsymbol{0}$$

By the spanning-overlap conditon, the rank of this matrix is $S+1$ (see proof of Proposition 3), thus, the kernel is reduced to zero, so that $(\boldsymbol{\rho}^i, \epsilon^i) = \boldsymbol{0}$. Market clearing for the securities then implies $\sum_{i \in \mathcal{I}} \bar{\boldsymbol{x}}_{\mathbf{1}}^i = \sum_{i \in \mathcal{I}} \boldsymbol{F}^i(\kappa^i, e^i)$, and summing over the agents' ASP budget constraints gives $\sum_{i \in \mathcal{I}} x_0^i = \sum_{i \in \mathcal{I}} \omega_0^i - \sum_{i \in \mathcal{I}} \kappa_0^i$, so that $(\bar{\boldsymbol{x}}, \bar{\boldsymbol{y}}, \bar{\boldsymbol{\kappa}}, \bar{\boldsymbol{e}}; \bar{\boldsymbol{\pi}})$ is an ASP equilibrium. By Proposition 2 $(\bar{\boldsymbol{x}}, \bar{\boldsymbol{y}}, \bar{\boldsymbol{\kappa}}, \bar{\boldsymbol{e}})$ is Pareto optimal. □

We conjecture that the spanning–overlap condition characterizes Pareto optimal weak-RCPP equilibria, in the same sense that the condition rank $\boldsymbol{V}(\bar{\boldsymbol{y}}) = S$ characterizes Pareto optimal equilibria when there is no moral hazard: for a "generic" economy a weak-RCPP equilibrium in which the security structure does not satisfy the spanning–overlap condition is not Pareto optimal. The precise sense in which the property is generic may, however, be delicate since the payoffs of the securities depend on $\bar{\boldsymbol{y}}$ and are, thus, endogenous.[16] Leaving aside this question, we now exhibit conditions on the characteristics of the economy under which there exist security structures satisfying the spanning–overlap condition.

5. Capital market securities and the spanning–overlap condition

The overlap condition is trivially satisfied if the equity contract of each firm can be replicated by the equity contracts of other firms: if $\bar{\boldsymbol{y}}^i = \sum_{j \neq i} \mu_j^i \bar{\boldsymbol{y}}^j$ then $\boldsymbol{\nu} = \sum_{j \neq i} \mu_j^i \bar{\boldsymbol{y}}^j$ and $\lambda_i^i = 1$ satisfies (ii). Thus, the spanning–overlap condition can be satisfied with equity markets alone if the subspace spanned by equity of any $I_1 - 1$ firms is $\mathbb{R}^S$: every firm is redundant and entrepreneur i can stay the sole owner of his firm, diversifying risks on the equity of other firms without distorting incentives. This case, while a theoretical possibility of the model, seems unlikely to be representative, for as soon as firms have idiosyncratic risks which make them different from other firms, such a situation can not arise and we need to look for securities other than equity to resolve the problem of risk sharing and incentives.

Assuming that security markets are not complete with just the equity of $I_1 - 1$ firms, let us now examine conditions under which security structures composed of standard capital market instruments—bonds, equity, indices and options—can satisfy the spanning–overlap condition. Note first that in general the shocks which affect the characteristics $(\boldsymbol{U}, \boldsymbol{\omega}, \boldsymbol{F})$ of an economy are of two kinds: those which affect the consumer side $(\boldsymbol{U}, \boldsymbol{\omega})$—consisting of shocks to the preferences $\boldsymbol{U} = (U^1, \ldots, U^I)$ and (date 1 part of) the endowments $\boldsymbol{\omega} = (\boldsymbol{\omega}^1, \ldots, \boldsymbol{\omega}^I)$ of the agents—and those which affect the production side $\boldsymbol{F} = (\boldsymbol{F}^1, \ldots, \boldsymbol{F}^I)$. Broadly speaking insurance and related markets handle the risks (shocks) on the consumer

[16] Even without moral hazard, when the payoffs of the securities are endogenous—for example when there are several goods and the payoffs depend on the spot prices—the sense in which the characteristics of an economy must be "generic" so as to prove the result is relatively subtle (see Magill and Shafer, 1990).

side of the economy, while the capital markets deal with the production risks. For each of these classes of risk, incentive problems created by moral hazard can arise. In this paper, we are not attempting to deal with risks arising on the consumer side: we assume that shocks do not affect agents' date 1 endowment streams ($\boldsymbol{\omega}_{\mathbf{1}}^i = 0, \forall i \in \mathcal{I}$), and we will now assume that they do not affect their preferences. In short, the focus is on risks that affect the firms' output streams $\boldsymbol{F} = (\boldsymbol{F}^1, \ldots, \boldsymbol{F}^I)$.

Definition 11. The state space $\mathcal{S}$ is *technological* if for any pair of states s, s' in $\mathcal{S}$ with $s \neq s'$, there exists a firm $i \in \mathcal{I}$ such that $F_s^i(\kappa^i, e^i) \neq F_{s'}^i(\kappa^i, e^i)$ for all $(\kappa^i, e^i) \in \mathbb{R}_{++}^2$.

The requirement that the state space $\mathcal{S}$ is technological means that (a) the only shocks which it contains are those which affect production, (b) the state space is non-redundant in that it only contains shocks which affect the output of some firm, and (c) production shocks are assumed to have a similar effect for all capital-effort combinations (κ^i, e^i). These conditions on the state space and technology are satisfied in Example 1 above, and in Example 2 provided that $F_s^i(\kappa^i, e^i)$ takes S^i different values for all (κ^i, e^i). Part (c) eliminates the possibility, illustrated in Example 3, that changing the effort level changes the states which lead to “high” output.

