general equilibrium with incomplete markets

An account is given of the principal concepts and results of general equilibrium with incomplete financial markets over a finite horizon, focusing on the generic existence, suboptimality and determinacy of equilibrium. Many results depend on the nature of the financial securities, whether they are real or nominal, nominal securities leading to the analysis of a class of monetary equilibrium models.

One of Adam Smith (1776)'s beautiful insights is that markets coordinate the activities of agents and lead to optimal allocations, even though agents act purely in their own self-interest. The idea was formalized in elegant form some 200 years later in the 1950s by Arrow, Debreu and McKenzie in the branch of economics which came to be known as *general equilibrium theory* (GE). The GE model, which involved a finite number of consumers, firms and goods, was static. Arrow (1953) and Debreu (1959) showed how the model could be extended to a setting with time and uncertainty by introducing an event-tree to describe the uncertainty, and a structure of markets in which contingent contracts for future delivery of commodities are traded at an initial date. Although this model, which has come to be known as the *Arrow-Debreu model* (AD), involves time and uncertainty in the characteristics of the economy, it is still essentially static: all trading is assumed to take place at an initial date, and at subsequent dates, promises are delivered but no new contractual commitments are made.

Spot-financial market equilibrium

In a striking paper Arrow (1953) showed that any AD equilibrium could be achieved by using an alternative and more realistic sequential system of markets, consisting of financial (Arrow security) markets and spot markets for goods at each date-event. An *Arrow security* purchased or sold at date t is a contract promising to deliver one unit of income in one of the possible contingencies that can occur at date $t+1$. If at each date-event there exists a complete set of such contracts, one for each contingency that can occur at the following date, then an AD equilibrium allocation can be achieved by a combination of these Arrow security markets for redistributing income, and spot markets for exchanging goods. When the Arrow securities are replaced by a general class of financial securities calling for the delivery of income or goods at future date-events, we obtain the concept of a *spot-financial market equilibrium*. In order that the allocation obtained with this structure of markets coincide with the allocation obtained with Arrow–Debreu contingent markets, two conditions must be satisfied: the financial markets must be *complete*, and agents must *correctly anticipate* at the initial date the spot prices of every good and the payoff of every security at every date-event in the future. This correct-anticipation condition is needed in order that the income that agents choose to bring forward by their holding of financial securities permits them to buy the bundle of goods that they had planned to consume when choosing their income transfers. To obtain such a well-coordinated outcome agents should have familiarity with the functioning of the markets, and some stationarity in the structure of the economy should prevail in order that agents can form such correct anticipations.

Removing the assumption of correct anticipations leads to the theory of *temporary equilibrium,* which focuses on the minimal conditions on agents'

expectations of future prices which permit current markets to clear. Maintaining the assumption of correct anticipations of future prices while dropping the assumption that financial markets are complete leads to the theory of general equilibrium with incomplete markets, GEI for short. The GEI model has served to improve our understanding of the relationship between the real, financial and monetary sectors of the economy by providing a common framework for studying traditional price theory, the theory of finance and monetary theory.

One of the significant contributions of general equilibrium to economic theory is that it has revealed the deep insights that an abstract and rigorous mathematical model can provide into the functioning of an economic system: rigour, abstraction and clarity of thought are the hallmarks of the GE approach. Three properties of an equilibrium – existence, optimality and determinacy – have provided the basic template for organizing the theory. Establishing existence ensures that the different components of the model fit together in a coherent way; the analysis of optimality evaluates the efficiency of the underlying market structure as a mechanism for allocating resources; establishing determinacy provides a measure of the ability of the model to predict the outcome of equilibrium. Following this programme in the theory of incomplete markets has required new mathematical techniques to analyse the properties of equilibrium. For, unlike in traditional GE, many properties of a GEI equilibrium are 'almost always true' but admit some counter-examples. For example, if the financial markets are incomplete, for almost all economies risk sharing will not be optimal: however there are some special economies studied in finance, like the mean-variance economies of the capital asset pricing model, in which the equilibrium is optimal with only bond and equity contracts which technically do not constitute a complete security structure. As a result the analysis of the GEI model relies heavily on the use of differential topology, which is the branch of mathematics ideally suited to study typical, or 'generic', properties of solutions to a system of equations.

