Econometrica Supplementary Material

SUPPLEMENT TO "A THEORY OF THE STAKEHOLDER CORPORATION"

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S.1. APPENDIX TO SECTION 2.4.1: ADDING FIRMS WITH DETERMINISTIC TECHNOLOGIES TO THE BENCHMARK MODEL

SUPPOSE THAT, in addition to the firm considered in the benchmark model of Section 2 (which now becomes firm 1), there are $J-1$ other firms (firms $2,\dots,J$) which use labor to produce the consumption good. These $J-1$ firms differ from firm 1 in that they are assumed to have deterministic technologies $f_2,\dots,f_J$ which are concave increasing functions of the quantity of labor used, and satisfy $f_i(0)=0$ and $f_i'(0)=\infty$, $i=2,\dots,J$; thus, the labor decision is their only decision. We assume that, when the technology f_g or f_b of firm 1 is realized at date 1, all firms act as price takers on the labor and good markets. As a result, there is no loss of generality in aggregating the $J-1$ deterministic firms in a single surrogate firm with a technology $\hat{f}$ defined by

$$\hat{f}(\hat{l}) = \max\bigl\{f_2(l_2)+\cdots+f_J(l_J) \,|\, l_2+\cdots+l_J=\hat{l}\bigr\},$$

where $\hat{f}$ is concave, increasing, and satisfies $\hat{f}(0)=0$ and $\hat{f}'(0)=\infty$. The agents—consumers, workers, and capitalists—are the same as in the benchmark model of Section 2. To justify that each firm maximizes its own profit, we assume that the two firms are owned by distinct subsets of shareholders. Let $\hat{\mathcal{E}}=(U,e,f,\gamma,\hat{f})$ denote the economy with preferences and endowments $(U^i,e^i)_{i=w,c,k}$ and technologies $(f,\gamma,\hat{f})$ for the firms, where all characteristics except for $\hat{f}$ are defined in Section 2.

A Pareto optimum of this economy is an allocation $(\pi^*,m^*,c^*,\ell^*,l^*,\hat{l}^*)$ that maximizes the sum of the agents' utilities

$$\max_{(\pi,m,c,\ell,l,\hat{l})\geq 0} \sum_{i=w,c,k}\left(m_0^i+\delta\sum_{s=g,b}\pi_s m_s^i\right)+\delta\sum_{s=g,b}\pi_s\bigl[u(c_s)-v(\ell_s)\bigr]$$

subject to the resource constraints for money, consumption, and labor

$$\text{(S.1)}\qquad \sum_{i=w,c,k} m_0^i+\gamma(\pi)=e_0,\qquad \sum_{i=w,c,k} m_s^i=e_1,$$

$$c_s=f_s(l_s)+\hat{f}(\hat{l}_s),\qquad \ell_s=l_s+\hat{l}_s,\qquad s=g,b.$$

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This is equivalent to finding $(c^*, \ell^*, \pi^*, l^*, \hat{l}^*)$ that solves

$$\max_{(c,\ell,\pi,l,\hat{l})\geq 0} e_0 - \gamma(\pi) + \delta \sum_{s=g,b} \pi_s\big[e_1 + u(c_s) - v(\ell_s)\big], \tag{S.2}$$

subject to the resource constraints (S.1). The maximum problem (S.2) decomposes into the choice, in each outcome $s = g, b$ (at date 1), of consumption–labor allocations $(c_s^*, \ell_s^*, l_s^*, \hat{l}_s^*)$ that maximize social welfare

$$W_s = u(c_s) - v(\ell_s), \tag{S.3}$$

subject to the resources constraints (S.1), and firm 1's choice of investment (at date 0), or more directly, the choice of the probability of success π^* that maximizes

$$\delta\big(\pi W_g^* + (1-\pi)W_b^*\big) - \gamma(\pi), \tag{S.4}$$

where W_g^*, W_b^* are the optimized values of (S.3). The first-order conditions for the choice of consumption–labor at date 1 are, for $s = g, b$,

$$u'\big(c_s^*\big)f_s'\big(l_s^*\big) = u'\big(c_s^*\big)\hat{f}'\big(\hat{l}_s^*\big) = v'\big(\ell_s^*\big), \quad c_s^* = f_s\big(l_s^*\big) + \hat{f}\big(\hat{l}_s^*\big), \ell_s^* = l_s^* + \hat{l}_s^*. \tag{S.5}$$

Since the social welfare W_s in each outcome s is a strictly concave function, there is a unique solution to the FOCs (S.5), which are necessary and sufficient for characterizing the optimal allocation. Since $f_g(l) > f_b(l)$ for all $l > 0$, $W_g(l, \hat{l}) = u(f_g(l) + \hat{f}(\hat{l})) - v(l + \hat{l}) > u(f_b(l) + \hat{f}(\hat{l})) - v(l + \hat{l}) = W_b(l, \hat{l})$ so that $W_g^* = \max_{(l,\hat{l})} W_g(l, \hat{l}) > W_b^* = \max_{(l,\hat{l})} W_b(l, \hat{l})$. This justifies our notation that "g" is the good social outcome. The FOC for the optimal choice of investment by firm 1 at date 0 is given by

$$\delta\big(W_g^* - W_b^*\big) = \gamma'\big(\pi^*\big), \tag{S.6}$$

and this has a unique solution π^* since γ' increases from 0 to ∞. Equation (S.6) requires that the marginal cost of increasing the probability of success equals the discounted social benefit of realizing the good rather than the bad outcome of firm 1.