When the state space is technological the vector of output $\bar{\boldsymbol{y}} = (\bar{\boldsymbol{y}}^1, \ldots, \bar{\boldsymbol{y}}^I)$ at an ASP equilibrium has the property that it distinguishes among the states, in the sense that for any two states s, s' with $s \neq s'$, $\bar{\boldsymbol{y}}_s \neq \bar{\boldsymbol{y}}_{s'}$. We want to show that this is the only property required to be able to exhibit a security structure composed of standard capital market securities which satisfies the spanning–overlap condition. To establish this result we will need some properties of options which we now derive.

Consider a security β with payoff stream $\boldsymbol{v}^\beta = (v_s^\beta)_{s\in\mathcal{S}} = (V^\beta(\bar{\boldsymbol{y}}_s))_{s\in\mathcal{S}}$, laying aside for the moment the fact that its payoff is a function of the output of the firms. If $\mathcal{M}^\beta$ denotes the index set for a collection of call options[17] on the security, let τ_m^β denote the striking price of option $m \in \mathcal{M}^\beta$ and let $\tau^\beta = (\tau_m^\beta)_{m\in\mathcal{M}^\beta}$ denote the vector of striking prices.

Definition 12. τ^β is said to be a *maximal* collection of options on security $\boldsymbol{v}^\beta$ if for every pair of states s, s' in $\mathcal{S}$ such that $v_s^\beta \neq v_{s'}^\beta$ (without loss of generality $v_s^\beta < v_{s'}^\beta$), there is a striking price $\tau_m^\beta, m \in \mathcal{M}^\beta$ such that $v_s^\beta < \tau_m^\beta < v_{s'}^\beta$.

Lemma 1. *Let $\boldsymbol{\tau}^\beta$ be a maximal collection of options on security β and let $J^\beta = \{\beta\} \cup \mathcal{M}^\beta$ denote the associated index set. If $\boldsymbol{v}^j$ denotes the payoff of security $j \in \mathcal{J}^\beta$, and if $\mathcal{V}^\beta = \langle \boldsymbol{v}^j, j \in \mathcal{J}^\beta \rangle$ is the subspace spanned by the securities in $\mathcal{J}^\beta$, then*

(a) $\mathbf{1} \in \mathcal{V}^\beta$.
(b) *Suppose that with each $j \in \mathcal{J}^\beta$ is associated a number $b_j \in \mathbb{R}$ such that*
  - if $j = \beta$, then $b_j \neq 0$,

[17] It suffices to restrict attention to call options since the payoff of a put can be replicated by a portfolio of a call with the same striking price, the security on which the options are written, and the bond—a property known as the put-call parity relation (see Cox and Rubinstein, 1985).

- if j and j' are two options with striking prices τ_j, τ'_j such that $v_s^\beta \notin (\tau_j, \tau_{j'}) \forall s \in S$, then $b_j = b_{j'}$,

then the striking prices $\boldsymbol{\tau}^\beta$ can be chosen (perturbed if necessary) such that there exist coefficients $(\lambda_j, j \in \mathcal{J}^\beta)$ satisfying

$$(i)\ \mathbf{1} = \sum_{j \in \mathcal{J}^\beta} \boldsymbol{v}^j \lambda_j, \qquad \text{(ii)} \sum_{j \in \mathcal{J}^\beta} b_j \lambda_j \neq 0.$$

Proof.

(a) Define the equivalence relation $s \approx_\beta s'$ if $v_s^\beta = v_{s'}^\beta$, and let $\Sigma^\beta = \mathcal{S}/\approx_\beta$ denote the quotient space for security β. Let $K = \#\Sigma^\beta$ denote the number of elements in Σ^β (number of different values taken by $\boldsymbol{v}^\beta$) and without loss of generality label the equivalence classes so that $v_{\sigma_1}^\beta < v_{\sigma_2}^\beta < \cdots < v_{\sigma_K}^\beta$. A maximal collection of options on $\boldsymbol{v}^\beta$ contains at least $K-1$ options with striking prices lying between the different values of $\boldsymbol{v}^\beta$. Without loss of generality, number the securities in $\mathcal{J}^\beta$ so that security 1 is the basic security $\boldsymbol{v}^\beta$, and securities $j = 2, \dots, K$ are the options with striking prices τ_j^β with $v_{\sigma_{j-1}}^\beta < \tau_j^\beta < v_{\sigma_j}^\beta$. The matrix of payoffs of securities in $\mathcal{J}^\beta$ includes

$$\tilde{\boldsymbol{V}}_\tau^\beta = \begin{bmatrix} v_{\sigma_1}^\beta & 0 & \dots & 0 \\ v_{\sigma_2}^\beta & v_{\sigma_2}^\beta - \tau_2^\beta & \dots & 0 \\ \vdots & \vdots & \dots & \vdots \\ v_{\sigma_K}^\beta & v_{\sigma_K}^\beta - \tau_2^\beta & \dots & v_{\sigma_K}^\beta - \tau_K^\beta \end{bmatrix}$$

which is of rank K. On the reduced state space Σ^β, markets are complete with equity and the $K-1$ options: thus, the constant vector $\tilde{\mathbf{1}} \in \mathbb{R}^K$ lies in $\langle \tilde{\boldsymbol{V}}_\tau^\beta \rangle$. Since the matrix of payoffs $\boldsymbol{V}_\tau^\beta$ of the securities in $\mathbb{R}^S$ is obtained from $\tilde{\boldsymbol{V}}_\tau^\beta$ by replicating appropriately the rows of $\tilde{\boldsymbol{V}}_\tau^\beta$, it follows that $\mathbf{1} \in \langle \boldsymbol{V}_\tau^\beta \rangle$.

