Unconventional Monetary Policy
and the
Safety of the Banking System

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ABSTRACT

This paper presents a simple general equilibrium model which simultaneously incorporates the banking sector and the monetary and macro-prudential policy of the Central Bank. Banks are viewed as intermediaries which channel funds from cash pools and depositors who insist on the complete safety of their funds, and investors who accept risks, to borrowers who invest in risky projects. Bank debt is rendered safe by the explicit or implicit guarantee of the government. The presence of cash pools which can either buy (short-term) government bills or lend to banks implies that the choice of an interest rate by the Central Bank determines the cost of funds for the banks. The government insurance of debt gives it an advantage over equity which implies that capital requirements are needed to limit bank leverage. The paper studies the possible monetary and prudential policies of the Central Bank and their effect on the banking equilibrium, for economies with a high demand for a safe asset—a notion precisely defined in the paper. We show that the conventional monetary and prudential tools, the interest rate and the capital requirements of banks, are not independent instruments, and that there is no choice of policy which can lead to a Pareto optimum. However enlarging the monetary policy toolkit by adding the payment of interest on bank reserves and QE policies can, in conjunction with appropriate capital requirements, restore the Pareto optimality of the banking equilibrium.

Keywords: Banking equilibrium, macro-prudential policy, capital requirements, unconventional monetary policy, Quantitative Easing, interest on reserves.

JEL classification numbers: D 50, D 61, E 58, G 21, G 28.

1   Introduction

The 2008 Financial Crisis had a significant impact on the Federal Reserve's monetary policy and on its prudential policy for the banking system. Prior to the crisis, capital requirements for banks were low, weakly monitored and readily by-passed through the creation of off-balance sheet special purpose vehicles. The focus of monetary policy was on the determination of the short-term interest rate, which, by current standards, was relatively high (an average of 3.5 % for the period 1998-2008) and the balance sheet of the Federal Reserve was of the order of $800 billion. After the crisis, banks and shadow banks were subject to significant increases in capital requirements, and banks, especially the systemically important banks, are now carefully monitored. The toolkit of the Federal Reserve has been expanded to include paying interest on the reserves of depository institutions and the Fed undertook several rounds of Quantitative Easing (asset purchases), which led to more than a fivefold increase in its balance sheet (currently $4.5 trillion). Over the post-crisis period (2008-2016) the interest rate has been essentially zero, and current yields to maturity on long-term bonds indicate that the market expects the interest rate to remain low.

The argument of this paper is that these changes in prudential and monetary policies are not independent, and are fundamentally related to the high demand for a safe asset. Prior to the crisis the high demand for safe and liquid debt on the wholesale money market, which greatly exceeded the supply of short-term Treasury Bills, led the banking sector to create substitute quasi-safe short-term debt through the use of securitization, collateral-backed loans and overnight maturity. The apparent safety provided by the collateral and short maturity, combined with the belief in a government safety net for the "too-big-to-fail" financial institutions, led to high leverage and hence fragility of the banking system so that, as Bernanke (2012) stressed, only a relatively small trigger (the collapse of the subprime mortgage market) was sufficient to touch off the crisis.

Since the crisis the focus of regulation has been on reducing the vulnerability of the system by imposing higher capital requirements and increased quality of the collateral, both of which restrict the capacity of the banking sector to use short-term debt to finance its operations. If the supply of short-term government debt is unchanged and if the capacity of the banking sector to absorb the demand for a safe asset is reduced, then the interest rate chosen by the Federal Reserve has to fall to re-establish equilibrium. Thus a prudential policy which seeks to increase the safety of the banking system must lead to a fall in the short-term interest rate, and to the extent that the change in prudential regulation is permanent, the downward pressure on the short-term interest rate will be permanent.