The definition of a competitive shareholder equilibrium of Definition 1 extends to this economy as follows:

DEFINITION S1: A *(reduced-form) shareholder equilibrium* of the economy $\hat{\mathcal{E}}$ is a vector of actions and prices $((\bar{\ell}, \bar{c}, \bar{\pi}, \bar{l}, \bar{\hat{l}}), (\bar{w}, \bar{p}))$ such that

(i) the consumption choice $\bar{c} = (\bar{c}_g, \bar{c}_b) \geq 0$ maximizes consumer's utility $\sum_{s=g,b} \frac{\bar{\pi}_s}{1+r}(u(c_s) - \bar{p}_s c_s)$;

(ii) the labor choice $\bar{\ell} = (\bar{\ell}_g, \bar{\ell}_b) \geq 0$ maximizes worker's utility $\sum_{s=g,b} \frac{\bar{\pi}_s}{1+r}(\bar{w}_s \ell_s - v(\ell_s))$;

(iii) firm 1's production plan $(\bar{\pi}, \bar{l}) = (\bar{\pi}, \bar{l}_g, \bar{l}_b) \geq 0$ maximizes shareholder value $\sum_{s=g,b} \frac{\pi_s}{1+r} R_s(l_s; \bar{w}_s, \bar{p}_s) - \gamma(\pi)$;

(iv) firm 2's production plan $(\bar{\hat{l}}) = (\bar{\hat{l}}_g, \bar{\hat{l}}_b) \geq 0$ maximizes shareholder value $\sum_{s=g,b} \frac{\bar{\pi}_s}{1+r} \hat{R}(\hat{l}_s; \bar{w}_s, \bar{p}_s)$;

(v) markets clear: $\bar{\ell}_s = \bar{l}_s + \bar{\hat{l}}_s$, $\bar{c}_s = f_s(\bar{l}_s) + \hat{f}(\bar{\hat{l}}_s)$, $s = g, b$.

It is easy to see that the FOCs at equilibrium imply that the FOCs (S.5) for the socially optimal choice of labor by the two firms are satisfied and

$$(\bar{c}, \bar{\ell}, \bar{l}, \bar{\hat{l}}) = (c^*, \ell^*, l^*, \hat{l}^*).$$

The remaining first-order condition for the choice of investment $\bar{\pi}$ which maximizes firm 1's shareholder value is

$$\text{(S.7)} \qquad \frac{1}{1+r}(\bar{R}_g - \bar{R}_b) = \gamma'(\bar{\pi}) \quad \text{if} \quad \bar{R}_g > \bar{R}_b, \quad \bar{\pi} = 0 \quad \text{otherwise,}$$

where $\bar{R}_s$ is the maximized profit of firm 1 in outcome s; this equation has a unique solution since $\gamma'(\pi)$ increases from 0 to ∞. Comparing (S.7) with (S.6), we see that, as in the benchmark model, if $W_g^* - W_b^* > \bar{R}_g - \bar{R}_b$, then $\bar{\pi} < \pi^*$ since γ' is increasing. The under-investment result extends to the economy $\hat{\mathcal{E}}$.

PROPOSITION S1: *There is under-investment in the shareholder equilibrium of the economy* $\hat{\mathcal{E}}$: $\bar{\pi} < \pi^*$.

PROOF: We prove that $W_g^* - W_b^* > \bar{R}_g - \bar{R}_b$, from which Proposition S1 follows. This proof generalizes the proof of Proposition 1 to take into account the second firm producing with technology $\hat{f}$.

Consider the parameterized family of production functions for firm 1

$$f(t, l) = t f_g(l) + (1-t) f_b(l), \quad t \in [0, 1],$$

where the parameter takes the production function continuously from the bad to the good technology. We associate with each $t \in [0, 1]$ a fictitious "t" spot economy at date 1 with the characteristics $(u, v, f(t, \cdot), \hat{f})$. The maximized social welfare for the t economy is

$$W(t) = \max\{u(c) - v(\ell) | c = f(t, l) + \hat{f}(\hat{l}), \ell = l + \hat{l}\}.$$

The solution $(c(t), \ell(t), l(t), \hat{l}(t))$ of this maximum problem is characterized by the equations

$$u'(c(t))f_2(t, l(t)) = v'(\ell(t)), \quad u'(c(t))\hat{f}'(\hat{l}(t)) = v'(\ell(t)), \tag{S.8}$$

$$c(t) = f(t, l(t)) + \hat{f}(\hat{l}(t)), \quad \ell(t) = l(t) + \hat{l}(t), \tag{S.9}$$

and this allocation can be induced by letting agents and firms make their choices on spot markets at prices

$$p(t) = u'(c(t)), \quad w(t) = v'(l(t)).$$

Let $R(t) = p(t)f(t, l(t)) - w(t)l(t)$ denote the (optimized) profit of firm 1 under these spot prices. We show that the function

$$D(t) = W(t) - R(t)$$

is strictly increasing on [0, 1]; this will imply that $D(1) = W_g^* - \bar{R}_g > D(0) = W_b^* - \bar{R}_b$ and hence establish the result. By the envelope theorem,

$$W'(t) = u'(c(t))f_1(t, l(t)),$$
$$R'(t) = p'(t)f(t, l(t)) + p(t)f_1(t, l(t)) - w'(t)l(t).$$