(b) Let $(b_1, \dots, b_K)$ with $b_1 \neq 0$ be the first K components of a vector as described in (b): b_k does not change if τ_k is changed as long as it remains in the interval $(v_{\sigma_k}^\beta, v_{\sigma_{k+1}}^\beta)$. The system of equations

$$\begin{bmatrix} \tilde{\boldsymbol{V}}_\tau^\beta \\ \boldsymbol{b} \end{bmatrix} \boldsymbol{\lambda} = \begin{bmatrix} \mathbf{1} \\ 0 \end{bmatrix} \tag{9}$$

is a system of $K+1$ equations in the K unknowns $\boldsymbol{\lambda}$, parameterized by the vector of striking prices $\boldsymbol{\tau}^\beta$. If $(b_2, \dots, b_K) = 0$ the result follows at once, since any solution of $\tilde{\boldsymbol{V}}_\tau^\beta \boldsymbol{\lambda} = \mathbf{1}$ satisfies $\lambda_k \neq 0$ for $k = 1, \dots, K$ so that $\boldsymbol{b}\boldsymbol{\lambda} = b_1 \lambda_1 \neq 0$. Suppose $(b_2, \dots, b_K) \neq 0$. Let us show that

$$\text{rank}\ D_{\lambda,\tau} \begin{bmatrix} \tilde{\boldsymbol{V}}_\tau^\beta \\ \boldsymbol{b} \end{bmatrix} \boldsymbol{\lambda} = K + 1$$

It suffices to prove that each equation can be controlled without perturbing the other equations by suitable choice of $(\mathrm{d}\boldsymbol{\lambda}, \mathrm{d}\boldsymbol{\tau})$. To perturb the first equation, choose $\mathrm{d}\lambda_1 \neq 0$. Since $b_1 \neq 0$ and $b_m \neq 0$ for some $m \neq 1$, choose $\mathrm{d}\lambda_m$ such that $b_1 \mathrm{d}\lambda_1 + b_m \mathrm{d}\lambda_m = 0$ so that the last equation still holds: set $\mathrm{d}\lambda_k = 0$ for $k \neq 1$ or m. Then Eq. (2) can be re-established by perturbing τ_2, Eq. (3) by perturbing $\tau_3, \dots$, equation K by perturbing τ_K. To perturb equation $k, 1 < k \leq K$, pick $\mathrm{d}\tau_k \neq 0$: then re-establish equation $k+1$ by choosing $\mathrm{d}\tau_{k+1}$ so that $\mathrm{d}\tau_{k+1}\lambda_{k+1} + \mathrm{d}\tau_k \lambda_k = 0$. Equations $k+2, \dots, K$ are re-established in the same way by choice of $\mathrm{d}\tau_{k+2}, \dots, \mathrm{d}\tau_K$. The last equation is unchanged since $\boldsymbol{\lambda}$ has not been changed. Finally, to control the last equation without changing the others, pick $m \geq 2$ such that $b_m \neq 0$ and choose $\mathrm{d}\lambda_m \neq 0$. To re-establish equation m choose $\mathrm{d}\tau_m$ such that $(v^{\beta}_{\sigma_m} - \tau_m)\mathrm{d}\lambda_m - \mathrm{d}\tau_m \lambda_m = 0$; in the same way choose $\mathrm{d}\tau_{m+1}$ to re-establish equation $m+1, \dots$, choose $\mathrm{d}\tau_K$ to re-establish equation K. Then by the transversality theorem[18] there is an open subset Ω^* of the space of parameters

$$\Omega = \{\boldsymbol{\tau} \in \mathbb{R}^{K-1} | v^{\beta}_{\sigma_1} < \tau_2 < v^{\beta}_{\sigma_2} < \cdots < \tau_K < v^{\beta}_{\sigma_K}\}$$

with $\Omega \backslash \Omega^*$ of measure zero, such that (9) has no solution for all $\boldsymbol{\tau} \in \Omega^*$. Since $\tilde{\boldsymbol{V}}^{\beta}_{\tau}\boldsymbol{\lambda} = \mathbf{1}$ has a solution, it follows that $\boldsymbol{b}\boldsymbol{\lambda} \neq 0$ which completes the proof. □

It is the combination of properties (a, b) of a maximal collection of options which makes them so convenient for simultaneously controlling risk sharing and incentives. For if entrepreneur i chooses inputs $(\bar{\kappa}^i, \bar{e}^i)$ and if $\boldsymbol{\tau}^i = (\tau^i_m, m \in \mathcal{M}^i)$ is a maximal collection of options on the equity $\boldsymbol{F}^i(\bar{\kappa}^i, \bar{e}^i)$, then there exist coefficients (λ^i_m) such that

$$F^i_s(\bar{\kappa}^i, \bar{e}^i)\lambda^i_i + \sum_{m \in \mathcal{M}^i} \max\{F^i_s(\bar{\kappa}^i, \bar{e}^i) - \tau^i_m, 0\}\lambda^i_m = 1, s \in \mathcal{S}$$

From the risk sharing point of view, agent i can obtain a riskless income stream either by trading on the sure bond or by a suitable portfolio of options and equity on his firm. However, from the incentive point of view the two methods are completely different: holding the bond will give the same payoff regardless of the effort of the entrepreneur, while the portfolio of options and equity will give a different and perhaps much less desirable result if entrepreneur i makes less effort than $\bar{e}^i$.