If the Central Bank does not supply a compensating increase in the supply of a safe asset, the

decrease in the interest rate required to re-establish equilibrium may be significant, and even lead to a negative interest rate. To avoid negative interest rates after the crisis, the Federal Reserve introduced a new tool of monetary policy, paying interest on reserves of depository institutions (IR), augmented by Reverse Purchase Agreements (Reverse Repo) to absorb funds from non-depository institutions such as money market funds. If the amount of reserves deposited with the Fed were small and the amount of securities that can be used for Reverse Repo were small, these policies would not significantly increase the supply of a safe asset. However if the other policy tool introduced after the crisis, Quantitative Easing (QE)— by which the Fed buys long-term securities which are paid by creating reserves— is carried out on a sufficiently large scale, then the magnitude of the Fed's balance sheet can be such that a combination of IR and Reverse Repo policies leads to a significant increase in the supply of a safe asset, decreasing the downward pressure on the short-term interest rate. The model presented in the paper shows that these new instruments of monetary policy are needed to complement an active prudential policy which seeks to enhance the safety of the banking sector. For, without these tools, the Central Bank would lack sufficient flexibility to choose an interest rate which coincides with the natural rate of interest, and investment would be impaired by the decrease in the short term-debt used by the banking system.

The model is a simple general equilibrium model of the economy that permits us to formalize the idea that there is a "high demand" for a safe and liquid asset, or equivalently the need for a large supply of short-term liquid debt. There are two dates and a single good. The economy consists of a private sector, with banks and agents providing funds to the banks, and a government sector consisting of the Treasury, which issues bonds and raises taxes, and the Central Bank in charge of monetary policy and the regulation of banks. The agents supplying funds to the banks are of three types: first retail depositors, who derive utility from using the payment system of banks; second cash pools which are surrogates in the model for the large institutional money market funds and other cash pools which supply the wholesale money market; and third risk neutral investors who supply equity to banks. The first two types of agents only buy completely safe assets, i.e. they are infinitely risk averse. Banks collect funds from these agents at date 0 and invest them in projects that yield a risky payoff at date 1. A typical bank in our model represents both a commercial bank which takes retail deposits and a shadow bank which borrows on the wholesale market against collateral.

The problem of the banking sector comes from the fact that banks act as intermediaries between two groups of agents (savers and borrowers) with conflicting needs. Depositors, both retail and institutional, want their funds to be available at short notice, while the projects of the entrepreneur/borrowers typically take time to mature. Depositors and short-term lenders also insist

that their funds be completely safe, while the business projects of the borrowers are necessarily risky and exposed to aggregate shocks, preventing banks from fully diversifying the risks of their loans. Thus to play their role of intermediation between savers and borrowers, banks have to perform two operations: *maturity transformation* (borrow short and lend long) and *risk transformation* (borrow safe and lend risky). The first type of transformation has been studied in the classic paper of Diamond-Dybvig (1983) and the subsequent literature. Our two-period model does not address this aspect of banking, but focuses on the problem risk transformation.

If society requires that banks act as effective intermediaries that channel funds from savers to investing entrepreneurs then a way has to be found for solving the basic conflict between the complete safety of deposits and the risk entailed in investing the funds in productive ventures. Experience has proved (see Gorton (2012)) that the resulting system can only work in a satisfactory way if it is backed by the government, whose role as a guarantor of banks' deposits and short-term liquid debt has significantly increased over the course of time. However explicit or implicit guarantees of bank deposits and short-term liquid debt give them an advantage over equity which, in a unregulated environment, banks exploit to create high leverage, leading to a high probability of bankruptcy and the occurrence of crises. Thus prudential regulation has to be put in place to limit bank leverage.

We also take as given that the need for macro management of the economy has led Central Banks to control the terms on which credit is made available in the economy by controlling the short-term interest rate. Thus in our model the Central Bank has a dual mandate: to determine *monetary policy* by choosing the (short-term) interest rate R^B and to determine *prudential policy* by choosing a capital requirement α for banks, where α is the proportion of investment funded by equity. We study the banking equilibrium associated with the choice of a pair of policies (R^B, α): the first result is that the two policy instruments are not independent. If the Central Bank uses the conventional monetary instrument R^B then there is a unique choice of capital requirement α which is compatible with equilibrium. Alternatively, if the Central Bank chooses to focus its attention on prudential policy and increases the capital requirement for banks, as in the post 2008 crisis era, then the interest rate must change (decrease) to reestablish equilibrium.