Thus $D'(t) = -p'(t)f(t, l(t)) + w'(t)l(t)$. Since (S.8) implies that the marginal products of labor are equalized, $f_2(t, l(t)) = \hat{f}'(\hat{l}(t))$, it follows that

$$p'(t) = u''(c(t))[f_1(t, l(t)) + f_2(t, l(t))(l'(t) + \hat{l}'(t))],$$
$$w'(t) = v''(\ell(t))(l'(t) + \hat{l}'(t)).$$

The change in the optimal allocation of labor $(l'(t), \hat{l}'(t))$ for the two firms can be obtained by differentiating the FOCs for the optimal allocation of labor (S.8). This gives the pair of linear equations

$$\begin{aligned} &u''(f_1 + f_2(l' + \hat{l}'))f_2 + u'(f_{21} + f_{22}l') - v''(l' + \hat{l}') = 0, \\ &u''(f_1 + f_2(l' + \hat{l}'))f_2 + u'\hat{f}''\hat{l}' - v''(l' + \hat{l}') = 0, \end{aligned} \tag{S.10}$$

where the arguments of the functions have been omitted to simplify notation. Solving these equations leads to

$$l' + \hat{l}' = \frac{-u''f_1f_2(f_{22} + \hat{f}'') - u'f_{21}\hat{f}''}{u'\hat{f}''f_{22} + (u''(f_2)^2 - v'')(f_{22} + \hat{f}'')}. \tag{S.11}$$

The denominator is positive since $f_{22}, \hat{f}'', u''$ are negative and v'' is positive, while the sign of the numerator is ambiguous. However, substituting this expression into $D'(t) = -u'' f_1 f + (v'' l - u'' f_2 f)(l' + \hat{l}')$ gives

$$D'(t) = \frac{1}{den}\big[u'' u' f \hat{f}'' (f_{21} f_2 - f_1 f_{22}) + u'' v'' f_1 \big(f_{22} + \hat{f}''\big)(f - f_2 l) - u' v'' \hat{f}'' f_{21} l\big],$$

where "*den*" is the positive denominator of $l' + \hat{l}'$. Since by concavity of f, $f - f_2 l > 0$, all the terms are positive and $D'(t) > 0$; thus, moving toward the good outcome constantly increases the welfare by more than the increase in profit. *Q.E.D.*

The calculations in the proof can be extended to see precisely what happens when the technology of firm 1 moves from its "bad" technology to its "good" technology. When moving from state b to state g:

(i) the sum of the surpluses of consumers of firms 1 and 2 increases ($p'(t) < 0$),

(ii) the sum of the surpluses of workers of firms 1 and 2 may increase or decrease (the sign of $w'(t)$ is ambiguous),

(iii) the profit of firm 2 decreases ($\hat{R}'(t) < 0$): improving the competitor's technology hurts the shareholders of firm 2.

For $s = g, b$, the difference $W_s^* - \bar{R}_s$ is the sum of the surpluses in (i)–(iii). The surprising part of the proof is that, despite the ambiguity of the sign of (ii) and the negative sign of (iii), the sum of these surpluses increases in moving from b to g: $W_b^* - \bar{R}_b < W_g^* - \bar{R}_g$. By the Hicks–Kaldor criterion, even though there may be some losers from the improvement in firm 1's technology, the winners can compensate the losers.

S.2. SECOND APPENDIX TO SECTION 2.4.1: GENERAL MODEL WITH J FIRMS

The under-investment result of Proposition S1 applies to a setting in which a dominant firm (firm 1) operates on spot markets for labor and output in parallel with a competitive fringe (represented by $\hat{f}$). We now extend this result to the more general setting where there are J firms, each of which makes an investment choice relative to its risky technology. In the general case where the J firms face different risks and have access to different technologies, we can show, by comparing first-order conditions, that a shareholder equilibrium is not Pareto optimal. But the under-investment result of Proposition 1 (or S1) is no longer always true. However, when the firms are sufficiently similar—in short, when we appeal to symmetry—the under-investment result can be extended to the case of J firms.