The GEI model

To set the stage for studying the properties of the GEI model, consider the simplest version of the model, a two-period ($t=0,1$) exchange economy with L commodities and I agents, where each agent is uncertain about his endowment of the goods at date 1. Let 'uncertainty' be expressed by assuming that 'nature' will draw one of S possible 'states of nature', say $s \in \{1,\ldots,S\}$, and though each agent does not know which state will be chosen, he does know what his endowment $\omega_s^i = (\omega_{s1}^i, \ldots, \omega_{sL}^i)$ will be if state s occurs. For convenience we label date 0 as state 0; then agent i's endowment is $\omega^i = (\omega_0^i, \omega_1^i, \ldots, \omega_S^i), i = 1, \ldots, I$. Agents can exchange goods and share their risks by trading on spot markets (one for each good in each state), and can redistribute their income over time and across states (thereby sharing risks) by trading on financial markets.

Let p_{sl} denote the spot price of good l in state s, and let $p_s = (p_{s1}, \ldots, p_{sL})$ denote the vector of spot prices in state s; then $p = (p_0, \ldots, p_s)$ denotes the vector of spot prices across all date-events in this two-period setting. A similar notation is used for allocations.

At date 0 there are also J securities ($j = 1, \ldots, J$) that agents can trade. Security j is a promise made at date 0 to pay V_s^j if state s occurs, where the payment V_s^j is measured in the unit of account of state s, $s = 1, \ldots, S$. We say that security j is *real* if it is a promise to deliver the value of a bundle of goods $A_s^j = (A_{s1}^j, \ldots, A_{sL}^j)$ in each state s so that $V_s^j = p_s A_s^j$. If the first good is

chosen as the numeraire for keeping accounts in each state, then $p_{s1} = 1$ and a security j that is real but which only delivers units of the first good is called a *numeraire* security: for such a security $V_s^j = A_{s1}^j, s = 1, \ldots, S$. Security j is said to be *nominal* if its payoff V_s^j is independent of the spot prices p_s. Whatever the type of the security, its price at date 0 is denoted by q_j, and the vector of all security prices is $q = (q_1, \ldots, q_J)$.

Each agent trades on the financial markets choosing a portfolio $z^i = (z_1^i, \ldots, z_J^i)$ of the securities. These transactions on the financial markets redistribute the agent's income across time and the states. The income acquired or sacrificed at date 0 is $-qz^i = -\sum_{j=1}^J q_j z_j^i$ (if $z_j^i < 0$, agent i sells security j, i.e. uses security j to borrow; if $z_j^i > 0$, agent i buys security j, i.e. uses security j to save). The income earned or due in state s is $V_s z^i = \sum_{j=1}^J V_s^j z_j^i$, where V_s denotes row s of the $S \times J$ matrix V of security payoffs. These income transfers serve to finance the excess expenditures $p_s(x_s^i - \omega_s^i)$ of the planned consumption stream $x^i = (x_0^i, x_1^i \ldots, x_S^i)$. Thus the agent's budget set, when current and anticipated prices are (p, q), is given by

$$\mathscr{B}(p, q, \omega^i) = \left\{ x^i \in \mathbb{R}_+^{L(S+1)} \middle| \begin{array}{l} p_0(x_0^i - \omega_0^i) = -qz^i, \quad z^i \in \mathbb{R}^j \\ p_s(x_s^i - \omega_s^i) = V_s z^i, \quad s = 1, \ldots, S \end{array} \right\}$$

Each agent i has a preference ordering over the consumption streams $x^i \in \mathbb{R}_+^{L(S+1)}$ which is represented by a utility function $u^i : \mathbb{R}_+^{L(S+1)} \to \mathbb{R}$ which is typically assumed to have 'nice' properties of strict quasi-concavity, monotonicity and smoothness.

An equilibrium of 'plans, prices, and price expectations' in Radner's (1972) terminology, also called a *spot-financial market equilibrium,* is defined as a pair of actions and prices $((\bar{x}, \bar{z}), (\bar{p}, \bar{q}))$ such that

(i) $(\bar{x}^i, \bar{z}^i)$ maximizes $u^i(x^i)$ over the budget set $\mathscr{B}(\bar{p}, \bar{q}, \omega^i), i = 1, \ldots, I$.
(ii) the spot markets clear: $\sum_{i=1}^I (\bar{x}_s^i - \omega_s^i) = 0, \ s = 0, \ldots, S$.
(iii) the financial markets clear: $\sum_{i=1}^I \bar{z}^i = 0$.