The use of debt to finance the banks' risky investments combined with the limited liability of banks' shareholders implies that banking equilibria can have a positive probability of bankruptcy. We show that the normative properties of banking equilibria depend on two fundamental parameters of the economy, which relate the risk of the technology to the relative supply of debt (by depositors/cash funds) and equity (by investors). The probability distribution of the risky return to the banks' investments determines a critical equity share $\widehat{\alpha}_c$ such that if the equity/investment ratio is

above $\widehat{\alpha}_c$ there is never bankruptcy, and if it is less there is a positive probability of bankruptcy. On the other hand, the characteristics of the economy—the preferences and endowments of the different types of providers of funds and the banks' investment technology—determine the share α^* of the investment contributed by investors in any Pareto optimal allocation. Every economy can then be shown to fall into one of two categories: those for which $\alpha^* \geq \widehat{\alpha}_c$, which we call *high-equity economies*, and those for which $\alpha^* < \widehat{\alpha}_c$, which we call *high-debt economies*.

In a high-equity economy, the constraint of a non-negative return for investors, due to limited liability, never binds and it is sufficient for the Central Bank to set the interest rate equal to the natural rate of return (the expected return on one-unit of investment by banks) to obtain a Pareto optimal choice of debt, equity and investment. In this type of economy government insurance of banks does not create an inefficiency if it is accompanied by an appropriate capital requirement. On the other hand in a high-debt economy in which a high share of investment is financed by depositors and cash pools, there is always a positive probability of bankruptcy. In such economies, which we take to represent many current economies, including the US, in which there is a high demand for a safe asset, there is no choice of conventional monetary and prudential policies which lead to a Pareto optimal allocation. In such economies there is a trade-off for Central Bank policy between setting the interest rate at the "right" level but directing too much debt towards the banks, which creates leverage and the risk of default, or setting the interest rate lower than the natural rate to ration the supply of debt and decrease leverage.

Instead of looking for a second best choice of policies (R^B, α) we show that in a high-debt economy the first best can be achieved if appropriate new instruments of monetary policy are introduced. To restore efficiency the Central Bank needs to find a way of decreasing debt and increasing bank capital in equilibrium so as to eliminate the possibility of bankruptcy. This can be achieved by the use of two unconventional instruments of monetary policy which have been introduced since the 2008 crisis: paying interest on reserves and buying securities from the private sector. If the Central Bank permits banks to place funds on reserve which receive interest and imposes a high capital requirement, then banks will be induced to park part of their funds at the Central Bank, avoiding their use for risky investment. However to keep investment at an appropriate level, equity has to replace debt as a source of bank financing. This can be achieved if the Central Bank uses the reserves to purchase the same risky securities as those purchased by investors, thereby increasing the supply of capital to the banks. The main result of the paper is that an appropriate combination of interest on reserves, asset purchases, and a well chosen policy (R^B, α), can achieve a Pareto optimal equilibrium for a high-debt economy.

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Relation to the Literature Our paper is related to several strands of the literature. First, the *safe asset* phenomenon: the high demand for safe and liquid debt with a focus on its potentially harmful consequences for the economy. The original literature comes from the international setting where "safe" means without risk of default, and is motivated by the excess demand for US government bonds from foreign countries trading in dollars which keeps US interest rates low (Caballero-Fahri-Gourinchas (2008, 2015)). Recently Caballero-Fahri (2016) present a closed economy version of of their model and focus on the consequence for equilibrium of the zero lower bound on the rate of interest (ZLB), which can become binding when the demand for a safe asset is sufficiently high. If we were to introduce the ZLB in our model and if it were binding in a banking equilibrium, then we would need to impose a quantity constraint on the amount of funds that can be lent by the cash pools and, in line with their results, we would find that consumption at date 0 is higher and investment (and hence output at date 1) lower than in an equilibrium without the zero lower bound. We do not consider the ZLB since our focus is on the consequence of a high demand for a safe asset for the vulnerability of the banking system (and hence the probability of crises) and on showing how the use of unconventional monetary policies can serve to increase the supply of a safe asset so that neither bankruptcy nor ZLB create inefficiency.