To keep notation simple, we focus on the case where $J=2$ and assume that the second firm now has a technology that is exposed to risk: if it invests $\hat{\gamma}(\hat{\pi})$ at date 0, it will use the technology $\hat{f}_g$ with probability $\hat{\pi}$ and the technology $\hat{f}_b$ with probability $1-\hat{\pi}$. We assume in addition that $(\hat{f}_g, \hat{f}_b) = (f_g, f_b)$ and $\hat{\gamma}(\hat{\pi}) = \gamma(\pi)$ (the symmetry assumption). There are now four possible outcomes $s=(s_1, s_2)$, with $s_1 \in \{g, b\}$ and $s_2 \in \{g, b\}$. We assume that the risks to which the firms are exposed are independent, so that the probability of the outcome $s=(s_1, s_2)$ is $\pi_s = \pi_{s_1}\hat{\pi}_{s_2}$. With this change in the definition of the outcome s, finding a Pareto optimal allocation still consists in finding a solution to (S.2) subject to the resource constraint (S.1), where firm 2's production function is now indexed by s ($\hat{f}_s = \hat{f}_g$ if $s_2 = g$, $\hat{f}_s = \hat{f}_b$ if $s_2 = b$). As in the benchmark model, the analysis can be decomposed into two steps: the first consists in finding the consumption–labor decision $(c_s^*, \ell_s^*, l_s^*, \hat{l}_s^*)$ which maximizes the social welfare $W_s = u(c_s) - v(\ell_s)$ for each s; the second consists in finding the optimal investments $(\pi^*, \hat{\pi}^*)$ which maximize the expected discounted welfare net of the cost of investment. The solution of the first problem is, as before, characterized by (S.5) where $(\hat{f}_s, \hat{f}')$ is replaced by $(\hat{f}_s, \hat{f}_s')$. On the other hand, the first-order conditions for the two firms' socially optimal investment choices $(\pi^*, \hat{\pi}^*)$ are now characterized by the pair of equations

$$\text{(S.12)}\qquad \big(W_{gg}^* - W_{bg}^*\big)\hat{\pi}^* + \big(W_{gb}^* - W_{bb}^*\big)\big(1-\hat{\pi}^*\big) = \frac{1}{\delta}\gamma'\big(\pi^*\big),$$

$$\big(W_{gg}^* - W_{gb}^*\big)\pi^* + \big(W_{bg}^* - W_{bb}^*\big)\big(1-\pi^*\big) = \frac{1}{\delta}\hat{\gamma}'\big(\hat{\pi}^*\big),$$

where W_s^* denotes the optimized social welfare in outcome $s \in \mathcal{S}$. Equation (S.12) is the generalization of (S.6) to the case where both firms make investment decisions at date 0. When the two firms have the same risks and the same technology, the first-order condition for the symmetric Pareto optimal investment π^* reduces to the single equation

$$\text{(S.13)}\qquad \big(W_{gg}^* - W_{bg}^*\big)\pi^* + \big(W_{gb}^* - W_{bb}^*\big)\big(1-\pi^*\big) = \frac{1}{\delta}\gamma'\big(\pi^*\big).$$

The increments in social welfare have the following intuitive submodularity property which serves to establish the uniqueness of the symmetric Pareto optimum.

LEMMA S1: $W_{gb}^* - W_{bb}^* > W_{gg}^* - W_{bg}^* > 0$.

PROOF: Consider the (t, τ) economy in which the production functions of the two firms are $f(t, l) = tf_g(l) + (1-t)f_b(l)$, $\hat{f}(\tau, \hat{l}) = \tau\hat{f}_g(\hat{l}) + (1-\tau)\hat{f}_b(\hat{l})$, and the consumers and workers have the characteristics (u, v). The maximum social welfare in the (t, τ) economy is

$$W(t,\tau) = \max\{u(c) - v(\ell) | c = f(t,l) + \hat{f}(\tau,\hat{l}), \ell = l + \hat{l}\}. \tag{S.14}$$

We show that $\frac{\partial^2 W}{\partial t \partial \tau} < 0$, which proves the lemma since it implies $W(1,1) - W(0,1) < W(1,0) - W(0,0) \Longleftrightarrow W^*_{gg} - W^*_{bg} < W^*_{gb} - W^*_{bb}$:

$$\frac{\partial W}{\partial \tau} = u'(c(t,\tau))\hat{f}_1(\tau, \hat{l}(t,\tau)),$$
$$\frac{\partial^2 W}{\partial t\,\partial \tau} = u''\left(f_1 + f_2\left(\frac{\partial l}{\partial t} + \frac{\partial \hat{l}}{\partial t}\right)\right)\hat{f}_1 + u'\hat{f}_{12}\frac{\partial \hat{l}}{\partial t},$$

where the arguments of the function in the second derivative have been omitted to simplify the expression. As in the proof of Proposition S1, $\frac{\partial l}{\partial t}$ and $\frac{\partial \hat{l}}{\partial t}$ can be calculated by differentiating the FOCs of the maximum problem (S.14). Calculations similar to those in the proof of Proposition S1 lead to

$$\begin{aligned} & u''\hat{f}_1\left(f_1 + f_2\left(\frac{\partial l}{\partial t} + \frac{\partial \hat{l}}{\partial t}\right)\right) \\ & \quad = u''\hat{f}_1 \frac{u' f_1 f_{22}\hat{f}_{22} - v'' f_1(f_{22} + \hat{f}_{22}) - u' f_2 f_{21}\hat{f}_{22}}{u' f_{22}\hat{f}_{22} + (u''(f_2)^2 - v'')(f_{22} + \hat{f}_{22})}, \end{aligned} \tag{S.15}$$