The market-clearing conditions (ii) for the agents' planned consumption vectors at date 1 (for $s = 1, \ldots, S$) are what Radner called an equilibrium of 'plans', since the planned consumptions of all agents are compatible, and the anticipated vector of prices p_s for each states s will be an equilibrium vector of spot prices if state s occurs (equilibrium of 'expectations'). Because agents trade at each date this is also called a *sequential equilibrium.*

If the rank of the payoff matrix V is S, so that all possible income transfers from date 0 to date 1 are feasible (at a cost), then we say that financial markets are *complete*. Otherwise, if rank $(V) < S$, then financial markets are *incomplete*, and the corresponding equilibrium is often called a GEI equilibrium.

Existence

If all securities are either numeraire or nominal securities, the budget set $\mathscr{B}(p, q, \omega^i)$ depends continuously on the prices (p, q). To prove existence of equilibrium with such securities, the main insight, over and above the techniques used in classical general equilibrium theory, is that the set of candidate equilibrium prices q for the securities must be restricted to no-arbitrage prices. A portfolio $z \in \mathbb{R}^J$ is called an arbitrage portfolio if $qz \leq 0$ (by selling some securities and buying others the portfolio has no cost) and $Vz \geq 0$ (the date 1 payoff is non-negative) with at least one inequality. An

arbitrage portfolio enables an agent to get something (a positive income in some state) without incurring any cost. q is a *no-arbitrage* vector of security prices if it does not admit an arbitrage portfolio.

Much of the modern theory of finance consists in exploring the consequences of no-arbitrage for the pricing of securities. The analysis centres around the following characterization of no-arbitrage, which is also fundamental for proving existence of an equilibrium in the GEI model. If V is a fixed matrix of payoffs for the securities, q is a no-arbitrage vector of security prices if and only if there exists a strictly positive vector of present-value prices $\pi = (\pi_1, \ldots, \pi_S)$ for income across the states at date 1 such that the price of each security is the present value of its payoff stream: $q_j = \sum_{s=1}^{S} \pi_s V_s^j, s = 1, \ldots, S$. By working with the present-value prices π, the standard Kakutani fixed point theorem can be used to prove existence of an equilibrium with numeraire or nominal securities.

When the securities are real a new difficulty appears, since the payoffs $V_s^j = p_s A_s^j$ depend on the spot prices, so that when the vector of spot prices p changes, the rank of the payoff matrix V can change, leading to discontinuities in the agents' demand functions. As Hart (1975) showed, this can lead to nonexistence of equilibrium. The same kind of discontinuity can appear in the multi-period model with long-lived securities even if the securities are nominal or numeraire. Overcoming this difficulty, i.e. showing that the economies which do not have equilibria are exceptional, has required sophisticated techniques of differential topology which we do not attempt to describe here. The first result was obtained by Duffie and Shafer (1985), and a survey of the methods used for proving generic existence is provided by Magill and Shafer (1991).

Optimality

Since there is a spot market – and thus a price – for each good in each state, the agents' rates of substitution among goods in each state are equalized. To obtain Pareto optimality the additional condition required is that the agents' present-value vectors

$$\pi^i = (\pi_s^i)_{s=1}^{S} = \left(\frac{\partial u^i / \partial x_{s1}^i}{\partial u^i / \partial x_{01}^i} \right)_{s=1}^{S}$$

for income across the states at date 1 (with good 1 as the numeraire) are equalized. The income transfers $(\tau^i)_{i \in I}$ needed for such equalization depend on the risk profiles of the agents' endowments $(\omega^i)_{i \in I}$. Pareto optimality can thus only be expected 'for sure' if any income transfer τ^i is achievable by the choice of a portfolio, i.e. if for any $\tau^i \in \mathbb{R}^S$ there exists $z^i \in \mathbb{R}^J$ such that $Vz^i = \tau^i$. This requires that rank $(V) = S$, namely complete markets. If markets are incomplete, although for particular endowment profiles the necessary income transfers can be achieved through the markets – for instance, the endowments could be Pareto optimal – it can be shown that for almost all endowment profiles $(\omega^i)_{i \in I}$, Pareto optimality is not achievable by a GEI equilibrium.

Since the GEI model involves an imperfection, Pareto optimality is too demanding a criterion. Constrained Pareto optimality, which respects the constraints on the possible income transfers, is a more useful benchmark for judging whether competitive markets lead to the best possible resource allocations given the constraints. An equilibrium allocation is *constrained Pareto optimal* if a 'planner' who can change agents' consumption and portfolios at date 0, but must otherwise let the existing markets induce the

allocation at date 1, cannot improve on the allocation. Surprisingly a GEI equilibrium is typically (generically in endowments and preferences) not constrained Pareto optimal. This property was first brought to light by Stiglitz (1982) and formally established by Geanakoplos and Polemarchakis (1986). The channel for the improvement is the change in relative prices at date 1 – the prices $(p_s)_{s=1}^{S}$ which clear the markets – induced by the change $(dz^i)_{i \in I}$ made by the planner in the agents' date 0 portfolios.