A separate literature has focused on relating the demand for safe debt to the emergence and functioning of the shadow banking sector (Gorton-Metrick (2010, 2012), Stein (2010)). These papers identify the run on the repo markets used in the shadow banking system when confidence on the asset-backed securities serving as collateral collapsed, as a key causal element of the 2008 Crisis (see also Bernanke (2012) and Rose (2015)). Poszar (2012, 2015) gives orders of magnitude for the different components of the demand for safe debt, showing that the sum greatly exceeds insured deposits and the supply of short term Treasury Bills. He also discusses the consequence of this high demand for the leverage of the shadow banking sector, in particular for broker-dealers and banks. From a historical perspective Gorton-Lewellen-Metrick (2012) show that the share of safe or quasi-safe debt issued by the financial sector has increased relative to the share of government bonds, even though the total supply has remained relatively stable as a percentage of GDP. In our paper we have attempted to capture the dramatic change of the banking system created by the emergence of shadow banks as a system parallel to standard commercial banks, by introducing cash pools as suppliers of safe debt alongside depositors, and assuming that banks can use the sure component of their payoff as collateral for their debt.

Our paper is close in spirit to Gennaioli-Shleifer-Vishny (2012, 2013) who model the relation between the demand for a safe asset and shadow banking, and point out the vulnerability of the resulting system. They assume that the agents demanding the safe asset are irrational and neglect

small probability disaster events, while we rationalize their behavior by the expectation of a bail-out by the government, but ex-ante this is essentially equivalent. Stein (2012) also models the demand for a safe asset as an asset that the banks can provide, but at the cost of a negative externality associated with the fire sale of their assets when their collateral value is called into question. Using the idea that the banking sector provides a surrogate safe asset with a negative externality, Greenwood-Hanson-Stein (2015, 2016) propose that the government, either the Treasury or the Central Bank, increase its provision of the safe asset to crowd out the provision by the banking sector, and argue that the most flexible approach is for the Federal Reserve to make more extensive use of Reverse Repurchase Agreements.

Second, our paper is related to the recent literature on the *regulation* of banks. Hanson, Kashyap and Stein (2011) provide an overview of prudential regulation, emphasizing the distinction between micro-prudential regulation which is partial equilibrium in nature, and macro-prudential regulation which takes into account broader general equilibrium effects. By this criterion, the recommendations underlying the Basel Accords (BCBS (2010)) and those of the Financial Stability Board (FSB (2012)) as well as those of many economists after the financial crisis (e.g. Admati-Hellwig (2013)), which were not based on general equilibrium models, were essentially micro-prudential in nature. Moreover, as pointed out by De Angelo-Stulz (2012) these recommendations for greatly increased capital requirements did not take into account that bank debt is an input for the banking process that differs from the inputs of standard corporations, so that capital structures with high debt may be more natural for banks than for standard corporations. Recently several macro DGSE models with the banking sector viewed as creating a friction in the transmission of funds from savings to investment, have been calibrated to assess the optimal capital requirements for banks (Christiano-Ikeda (2013), Corbae-D'Erasmo (2014), Nguyen (2014), Begenau-Landvoigt (2016)). On a more theoretical level Benigno-Robatto (2016) model the liquidity value of short-term bank debt (deposits) by a cash-in-advance constraint and show that there can be several equilibria depending on whether or not bankruptcy occurs. The role of a capital requirement is then to select the Pareto optimal equilibrium without bankruptcy.