When there are several firms, adding a maximal collection of options for each firm (i.e. using only simple options) may not suffice to complete the markets, and complex options may be required to obtain the requisite spanning, as shown by the following example.

Example 6. Suppose there are four states and two firms with production functions satisfying: for firm 1, for all $(\kappa^1, e^1) \gg 0$ and $\boldsymbol{y}^1 = \boldsymbol{F}^1(\kappa^1, e^1)$, $y^1_1 = y^1_2 > y^1_3 = y^1_4$, while for firm 2, for all $(\kappa^2, e^2) \gg 0$ and $\boldsymbol{y}^2 = \boldsymbol{F}^2(\kappa^2, e^2)$, $y^2_1 = y^2_3 > y^2_2 = y^2_4$. This suggests a state space $\mathcal{S} = \{\alpha, \beta\} \times \{\gamma, \delta\}$ in which (α, β) are the good and bad states for firm 1, while (γ, δ) are the good and bad states for firm 2. Let $(\bar{\boldsymbol{y}}^i, \bar{\kappa}^i, \bar{e}^i)$, $i = 1, 2$, be the production part of a Pareto optimal allocation with $\bar{\boldsymbol{y}} \gg 0$. If there is one option for firm 1 with striking price τ^1 such that $y^1_h > \tau^1 > \bar{y}^1_l$ (where $\bar{y}^1_h = \bar{y}^1_1 = \bar{y}^1_2$, $\bar{y}^1_l = \bar{y}^1_3 = \bar{y}^1_4$) and one option

[18] See Magill and Quinzii (1996, Theorem 11.3).

for firm 2 with striking price τ^2 such that $\bar{y}_h^2 > \tau^2 > \bar{y}_l^2$ (using similar notation) then the payoff matrix $\boldsymbol{V}(\bar{\boldsymbol{y}})$ is

$$\boldsymbol{V}(\bar{\boldsymbol{y}}) = \begin{bmatrix} y_h^1 & y_h^1 - \tau^1 & \bar{y}_h^2 & \bar{y}_h^2 - \tau^2 \\ y_h^1 & y_h^1 - \tau^1 & \bar{y}_l^2 & 0 \\ y_l^1 & 0 & \bar{y}_h^2 & \bar{y}_h^2 - \tau^2 \\ y_l^1 & 0 & \bar{y}_l^2 & 0 \end{bmatrix}$$

The securities of firm 1 span the two-dimensional subspace $\mathcal{V}^1(\bar{\boldsymbol{y}}) = \{\boldsymbol{v} \in \mathbb{R}^4 | v_1 = v_2, v_3 = v_4\}$, while the securities of firm 2 span the two-dimensional subspace $\mathcal{V}^2(\bar{\boldsymbol{y}}) = \{\boldsymbol{v} \in \mathbb{R}^4 | v_1 = v_3, v_2 = v_4\}$. Each of these subspaces contains $\mathbf{1}$ so that $\mathcal{V}^1(\bar{\boldsymbol{y}}) + \mathcal{V}^2(\bar{\boldsymbol{y}})$ is of dimension three. A complex option on an index of the two equities (for example the portfolio $2y^1 + y^2$ with striking price τ such that $2\bar{y}_l^1 + \bar{y}_h^2 > \tau > 2y_l^1 + \bar{y}_l^2$, or such that $2\bar{y}_h^1 + \bar{y}_h^2 > \tau > 2\bar{y}_h^1 + \bar{y}_l^2$) completes the markets.

We can now show that, if the state space is technological, it is possible to associate with each Pareto optimal allocation a security structure consisting of standard capital market securities which satisfies the spanning–overlap condition. By Proposition 3 this implies that, if the state space is technological, any Pareto optimal allocation can be obtained as a weak-RCPP equilibrium.

Proposition 5 (ASP implies weak-RCPP). *If $(\bar{\boldsymbol{x}}, \bar{\boldsymbol{y}}, \bar{\boldsymbol{\kappa}}, \bar{\boldsymbol{e}}; \bar{\boldsymbol{\pi}})$ is an ASP equilibrium of an economy $\mathcal{E}(\boldsymbol{U}, \boldsymbol{\omega}_0, \boldsymbol{F})$ for which the state space is technological, then there exists a security structure $\boldsymbol{V}$ consisting of standard capital market securities and a vector of portfolios $\bar{\boldsymbol{z}} = (\bar{z}^1, \dots, \bar{z}^I)$ such that $(\bar{\boldsymbol{x}}, \bar{\boldsymbol{y}}, \bar{\boldsymbol{e}}, \bar{\boldsymbol{\kappa}}, \bar{\boldsymbol{z}}; \bar{\boldsymbol{\pi}})$ is a weak-RCPP equilibrium of $\mathcal{E}(\boldsymbol{U}, \boldsymbol{\omega_0}, \boldsymbol{F}, \boldsymbol{V})$.*