This phenomenon can be seen most simply in a model in which relative prices at date 1 fall out directly as the marginal products of a neo-classical production function. Suppose that at date 0 there is a stock of a single good which can either be consumed or carried over to date 1 as capital input. At date 1 a firm uses this capital and labour to produce a consumption good, with a constant-returns production function $y = F(K, L)$. Each agent has the same initial endowment ω_0 of the good at date 0 and has a risky labour endowment. At the beginning of date 1, nature draws n agents who are given ℓ_b units of (effective) labour, the remaining $I-n$ agents being given ℓ_g units of labour, with $\ell_b < \ell_g$. There are thus $I!/(n!(I-n)!)$ aggregate states of nature, all equiprobable, which differ from one another by the names of the agents who have the good and the bad draw for their labour endowment. In every state the total supply of labour is the same, $L = n\ell_b + (I - n)\ell_g$. From the point of view of the agents all states in which they have a good draw are equivalent, so that each agent perceives a probability $\rho = n/I$ of having a bad draw and $1-\rho$ of having a good draw. There are no insurance markets against these labour risks: there is only one security (capital) to transfer income to the two possible outcomes next period, thus markets are incomplete.

Assuming that all agents have the same utility function $U(x_0, x_1) = u(x_0) + \beta E(u(x_1))$ where $0 < \beta \le 1$ and $x_1 = (x_b, x_g)$, the equilibrium (k, w, R) is characterized by the budget equations, FOCs and market clearing equations

$$x_0 = \omega_0 - k, x_b = w\ell_b + Rk, x_g = w\ell_g + Rk,$$
$$u'(x_0) = \beta(\rho u'(x_b) + (1 - \rho)u'(x_g))R$$

$$w = F_L(K, L), \quad R = F_K(K, L),$$
$$K = kI, \quad L = (\rho\ell_b + (1 - \rho)\ell_g)I$$

Suppose a planner changes the investment k chosen by the typical agent at date 0 by dk; then

$$dx_0 = -dk, \quad dx_b = dw\ell_b + dRk + Rdk,$$
$$dx_g = dw\ell_g + dRk + Rdk$$

which induces a change in the wage and rental rates

$$dw = F_{LK}(K, L)Idk, \quad dR = F_{KK}(K, L)Idk$$

Substituting (dx_0, dx_b, dx_g) the *direct effect* of the change dk is zero in view of the first-order condition for the optimal choice of k, but the *price effects* remain so that

$$dU = \beta\big[(\rho u'(x_b)\ell_b + (1 - \rho)u'(x_g)\ell_g)dw$$
$$+ (\rho u'(x_b) + (1 - \rho)u'(x_g))kdR\big]$$

Let $\bar{\ell} = \rho\ell_b + (1 - \rho)\ell_g$ denote the mean labour endowment. Since F_K is homogenous of degree 0, $dw\bar{\ell} + dRk = 0$. The terms in dw and dR would cancel if $u'(x_b) = u'(x_g)$ i.e. in the case of complete insurance markets. In the absence of insurance markets, $u'(x_b) \neq u'(x_g)$ and $dU \neq 0$. dU can be written as

$$dU = \beta(E(u'(x_1)\ell_1)dw + E(u'(x_1)kdR))$$
$$= \beta(E(u'(x_1))E(\ell_1)dw + cov(u'(x_1, \ell_1))dw + E(u'(x_1))kdR)$$
$$= \beta cov(u'(x_1), \ell_1)dw$$

Since u' is decreasing, it follows that $cov(u'(x_1), \ell_1) < 0$. A change $dk < 0$, which implies $dw < 0$, leads to an increase in welfare, $dU > 0$.

Reducing saving at date 0 increases date 0 consumption and reduces consumption at date 1, and to terms of first order, the direct effect of the change in consumption is zero, since agents have optimized on their choice of saving at equilibrium. But the price of capital increases and the price of labour decreases, shifting the representative agent's income away from the risky labour income ($w\ell_b$, $w\ell_g$) and towards the sure return (kR, kR) on capital. The price effect reduces the variability of date 1 consumption, improving the welfare of the representative agent. The change in prices (partially) replaces the insurance market which is missing.