which is negative since the numerator and the denominator of the fraction on the right hand side are positive. From the calculation in the proof of Proposition S1, we also deduce

$$\frac{\partial \hat{l}}{\partial t} = \frac{1}{u'\hat{f}_{22}}\left((v'' - u''(f_2)^2)\left(\frac{\partial l}{\partial t} + \frac{\partial \hat{l}}{\partial t}\right) - u'' f_1 f_2\right),$$

which, after substituting the value of $\frac{\partial l}{\partial t} + \frac{\partial \hat{l}}{\partial t}$, gives

$$\frac{\partial \hat{l}}{\partial t} = \frac{-u' f_{21}\hat{f}_{22}(v'' - u''(f_2)^2) - u'u'' f_1 f_2 f_{22}\hat{f}_{22}}{u'\hat{f}_{22} den},$$

where "*den*" is the positive denominator in (S.15). The numerator of the fraction is positive, *den* is positive, and since $\hat{f}_{22} < 0$, $\frac{\partial \hat{l}}{\partial t} < 0$. This property is intuitive: if the productivity of firm 1 increases, the amount of labor used by firm 2 in the efficient allocation decreases. Thus, the two terms in $\frac{\partial^2 W}{\partial t \partial \tau}$ are negative and the result follows. *Q.E.D.*

Lemma S1 asserts that the increment in social welfare when firm 1 has a good rather than a bad outcome is greater when the other firm has the outcome "b" rather than "g," since firm 1 adds its production to the smaller production by firm 2. The existence and uniqueness of a symmetric Pareto optimum follows at once by noting that the function

$$\phi(\pi) = (W^*_{gg} - W^*_{bg})\pi + (W^*_{gb} - W^*_{bb})(1-\pi) - \frac{1}{\delta}\gamma'(\pi)$$

satisfies $\phi(0) > 0$, $\phi(\pi) \to -\infty$ as $\pi \to 1$, and $\phi'(\pi) < 0$ by Lemma 1 and $\gamma'' > 0$. Since ϕ is continuous, there is a unique π^* satisfying $\phi(\pi^*) = 0$.

The concept of a (reduced-form) shareholder equilibrium (Definition S1) extends in a natural way to this new setting where both firms have risks: the maximum problem of firm 2 ((iv) in Definition S1) now involves choosing a probability $\hat{\pi}$ at date 0 and a production plan in each outcome $s \in \mathcal{S}$ at date 1. As before, profit maximization and optimal choices of consumers and workers on spot markets at date 1 lead to an optimal consumption–labor allocation for each outcome $s \in \mathcal{S}$. The first-order conditions for the optimal choices of investment $(\bar{\pi}, \bar{\hat{\pi}})$ by the firms which maximize shareholder values are given by

$$\text{(S.16)} \qquad \begin{aligned} &(\bar{R}^1_{gg} - \bar{R}^1_{bg})\bar{\hat{\pi}} + (\bar{R}^1_{gb} - \bar{R}^1_{bb})(1-\bar{\hat{\pi}}) \leq \frac{1}{\delta}\gamma'(\bar{\pi}), \quad = \quad \text{if} \quad \bar{\pi} > 0, \\ &(\bar{R}^2_{gg} - \bar{R}^2_{gb})\bar{\pi} + (\bar{R}^2_{bg} - \bar{R}^2_{bb})(1-\bar{\pi}) \leq \frac{1}{\delta}\hat{\gamma}'(\bar{\hat{\pi}}), \quad = \quad \text{if} \quad \bar{\hat{\pi}} > 0, \end{aligned}$$

where $\bar{R}^1_s$ and $\bar{R}^2_s$ denote the maximized profit of firms 1 and 2 given the spot prices $(\bar{p}_s, \bar{w}_s)$. Equation (S.16) is the generalization of (S.7) to the setting where both firms make investment decisions at date 0. At a symmetric equilibrium $\bar{R}^1_{bg} = \bar{R}^2_{gb}$, $\bar{R}^1_{gg} = \bar{R}^2_{gg}$, $\bar{R}^1_{bb} = \bar{R}^2_{bb}$ so that the common choice of investment, which for simplicity we still denote by $\bar{\pi}$, is characterized by the FOC

$$\text{(S.17)} \qquad (\bar{R}^1_{gg} - \bar{R}^1_{bg})\bar{\pi} + (\bar{R}^1_{gb} - \bar{R}^1_{bb})(1-\bar{\pi}) \leq \frac{1}{\delta}\gamma'(\bar{\pi}), \quad = \quad \text{if} \quad \bar{\pi} > 0.$$

Establishing a monotone ranking of the solutions of the first-order conditions (S.12) at a Pareto optimum and at an equilibrium (S.16) in the general case is difficult; however, when the firms are similar, the submodularity property makes it possible to compare the solutions of (S.13) and (S.17), and this leads to the following generalization of Proposition S1.

PROPOSITION S2: *In any symmetric shareholder equilibrium of an economy with J firms, there is under-investment*: $\bar{\pi} < \pi^*$.