Two papers, Allen-Carletti-Marquez (ACM) (2015) and Hellwig (2015b) present two-period general equilibrium models in which, as in our framework, banks channel funds from depositors and investors to risky productive investment. Each paper draws on a different surrogate device to make a distinction between "depositors" who provide debt and "investors" who provide equity. ACM use restricted participation: depositors are risk neutral but do not have access to the equity market. Hellwig uses a representative-agent model in which deposits provide additional utility above their purchasing power, which he calls a "warm-glow" utility: this is akin to the convenience

yield of deposits in our model. The main difference between these papers and ours is a difference in perspective. They study unregulated banking equilibria, focusing on the capital-debt ratio in equilibrium and subsequently, the existence of Pareto improving interventions by a regulator. To keep our paper to a reasonable length we have omitted the description of the unregulated banking equilibrium for our economy, which would be inefficient in a high-debt economy, and take as given that the government steps in with insurance and regulation of the banking sector to lessen the probability of occurrence of crises. This permits us to focus on the relation between the monetary and prudential policies of the Central Bank, a subject which has not received much attention with the exception of the recent papers of Greenwood-Hanson-Stein (2015, 2016) and the broad conceptual discussions of Hellwig (2014, 2015a).

Macro-monetary models which study unconventional policies, in particular QE, have typically been based on moral hazard frictions for the intermediaries (Reis (2009), Gertler-Karadi (2011)). In these models, since incentive constraints limit the amount of funds lent to banks, prudential policies in the form of capital requirements are not desirable for efficiency: the emphasis is on QE policies which compensate for an inefficiently low level of lending by the banking sector. However, as in our model, the use of QE and interest on reserves increases the size and risk of the Central Bank's balance sheet, and recent papers (Reis (2015), Hall-Reis (2015)) study potential default of the Central Bank, a possibility which we exclude by assuming that the Central Bank is fully backed by the fiscal authority.

The rest of the paper is organized as follows. Section 2 shows the fundamental importance of capital regulation in the analysis of bank profit maximization in the presence of government insurance of deposits. Section 3 completes the description of the model, defines a competitive banking equilibrium and studies the monetary-prudential policies compatible with such an equilibrium. Section 4 studies the normative properties of banking equilibria, showing that whether or not there exists a Pareto optimal banking equilibrium depends on the relation between the two fundamental ratios, the critical and natural equity-to-debt ratios of the economy. Section 5 introduces unconventional monetary policies and shows that if the Central Bank adds to the conventional tools a combination of paying interest on reserves and purchasing assets in the private sector—which we call an IRAP policy—then a well-chosen combination of policies can lead to a Pareto optimal banking equilibrium in an economy with high demand for a safe asset. Section 6 extends this optimality result to a richer model in which in addition to the bank-financed production sector, there is a sector consisting of firms financed directly by the capital market. Section 7 concludes.

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2 Bank Profit Maximization and Capital Regulation

Consider a two-date $(t = 0, 1)$ economy with a continuum of mass 1 of identical banks that collect funds from savers at date 0 and invest the funds in productive projects with risky per-unit payoff $\tilde{a}$ at date 1; that is, we bypass the intermediate step where banks lend their funds to entrepreneurs who undertake risky projects and reimburse the bank when the projects succeed. The random variable $\tilde{a}$ has support $A = [\underline{a}, \infty)$ with $\underline{a} > 0$, and continuous density f with $f(\underline{a}) = 0$. For convenience we extend f to a continuous function defined on $[0 \ \infty)$ by defining $f(a) = 0$ if $0 \leq a \leq \underline{a}$. We assume that the banks' investment has constant returns and that all banks have perfectly correlated payoffs. The last property implies that the banking sector can be modeled as a single representative bank that behaves competitively.