A reduction dk in the agents' savings can also be achieved if the planner imposes a tax t on saving and redistributes the proceeds lump sum ($T = kt$) to the agents. The property of constrained suboptimality of a GEI equilibrium suggests that appropriate taxes on securities could be used to improve on the allocation achieved with incomplete markets. However Citanna, Polemarchakis and Tirelli (2006) have shown that to be sure to achieve a Pareto improvement in this way the number of securities (J) must exceed the number of agents (I), since the needed reallocations can only be achieved for sure if there are as many instruments (taxes) as objectives (agents' utilities).

Determinacy

The study of determinacy of equilibrium has served to uncover important differences between economies in which securities are nominal and those in which they are real. The study of economies with nominal securities led to the realization that monetary considerations need to be incorporated as an integral part of the sequential model.

In an economy in which securities are real, since the payoff of each security is proportional to the spot prices, doubling spot prices in a state doubles the payoffs of the securities, leaving agents' budget sets unchanged $(p_s(x_s^i - \omega_s^i) = \sum_{j=1}^{J} p_s A_s^j z^i)$. Thus price levels do not matter, and as in the standard GE model, the spot prices in each state can be normalized (e.g. $p_{s1} = 1, s = 0, \ldots, S$). Using arguments of differential topology analogous to those developed for the GE model it can be shown that generically (in endowments) an economy has only a finite number of equilibria – in short, with real assets GEI allocations are determinate.

When the securities are nominal, since the payoffs are independent of the spot prices, price levels matter. Doubling the price level in state s halves the purchasing power of the income promised by the assets in this state $(p_s(x_s^i - \omega_s^i) = \sum_{j=1}^{J} V_s^j z^i)$. This reasoning is insufficient to conclude that agents will be affected: for if agents correctly anticipate the 'doubling of the price level in state s' then they may adapt their portfolios accordingly to annul the effect. This is where the incompleteness of the security structure enters the picture. If the financial markets are complete, any change in the price levels across the states at date 1 can be 'undone' by a corresponding change in the portfolio chosen at date 0, so that again equilibrium allocations do not depend on price levels. If markets are incomplete, some changes in price levels cannot be 'undone' by changes in the agents' choices of portfolios, so that equilibrium allocations are different with different price

levels. Thus if the security structure consists of nominal securities and is incomplete, and if a GEI equilibrium is defined by conditions (i), (ii) and (iii) above so that nothing determines price levels, then there is a continuum of equilibrium outcomes. This property was first noted by Cass (1989), and the precise characterization of the dimension of the manifold of equilibria was studied by Balasko and Cass (1989) and Geanakoplos and Mas-Colell (1989).

Magill and Quinzii (1992) argued that a nominal contract is a promise to make a deferred payment of a sum of money, and that such promises only come to be made in an economy in which money is already used as a medium of exchange and a unit of account. What is needed therefore is a way of introducing money as a medium of exchange in the GEI model so that price levels are determined by the monetary side of the economy. They introduce a highly stylized (some might say 'brute force') model in which Clower's (1967) idea that only money can buy goods leads to a system of $S+1$ quantity theory equations

$$\sum_{i=1}^{I} p_s x_s^i = M_s, \quad s = 0, 1, \ldots, S$$

asserting that the demand for money for transactions must equal the supply of money M_s in each state – the vector $M = (M_0, M_1, \ldots, M_S)$ defining the monetary policy. When agents correctly anticipate the monetary policy and markets are complete, monetary policy does not affect the equilibrium allocation – a change from M to M' just leads to a change of portfolios financing the same allocation – but if markets are incomplete, different monetary policies lead to different allocations. The indeterminacy in the GEI model without price level determination becomes the property that, with nominal assets and incomplete markets, correctly anticipated monetary policy has real effects.

The need to introduce money explicitly into the GEI model with nominal assets has prompted the development of monetary models which are closer to the cash-in-advance models of macroeconomics, in which the interest cost of holding money (seignorage tax) is explicitly modelled. This has led to interesting ways of examining the structure of a monetary equilibrium model over a finite horizon (Dubey and Geanakoplos, 2003; Drèze and Polemarchakis, 2000) and to exploring the conditions (nonRicardian versus Ricardian) under which monetary and fiscal policies do or do not determine price levels (Nakajima and Polemarchakis, 2005).

In this short entry we have focused on properties of the GEI model in the simplest two-period or finite-horizon exchange setting. A more complete analysis of this model can be found in Magill-Quinzii (1996). A host of interesting new issues arise when the model is extended to an infinite horizon and in addition is extended to incorporate default, which bring to light the close connexion between GEI and macroeconomics.

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See also

Arrow-Debreu model of general equilibrium;
computation of general equilibria;
computation of general equilibria (new developments);
general equilibrium.

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