Proof. It suffices to prove that there exists a security structure $\boldsymbol{V}$ which satisfies the spanning–overlap condition. Since, by Assumption $\mathcal{C}$(v), $(\kappa^i, e^i) \gg 0$ for all $i \in \mathcal{I}_1$ and since the state space $\mathcal{S}$ is technological, the vector of firms' outputs distinguishes among states: for $s \neq s'$, $\bar{\boldsymbol{y}}_s \neq \bar{\boldsymbol{y}}_{s'}$. By Ross (1976) there exists an index with weights α_i for $i \in \mathcal{I}_1$ such that if $s \neq s'$ then $\sum_{i \in \mathcal{I}_1} \alpha_i \bar{y}_s^i \neq \sum_{i \in \mathcal{I}_1} \alpha_i \bar{y}_{s'}^i$. Consider a security structure $\boldsymbol{V}$, composed of

- the sure bond $\mathbf{1}$,
- the index $\sum_{i \in \mathcal{I}_1} \alpha_i y^i$ and a maximal collection of call options on the index at $\bar{\boldsymbol{y}}$ with striking prices $(\tau_m^\alpha)_{m \in \mathcal{M}^\alpha}$, and such that no striking price takes the value $\sum_{i \in \mathcal{I}_1} \alpha_i \bar{y}_s^i$ for any $s \in \mathcal{S}$,
- for each $i \in \mathcal{I}_1$ the equity of firm i and a maximal collection of call options $(\tau_m^i)_{m \in \mathcal{M}^i}$ on this equity at $\bar{\boldsymbol{y}}^i$, with no striking price taking the value $\bar{y}_s^i$ for any $s \in \mathcal{S}$.

With the equity of firm i associate the scalar $\bar{b}_i^i = \sum_{s \in \mathcal{S}} \bar{\pi}_s (\partial F_s^i(\bar{\kappa}^i, \bar{e}^i)/\partial e^i)$ which is positive, since $\bar{b}_i^i = -(\partial u_1^i(\bar{x}_1^i, \bar{e}^i)/\partial e^i)/u^{i\prime}{}_0(\bar{x}_0^i)$ which is positive by Assumption $\mathcal{C}$(iii). To each option on the equity of firm i with striking price τ_m^i associate the scalar

$$\bar{b}^i_m = \sum_{\{s\in\mathcal{S}\mid \bar{y}^i_s > \tau^i_m\}} \bar{\pi}_s \frac{\partial F^i_s}{\partial e^i}(\bar{\kappa}^i, \bar{e}^i)$$

Then by Lemma 1, the striking prices $(\tau^i_m)_{m\in\mathcal{M}^i}$ can be chosen so that there exist $(\lambda^i_j)_j \in \mathcal{J}^i$ such that $\mathbf{1} = \sum_{j\in\mathcal{J}^i} V^j(\bar{\boldsymbol{y}}^i)\lambda^i_j$ and $0 \neq \sum_{j\in\mathcal{J}^i} \bar{b}^i_j\lambda^i_j$, where $\mathcal{J}^i = \{i\} \cap \mathcal{M}^i$ is the index set of the securities of firm i, and where $\bar{b}^i_j = \bar{b}^i_m$ when security j is the option with striking price τ^i_m, $m \in \mathcal{M}^i$.

The payoff matrix $V(\bar{\boldsymbol{y}})$ has rank S, since it has rank S with only the index and the options on the index. The security structure is differentiable at $\bar{\boldsymbol{y}}$ since no striking price coincides with the value taken by any security, and the overlap condition (ii) of Proposition 3 is satisfied with $\boldsymbol{\nu}^i = \mathbf{1}$, $\forall i \in \mathcal{I}_1$. By Proposition 3 there exists $\bar{z}$ such that $(\bar{\boldsymbol{x}}, \bar{\boldsymbol{y}}, \bar{\boldsymbol{e}}, \bar{\boldsymbol{\kappa}}, \bar{z}; \bar{\boldsymbol{\pi}})$ is a weak-RCPP equilibrium of $\mathcal{E}(\boldsymbol{U}, \boldsymbol{\omega}_0, \boldsymbol{F}, \boldsymbol{V})$. □

Proposition 5 leads to an existence theorem for weak-RCPP equilibria of an economy with a technological state space.

Corollary 6 (Existence of weak-RCPP equilibria). *If the state space is technological, then for any $\boldsymbol{\omega}_0 \in \mathbb{R}^I_+$, $\boldsymbol{\omega}_0 \neq 0$, there is a security structure $\boldsymbol{V}$ such that the economy $\mathcal{E}(\boldsymbol{U}, \boldsymbol{\omega}_0, \boldsymbol{F}, \boldsymbol{V})$ has a weak-RCPP equilibrium which is Pareto optimal.*

In order that security structures based on the realized outputs of firms generate complete markets, it is essential that distinct states lead to distinct outputs, namely that the spate space be technological. Note, however, that this property does not imply that states of nature could be deduced from observed outputs. The difficulty in deducing a state of nature from the observation of the firms' outputs comes from the fact that many different combinations of effort levels and states can lead to the same outcome ($F^i_s(\kappa^i, e^i) = F^i_s(\kappa^i, e^{i'})$ for $s \neq s'$ and $e^i \neq e^{i'}$), making it impossible to prove that a given state s has occurred when effort e^i cannot be verified. The assumption that *for a given level of effort*, distinct states lead to distinct outcomes ($F^i_s(\kappa, e) \neq F^i_{s'}(\kappa, e)$ for some i) does not alter this basic difficulty: it simply expresses the fact that firms are subject to shocks of different severities, some more favorable than others, and that for any given effort level a bad shock always produces a worse outcome than a favorable shock.