PROOF: The proof of Proposition S1 consisted in showing that $W_g^* - W_b^* > \bar{R}_g - \bar{R}_b$ when firm 2 has a fixed technology. This implies that, for any realization of the technology of firm 2,

$$W^*_{gs_2} - W^*_{bs_2} > \bar{R}^1_{gs_2} - \bar{R}^1_{bs_2}, \quad s_2 = g, b. \tag{S.18}$$

We want to prove that $\bar{\pi} < \pi^*$. Suppose by contradiction that $\bar{\pi} \geq \pi^*$. Since π^* is positive, this implies that $\bar{\pi} > 0$, and thus that (S.17) holds with equality. Then $\gamma'(\bar{\pi}) \geq \gamma'(\pi^*)$ and by (S.13) and (S.17),

$$\begin{aligned}
&\big(\bar{R}^1_{gg} - \bar{R}^1_{bg}\big)\bar{\pi} + \big(\bar{R}^1_{gb} - \bar{R}^1_{bb}\big)(1-\bar{\pi}) \\
&\quad \geq \big(W^*_{gg} - W^*_{gb}\big)\pi^* + \big(W^*_{bg} - W^*_{bb}\big)\big(1-\pi^*\big) \\
&\quad \geq \big(W^*_{gg} - W^*_{gb}\big)\bar{\pi} + \big(W^*_{bg} - W^*_{bb}\big)(1-\bar{\pi}),
\end{aligned}$$

where the second inequality follows from Lemma S1: the convex combination with weights $(\bar{\pi}, 1-\bar{\pi})$ puts less weight on the larger term $(W^*_{bg} - W^*_{bb})$ than the convex combination with weights $(\pi^*, 1-\pi^*)$. The resulting inequality between expected profit and expected welfare increments contradicts (S.18); thus, $\bar{\pi} < \pi^*$. The proof is readily extended to the case $J > 2$ and is left to the reader. *Q.E.D.*

S.3. APPENDIX TO SECTION 2.4.2: UNDER-INVESTMENT WITH MONOPOLISTIC PRICING ON SPOT MARKETS

In a monopolistic equilibrium, the firm knows the demand function $c(p) = u'^{-1}(p)$ of the consumers, which implies a revenue $Q(c) = cu'(c)$ as a function of the amount c sold on the market, and knows the supply function $\ell(w) = v'^{-1}(w)$ of the workers, which yields a cost $C(l) = lv'(l)$ as a function of the amount l of labor used by the firm. We make the following standard assumptions to ensure that the profit-maximizing problem of the monopoly on the spot markets has a solution.[1]

ASSUMPTION M: (i) *The revenue function Q is concave, differentiable on* $[0,\infty)$, *and* $Q'(0) > 0$. (ii) *The cost function C is increasing, differentiable, and convex on* $[0,1)$, $C'(0) = 0$, *and* $C(1) = +\infty$.

[1]If the utility function is a power function, that is, $u(c) = \frac{1}{\alpha}c^\alpha$, then Assumption M(i) restricts α to satisfy $0 < \alpha \leq 1$. If the utility function generates a linear demand function, then Assumption M(i) is satisfied. The last two properties of C can be deduced from the assumptions made on v and are included in Assumption M for convenience.

In a monopolistic equilibrium, the firm chooses the quantities (y_s, l_s) in outcome s and the probability of success π so as to maximize the shareholder value

$$\text{(S.19)} \qquad \mathrm{SV}^m(\pi, y, l) = \frac{1}{1+r}\left(\sum_{s=g,b} \pi_s\big(Q(y_s) - C(l_s)\big)\right) - \gamma(\pi)$$

under the constraint that $y_s = f_s(l_s)$, $s = g, b$. As in the case of the competitive firm, the maximum problem decomposes into two parts: maximizing the spot profit $R_s^m = Q(y_s) - C(l_s)$ under the technology constraint in each outcome s, and choosing the investment level which maximizes $\frac{1}{1+r}(\pi\bar{R}_g^m + (1-\pi)\bar{R}_b^m) - \gamma(\pi)$, anticipating the value $\bar{R}_s^m$ of the maximized spot profit.

DEFINITION S2: A *monopolistic shareholder equilibrium* of the economy $\mathcal{E}$ is a vector of actions and prices $((\bar{c}^m, \bar{\ell}^m, \bar{\pi}^m, \bar{l}^m), (\bar{p}^m, \bar{w}^m))$ such that

(i) the firm's production plan $(\bar{\pi}^m, \bar{l}^m) = (\bar{\pi}^m, \bar{l}_g^m, \bar{l}_b^m) \geq 0$ maximizes shareholder value (S.19) subject to the technology constraints,

(ii) $\bar{c}_s^m = f_s(\bar{l}_s^m)$, $\bar{p}_s^m = u'(\bar{c}_s^m)$, $\bar{\ell}_s^m = \bar{l}_s^m$, $\bar{w}_s^m = v'(\bar{\ell}_s^m)$, $s = g, b$.