Banks are created at date 0 by raising equity (E) and debt (D), and investing the funds $K = E + D$ to get the risky payoff $K\tilde{a}$ at date 1. We refer to K as the productive investment of the banking sector or as the assets of a bank, when we think of the balance sheet of the representative bank. In the model the equity to asset ratio $\frac{E}{K}$ represents the capital-asset ratio which is targeted by banking regulators to improve the safety of the banking sector. The investors who provide the equity are taken to be risk neutral, to have limited liability and to discount profit at the rate R^E, which will be determined endogenously in equilibrium. The suppliers of debt (who are described later) are infinitely risk averse. This means that they will only provide funds if the repayment of the funds at date 1 is either backed by safe collateral or, if the debt exceeds the amount of safe collateral, is backed by explicit or implicit government insurance. Thus we assume that in order to make the system work with infinitely risk averse debt suppliers, the government provides the requisite insurance, either directly as deposit insurance or indirectly by rescuing the banking sector when it is in danger of failing. The consequence of the insurance is that the interest rate R that a bank pays on its debt does not depend on its debt-equity choice. Let $\hat{a}$ defined by

$$K\hat{a} = RD$$

denote the threshold per unit payoff which suffices to reimburse the promised repayment on the bank's debt D. If $\hat{a} \leq \underline{a}$ the debt is safe even without insurance and the bank is never bankrupt. If $\hat{a} > \underline{a}$, since shareholders have limited liability, the bank pays its debt only if its realized payoff is at least at the threshold, $a \geq \hat{a}$, and is bankrupt if $a < \hat{a}$. In both cases the expected profit of the bank's shareholders can be written as

$$\int_{\hat{a}}^{\infty} (Ka - RD)f(a)da - R^E E \tag{1}$$

since if $\hat{a} < \underline{a}$, $f(a) = 0$ for $a \in [\hat{a}, \underline{a}]$. The bank chooses its investment and financing $(K, E, D, \hat{a})$

to maximize (1) subject to $K = E + D$, $K\hat{a} = RD$. Using these two relations the bank's maximum problem reduces to choosing $(E, D, \hat{a})$ to maximize

$$\int_{\hat{a}}^{\infty} (E + D)(a - \hat{a})f(a)da - R^E E$$

subject to $(E + D)\hat{a} = RD$, $E \geq 0$, $D \geq 0$. The government's insurance of the reimbursement of the bank's debt and the associated constancy of the bank's borrowing rate R has the following important consequence.

Proposition 1. *The problem of choosing $E \geq 0$, $D \geq 0$ to maximize the bank's expected profit (1) subject to $K = E + D$ and $K\hat{a} = RD$, has no solution.*

Proof. Suppose $E = 0$, $D > 0$ then $\hat{a} = R$ and the banks profit is $D \int_R^{\infty} (a - R)dF(a)$ which, if there is positive probability that $a > R$, tends to infinity as $D \to \infty$. □

This result stands in sharp contrast to the result that holds under the standard assumption of finance that the price of the risky debt of a corporation with limited liability is the present value of the income stream that it delivers, and thus depends on the probability that the debt is reimbursed. In that setting the bank's profit maximizing problem has a solution—in fact infinitely many solutions, all with zero expected profit and an indeterminate debt-equity ratio[1]: this is the Modigliani-Miller theorem.

The government's insurance of the bank's debt makes it, in the terminology of Gorton (2010), 'information insensitive' in the sense that the interest rate that lenders require is not tied to the riskiness of the debt, so that lenders do not feel the need to get information on the financing and investment strategies of the bank. This lack of dependence of the interest rate on the riskiness of the loan tends to make debt look 'cheap' and leads to strategies where equity tends to zero and debt tends to infinity. This conforms with the commonly held view of bankers that debt is cheaper than equity as a source of funds and suggests (what experience confirms) that bankers, if left to their own choices, choose financing strategies with a lot of debt. As a result regulators typically