In the model that we have outlined, from a theoretical point of view, there are two types of contracts that can be introduced: contracts based on states or contracts based on the realized outputs of firms. If we neglect transactions costs, then a system of contracts based on states solves the resource allocation problem, since in conjunction with private property (sole ownership) it leads to a Pareto optimal outcome (Proposition 2). Our basic hypothesis however, is that the transactions costs of operating a system based on primitive causes is very high: it would be tantamount to itemizing all the future shocks to their business environment that corporations typically report in their quarterly (10Q) and annual (10K) reports, with sufficient precision so as to write verifiable contracts contingent on their occurrence. The verification (litigation) costs of operating such a system would be enormous, and it is for this reason that we have called an ASP equilibrium an *artificial* equilibrium: a highly eleborate system of contracts based on primitive causes is not the type of financial structure that we observe in the real world. In practice society has adopted the second system of contracts based on realized outcomes, since the costs of verifying output (profit) are essentially zero.

Once equity markets are in place, introducing options on the equity, or indices on the equity and options on the indices, are low-cost securities to introduce—and incur no verification costs. It is, thus, of interest and importance to know (Proposition 5) that a system of markets based on such securities can, under favorable circumstances (the state space is technological), achieve the same outcome as the ideal (in a world where state verification is costless) system of contingent markets (ASP).

Combining Proposition 4 and the proof of Proposition 5, we can deduce that in an economy with a technological state space, a weak-RCPP equilibrium is inefficient only if some securities which would be useful to complete the markets (indices and options on indices) or would be useful for incentive purposes (simple options) are missing. Thus, if we assume that indices and options, which are essentially costless to introduce, are brought into the market when they are needed, weak-RCPP equilibria are Pareto optimal. What remains to be studied are conditions under which a weak-RCPP is an RCPP equilibrium, i.e. when the first-order approach gives a correct result. This happens when the effort level satisfying the first-order conditions for problem (E) which maximizes the overall utility of entrepreneur i in the budget set (8) is also the solution to problem (E), i.e. maximizes the date 1 utility of the entrepreneur, net of the cost of effort, once the financial variables are fixed. We have examined a large family of economies satisfying the assumptions of Proposition 5, and in each case we found that there is a profile of striking prices (typically many profiles) for the options for which the weak-RCPP equilibrium which decentralizes the ASP allocation is an RCPP equilibrium: there are also profiles for which the weak-RCPP is not an RCPP equilibrium. The example that follows is typical and illustrates the apparent difficulty of establishing a general result due to the inherent non-convexities introduced by options.

Example 7. Consider an economy with two agents, an entrepreneur (agent 1) and an investor (agent 2), three states of nature of equal probability, the output across the states being given by the production function

$$\boldsymbol{F}^1(\kappa^1, e^1) = \sqrt{\kappa^1 e^1}\,\boldsymbol{\eta} \quad \text{with } \boldsymbol{\eta} = (25, 23, 20)$$

The utility functions are given by $U^i(\boldsymbol{x}^i, e^i) = u_0^i(x_0^i) + u_1^i(\boldsymbol{x}_1^i) - c(e^i)$, $i = 1, 2$, with

$$u_0^i(x_0^i) = \sqrt{a^i + x_0^i}, \quad u_1^i(\boldsymbol{x}_1^i) = \frac{\delta}{3}\sum_{s=1}^{3}\sqrt{a^i + x_s^i}, \quad \delta = 0.9, \quad c(e^i) = (e^i)^2$$

and the initial wealth of each agent is $\omega_0^1 = 30$, $\omega_0^2 = 270$. When $a^1 = a^2 = 0$, the sole-proprietorship contingent-market equilibrium is given in the table below

	SPCM equilibrium								
	$\bar{x}_0^i$	$\bar{x}_1^i$	$\bar{x}_2^i$	$\bar{x}_3^i$	$\bar{\kappa}^i$	$\bar{e}^i$	$\bar{y}_1^i$	$\bar{y}_2^i$	$\bar{y}_3^i$
Agent 1	65	124	114	99	112	1.86	360	331	288
Agent 2	124	236	331	288	0	0	0	0	0

To decentralize this Pareto optimum as a weak-RCPP equilibrium with a market structure consisting of equity with payoff $\bar{\boldsymbol{y}}$ (security 1), a bond with payoff $\mathbf{1} = (1, 1, 1)$ (security 2), and options with payoffs $V^j(\bar{y}_s) = \max\{\bar{y}_s - \tau_j, 0\}$, $j \in \mathcal{M}$, we need to solve the system of equations

$$\bar{x}_s^1 = \bar{y}_s\, z_1^1 + z_2^1 + \sum_{j\in\mathcal{M}} V^j(\bar{y}_s) z_j^1, \quad s = 1, 2, 3,$$
$$c'(\bar{e}^1) = \sum_{s=1}^{3} \frac{\partial u_1^1}{\partial x_s^1}(\bar{\boldsymbol{x}}^1) \frac{\partial F_s^1}{\partial e}(\bar{\kappa}^1, \bar{e}^1) \left(z_1^1 + \sum_{j\in\mathcal{J}_s} z_j^1 \right) \tag{10}$$