Note that (ii) in Definition S2 ensures that the consumers and workers maximize their utility given the prices $\bar{p}^m$ and $\bar{w}^m$. Under Assumption M, a monopolistic equilibrium exists and is unique. It can be summarized by the investment-labor choice $(\bar{\pi}^m, \bar{l}^m)$ which maximizes shareholder value (S.19). The first-order condition for the maximization of the spot profit in outcome s is

$$\text{(S.20)} \qquad Q'\big(f_s(l_s)\big)f'(l_s) = C'(l_s).$$

Since C' increases from 0 to ∞ and $Q'(0) > 0$, there is a unique solution $\bar{l}_s^m$ to this equation, and $\bar{R}_s^m = Q(f(\bar{l}_s^m)) - C(\bar{l}_s^m)$. The optimal choice of investment $\bar{\pi}^m$ satisfies the first-order condition

$$\text{(S.21)} \qquad \frac{1}{1+r}\big(\bar{R}_g^m - \bar{R}_b^m\big) = \gamma'\big(\bar{\pi}^m\big), \quad \text{if} \quad \bar{R}_g^m > \bar{R}_b^m, \quad \pi^m = 0, \quad \text{otherwise},$$

which has a unique solution since γ' increases from 0 to ∞.

Suppose that a planner, rather than the firm, chooses the investment to maximize the social welfare, subject to the constraint that the consumers and workers have to trade on the spot markets at the prices determined by the firm. Then the planner will maximize

$$\text{(S.22)} \qquad \frac{1}{1+r}\big(\pi\overline{W}_g^m + (1-\pi)\overline{W}_b^m\big) - \gamma(\pi),$$

where

$$\overline{W}_s^m = \big(u(\bar{c}_s^m) - \bar{p}_s^m \bar{c}_s^m\big) + \big(\bar{w}_s^m \bar{\ell}_s^m - v(\bar{\ell}_s^m)\big) + \bar{R}_s^m = u(\bar{c}_s^m) - v(\bar{\ell}_s^m).$$

Hence the constrained planner's optimal choice π^{m*} satisfies the first order condition:

$$\text{(S.23)} \quad \frac{1}{1+r}\big(\overline{W}_g^m - \overline{W}_b^m\big) = \gamma'\big(\pi^{m*}\big), \quad \text{if} \quad \overline{W}_g^m > \overline{W}_b^m, \qquad \pi^{m*} = 0, \quad \text{otherwise.}$$

As in the competitive case, the comparison between the firm's and the planner's optimal choice of investment amounts to comparing the increment to welfare with the increment to profit between the good and the bad outcome. We show that under-investment continues to hold if we make the following additional assumptions on the firm's technology and on the demand side of the economy.

ASSUMPTION T: *The firm's technology is given by*

$$f_s(l) = \varepsilon_s f(l), \quad s = g, b,$$

where $\varepsilon_g > \varepsilon_b \geq 0$ *and* $f(\cdot)$ *is increasing, differentiable, concave,* $f(0) = 0$, $f'(0) = \infty$.

ASSUMPTION E: *The elasticity of the marginal revenue* $Q(\cdot)$ *is less than* 1

$$\frac{-Q''(c)c}{Q'(c)} < 1 \quad \textit{when} \quad Q'(c) \neq 0.$$

Assumption T is the assumption of *multiplicative uncertainty*—frequently used in general equilibrium and macro—which facilitates comparison of the technologies in the two outcomes. Assumption E bounds the response of the firm's marginal revenue to a decrease in its output; we know of no case where Assumption M is satisfied and the elasticity is greater than 1.

PROPOSITION S3: *If Assumptions* M, T, E *are satisfied, then there is under-investment in the monopolistic shareholder equilibrium of* $\mathcal{E}$: $\bar{\pi}^m < \pi^{m*}$.

PROOF: Let us show that

$$\overline{W}_g^m - \overline{W}_b^m > \bar{R}_g^m - \bar{R}_b^m,$$

from which the result follows.

As in the proof of Lemma 2, consider the function $f(t,l) = tf_g(l) + (1-t)f_b(l)$ and the t spot economy in which the production function is $f(t,l)$. Let

$$R^m(t) = \max_{l\geq 0}\{Q(f(t,l)) - C(l)\}$$

be the maximum profit of the monopolist on the spot markets of the t economy, and let $l^m(t)$ denote the labor choice which maximizes this profit. Note that $R^m(1) = \bar{R}^m_g$ and $R^m(0) = \bar{R}^m_b$. Let

$$W^m(t) = u(f(t, l^m(t))) - v(l^m(t))$$

be the total welfare in the t spot economy and

$$D^m(t) = W^m(t) - R^m(t)$$

denote the surplus of the consumers and workers. We want to show that $D^m(t)$ is increasing in t, which implies that $\overline{W}^m_g - \bar{R}^m_g > \overline{W}^m_b - \bar{R}^m_b$. Applying the envelope theorem to calculate the derivative of l^m, we find that

$$\frac{dD^m(t)}{dt} = (u' - Q')f_1 + \frac{dl^m(t)}{dt}(u'f_2 - v'), \tag{S.24}$$

where the arguments of the functions at the optimal choice of the monopolist in the t economy have been omitted for simplicity. Calculating the derivative of l^m by differentiating the first-order condition

$$Q'(f(t, l^m(t))f_2(t, l^m(t))) = C'(l^m(t)) \tag{S.25}$$

which defines it, gives

$$\frac{dl^m(t)}{dt} = -\frac{Q''f_1f_2 + Q'f_{12}}{Q''f_2^2 + Q'f_{22} - C''}.$$

The denominator of the fraction is negative since $Q'' < 0$ by Assumption M, Q' is positive at the optimum as shown by (S.25), and $C'' > 0$ by Assumption M. The sign of the numerator is, however, ambiguous since $Q'' < 0$, $f_1 > 0$, $f_2 > 0$, $Q' > 0$, and $f_{12} > 0$. As in the competitive case, we cannot be sure that the firm uses more labor in the good than in the bad outcome.

Inserting the expression for $\frac{dl^m(t)}{dt}$ in (S.24) gives

$$\frac{dD^m(t)}{dt} = \frac{1}{den}\big[(u' - Q')f_1(Q''f_2^2 + Q'f_{22} - C'') - (u'f_2 - v')(Q''f_1f_2 + Q'f_{12})\big], \tag{S.26}$$

where *den* is the negative denominator of $\frac{dl^m(t)}{dt}$. The numerator of (S.26) can be written as

$$N = Q'' f_1 f_2 \bigl(-Q' f_2 + v'\bigr) + \bigl(u' - Q'\bigr) f_1 \bigl(Q' f_{22} - C''\bigr) - Q' f_{12} \bigl(u' f_2 - v'\bigr).$$

Since $Q' = u'' f + u'$, $u' - Q' = -u'' f > 0$. Replacing Q' and C' by their values in function of u and v in the FOC (S.25) gives $u' f_2 - v' = l v'' - u'' f f_2$ and $-Q' f_2 + v' = -v'' l$. Thus N can be rewritten as

$$N = -l v'' \bigl[Q'' f_1 f_2 + Q' f_{12}\bigr] - u'' f \bigl[Q' f_1 f_{22} - C'' f_1 - Q' f_{12} f_2\bigr].$$

The second term is negative. Thus, if $Q'' f_1 f_2 + Q' f_{12} > 0$, N is negative. Under Assumption T, $f(t, l^m(t)) = (t\varepsilon_g + (1-t)\varepsilon_b) f(l^m(t))$, $f_1(t, l^m(t)) = (\varepsilon_g - \varepsilon_b) f(l^m(t))$, $f_2(t, l^m(t)) = (t\varepsilon_g + (1-t)\varepsilon_b) f'(l^m(t))$, $f_{12}(t, l^m(t)) = (\varepsilon_g - \varepsilon_b) f'(l^m(t))$. $Q'' f_1 f_2 + Q' f_{12} > 0$ is equivalent to

$$\text{(S.27)} \quad \frac{Q''\bigl(f\bigl(t, l^m(t)\bigr) f\bigl(t, l^m(t)\bigr)\bigr)}{Q'\bigl(f\bigl(t, l^m(t)\bigr)\bigr)} > -\frac{(\varepsilon_g - \varepsilon_b) f'\bigl(l^m(t)\bigr)}{\bigl(t\varepsilon_g + (1-t)\varepsilon_b\bigr) f'\bigl(l^m(t)\bigr)} \frac{\bigl(t\varepsilon_g + (1-t)\varepsilon_b\bigr) f\bigl(l^m(t)\bigr)}{(\varepsilon_g - \varepsilon_b) f\bigl(l^m(t)\bigr)} = -1.$$

Under Assumption M, Q'' is negative. Since the elasticity is calculated at the optimum of the monopoly in the t economy, $Q' > 0$. Under Assumption E, the inequality (S.27) holds, $N < 0$, and $\frac{dD^m(t)}{dt} < 0$, which proves the proposition. *Q.E.D.*

S.4. APPENDIX TO SECTION 4: PROOF OF LEMMA 1

Since the allocation of labor is optimal and spot prices are competitive in a stakeholder equilibrium, to prove $\hat{l}_b^{\text{st}} > \hat{l}_g^{\text{st}}$ it is sufficient to prove that $\hat{l}'(t) < 0$, where $\hat{l}(t)$ is the optimal choice of labor by firm 2 in the artificial t economy introduced in the proof of Proposition S1. It follows from (S.10) that

$$\hat{l}' = \frac{\bigl(v'' - u''(f_2)^2\bigr)\bigl(l' + \hat{l}'\bigr) - u'' f_1 f_2}{u' \hat{f}''}.$$

Inserting the value of $l' + \hat{l}'$ into (S.11) leads to

$$\hat{l}' = \frac{-u' f_{21} \hat{f}'' \bigl(v'' - u''(f_2)^2\bigr) - u' u'' f_1 f_2 \hat{f}'' f_{22}}{u' \hat{f}'' den} < 0,$$

where *den* denotes the positive denominator of (39). Thus $\hat{l}(1) = \hat{l}_g < \hat{l}(0) = \hat{l}_b$, which proves the lemma.

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