where $\mathcal{J}_s$ denotes the collection of options which are in the money in state s. The system admits a solution as soon as there is an option, call it option 1 (and security 3), with a striking price τ_1 such that $\bar{y}_2 < \tau_1 < \bar{y}_1$ and a second option, option 2 (and security 4), with a striking price τ_2 such that $\bar{y}_3 < \tau_2 < \bar{y}_2$. For most values of (τ_1, τ_2), the solution of the system (10) is such that $z_1^1 > 1$ and z_2^1 is large and negative: this is more readily interpreted by introducing a third option, option 3 (and security 5), with striking price $\tau_3 < \bar{y}_3$, which has the same marginal effect as equity but automatically subtracts the income $\tau_3 z_5^1$ from the income that the entrepreneur receives from the firm. For each combination $\tau = (\tau_1, \tau_2, \tau_3)$ of the striking prices satisfying the relevant inequalities and any $z_1^1 = \theta$, there is a solution to the system of Eq. (10) which gives the financial variables $\boldsymbol{z}^1(\tau, \theta)$ of a weak-RCPP equilibrium. To study if these equilibria are RCPP equilibria, we have computed the maximum of the function

$$W^1_{(\tau,\theta)}(e^1) = u_1^1 \left(\boldsymbol{F}^1(\bar{\kappa}^1, e^1)\theta + \sum_{j=2}^{5} V^j(\bar{\kappa}^1, e^1)\, z_j^1(\tau, \theta) \right) - c(e^1)$$

for different values of τ, fixing θ at 0.4.[19] Many, but not all, values of τ give a maximum at $\bar{e}^1$. Some lead to an optimal effort which is smaller and some to an optimal effort which is larger than $\bar{e}^1$. Fig. 1 illustrates the three cases, and shows the incentive schedule $\phi(y)$ (i.e the date 1 consumption of agent 1 as a function of the realized output y) associated with the corresponding values of $\boldsymbol{z}^1(\tau, \theta)$. As the figure shows, the function $W^1_{(\tau,\theta)}(e^1)$ is far from being concave and has a complicated structure because of the changes of regime induced by options successively entering into the money in the different states when effort is increased.[20] Varying the parameters of the model—for example the risk composition $\boldsymbol{\eta}$ of the firm's output or the parameters a^1 and a^2 of agents' risk aversion—leads in each case to the same conclusion: it is always possible to find striking prices which induce a reward structure such that the entrepreneur chooses the optimal level of effort $\bar{e}^1$, but not all striking prices work. If the entrepreneur is risk tolerant or if the technology is not very risky, the reward schedules which work tend to be increasing; when the investor is risk tolerant

[19] We take a grid $\tau_{1_i} = \bar{y}_2 + (i/10)(\bar{y}_1 - \bar{y}_3)$, $\tau_{2_j} = \bar{y}_3 + (j/10)(\bar{y}_2 - \bar{y}_1)$, $\tau_{3_k} = (k/10)(\bar{y}_3)$, for $i, j, k \in \{1, \dots, 9\}$.

[20] In the fixed outcome case, assumptions have been found under which the agent's maximum problem is convex so that the first-order approach yields the solution to the global effort-choice problem (Rogerson, 1985; Jewitt, 1988). We have not found an equivalent way of convexifying an entrepreneur's choice problem in this model.

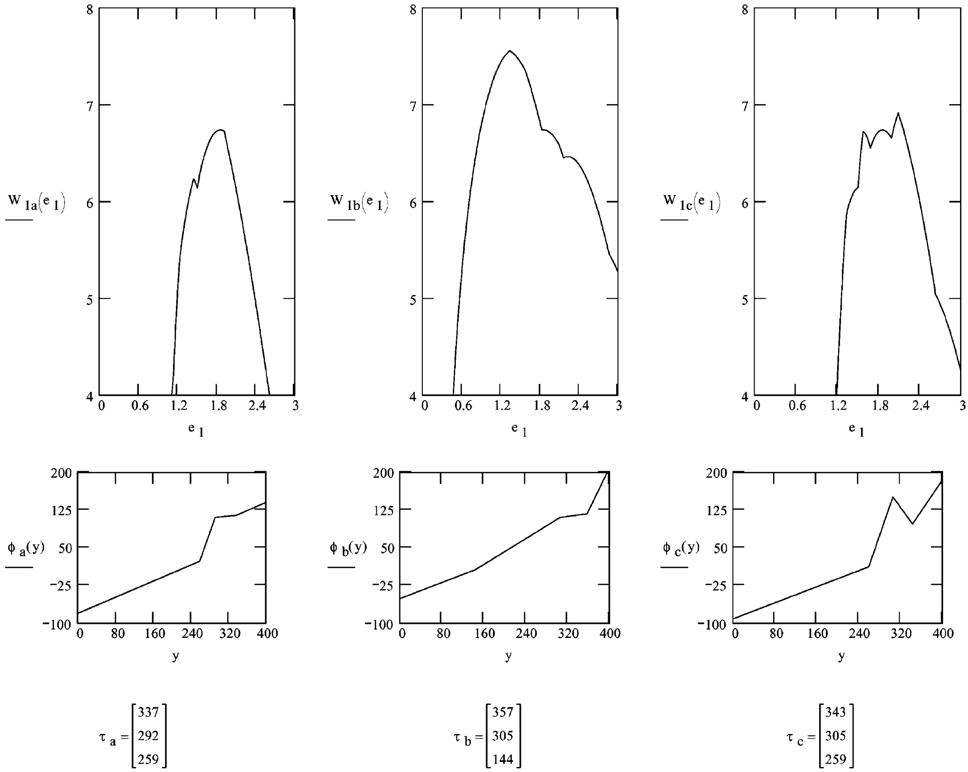

Fig. 1. The date 1 utility of the entrepreneur, net of his cost of effort, as a function of effort for the portfolio of options which solves the system (37), for different combinations of striking prices. In case (a) the optimal effort is at the Pareto optimal level $e = 1.86$ and the weak-RCPP is an RCPP; with the striking prices of case (b) the effort optimal for agent 1 is below the Pareto optimal level, above in case (c).

and insures the entrepreneur in the Pareto optimal allocation, the reward schedules have a decreasing portion as in Fig. 1c. It is interesting to note that the entrepreneur can be induced to make the optimal effort even when the investor is essentially risk neutral and bears all the risks of the economy. For example with $\boldsymbol{\eta} = (25, 15, 10)$ and $a^1 = 0$, $a^2 = 10{,}000$, the SPCM equilibrium is

	SPCM equilibrium								
	$\bar{x}_0^i$	$\bar{x}_1^i$	$\bar{x}_2^i$	$\bar{x}_3^i$	$\bar{\kappa}^i$	$\bar{e}^i$	$\bar{y}_1^i$	$\bar{y}_2^i$	$\bar{y}_3^i$
Agent 1	66	67	66	66	96	1.72	322	193	129
Agent 2	137	254	127	63	0	0	0	0	0

so that the entrepreneur's consumption is essentially constant across states. Achieving the first best allocation in this case, while impossible in the standard model where effort affects the probability of the outcomes but the outcomes are fixed, is possible in the present model

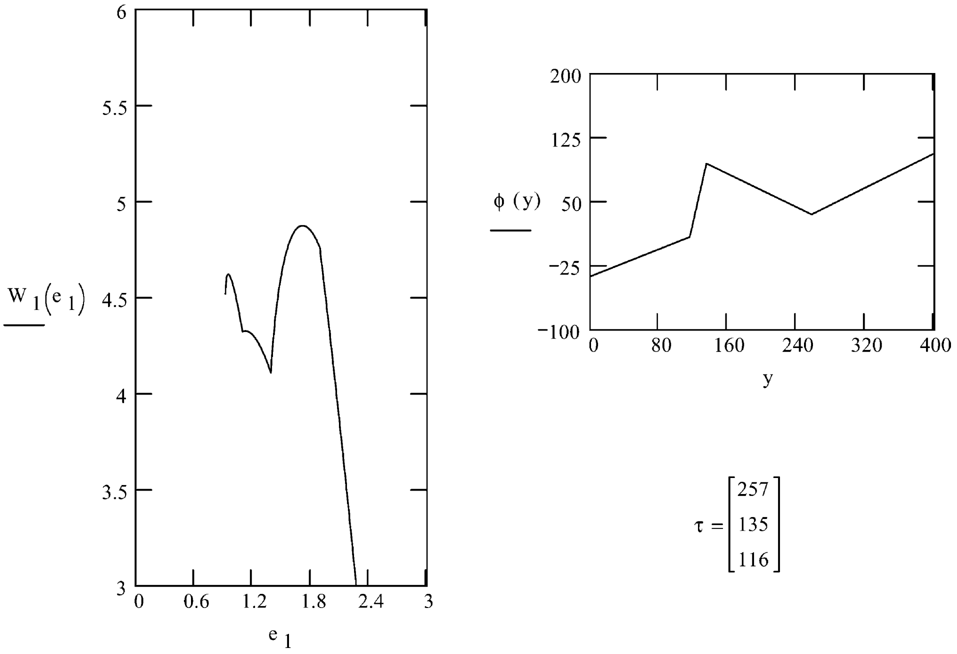

Fig. 2. A reward schedule which induces the entrepreneur to choose the optimal effort in the case where the investor risk is neutral.

where effort influences the outcome in each state. A reward schedule which induces the optimal effort of the entrepreneur for this case is shown in Fig. 2.

The family of examples that we have studied is encouraging since it shows that options, which are now extensively used for incentive contracts of top executives,[21] can lead to an efficient allocation of risk and incentives. But it also shows that the result is sensitive to the exact form of the incentive package, which makes it hard to obtain general results. At the moment we have not found general conditions under which the First and Second Theorems of Welfare Economics can be proved for RCPP equilibria, and we leave this question for future research.

Acknowledgements

An earlier version of this paper circulated under the title “Equity, Options and Efficiency in the Presence of Moral Hazard”. We are grateful to numerous participants in seminars at Northwestern University, MIT, Stanford University, Carnegie Mellon University, UC Berkeley, UC San Diego, UC Riverside, University of Minnesota, University of

[21] In the last 10 years, options have come to represent about 40% of the compensation of CEOs in the largest companies in the US (see Murphy, 1999).

Pennsylvania, UC Davis, and the California Institute of Technology for helpful comments. We would also like to thank several anonymous referees for their constructive criticism and suggestions which further helped us improve the scope and clarity of the paper: all remaining imperfections are attributable to the authors.

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