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[1]Assume for simplicity that $\underline{a} = 0$. In a standard finance framework where agents provide funds to the bank and perceive the debt to be risky, the bank's profit maximizing problem would be

$$\max_{(E,D',K,\hat{a})} \int_{\hat{a}}^{\infty} K(a - \hat{a})\mu(a)f(a)da - E$$

subject to (i) $K = E + q_D D'$; (ii) $K\hat{a} = D'$ and (iii) $q_D = \int_0^{\hat{a}} \frac{Ka}{D'}\mu(a)f(a)da + \int_{\hat{a}}^{\infty} \mu(a)f(a)da$ where D' is the amount of debt to be reimbursed at date 1 and $\mu(a)_{a \in A}$ is the stochastic discount factor, which is constant and equal to $\frac{1}{R^E}$ if investors are risk neutral. It is easy to check that a necessary condition for this problem to have a solution is $1 = \int_0^{\infty} a\,\mu(a)dF(a)$ and that any $E > 0$, $D' > 0$, K, $\hat{a}$ satisfying (i) and (ii) give zero profit and give a solution to the bank's problem, where the price of debt is given by (iii). This is just the Modigliani-Miller theorem in a setting with constant returns.

set minimum equity-to-asset ratios for banks. In the setting of our model this suggests imposing a constraint of the form

$$E \geq \overline{\alpha} \, K, \quad 0 < \overline{\alpha} < 1$$

where $\overline{\alpha}$ is the capital requirement of the bank. The profit maximizing problem of the representative bank thus becomes to choose $(E, D, \hat{a})$ to maximize

$$\Pi = \int_{\hat{a}}^{\infty} (E + D)(a - \hat{a}) f(a) da - R^E E$$

subject to $(E + D)\hat{a} = RD$, $E \geq \overline{\alpha}(E + D)$, $E \geq 0$, $D \geq 0$. The capital constraint $E \geq \overline{\alpha}(E + D)$ suggests that instead of using equity and debt (E, D) as the bank's choice variables we use (E, α), namely the bank's equity[2] E and its equity-asset ratio $\alpha = \frac{E}{E+D}$: this simple change of variable greatly simplifies the analysis of the bank's decision problem. The variables $(D, \hat{a})$ can then be recovered from (E, α) since $D = (\frac{1-\alpha}{\alpha})E$ and $\hat{a} = (1 - \alpha)R$. Since $E + D = \frac{E}{\alpha}$, if we define the function

$$\Phi(\alpha; R) = \frac{1}{\alpha} \int_{(1-\alpha)R}^{\infty} (a - (1 - \alpha)R) f(a) da, \quad 0 < \alpha \leq 1 \tag{2}$$

then the bank's profit can be decomposed into the product

$$\Pi(E, \alpha) = E \cdot (\Phi(\alpha; R) - R^E), \quad E \geq 0, \ 0 < \alpha \leq 1.$$

The function $\Phi(\alpha; R)$ defines the bank's *expected rate of return on equity* when its equity ratio is α and it faces the interest rate R on its debt. It can also be useful to view Φ as a function of the face value $\hat{a} = \frac{DR}{E+D} = (1 - \alpha)R$ of the bank's debt (per unit of investment). If we let $r(\hat{a}) = \int_{\hat{a}}^{\infty} (a - \hat{a}) f(a) da$ then $\Phi(\alpha, \frac{\hat{a}}{1-\alpha}) = \frac{1}{\alpha} r(\hat{a})$, where $r(\hat{a})$ is the expected return per unit of investment. $r(\hat{a})$ can be viewed as the value of a call option on one unit of the bank's assets with exercise price equal to the face value $\hat{a}$ of the debt : this expresses the property of "limited liability" of the bank—that it defaults on its debt $\hat{a}$ when it cannot pay $(a < \hat{a})$. The bank's return on equity $\Phi(\alpha, \frac{\hat{a}}{1-\alpha})$ is a levered multiple of its return to investment, $\Phi(\alpha, \frac{\hat{a}}{1-\alpha}) = \frac{1}{\alpha} r(\hat{a})$ where $\frac{1}{\alpha}$ is the bank's *equity leverage*.[3] Using the function $\Phi(\alpha, R)$ the bank's maximum problem expressed in terms of
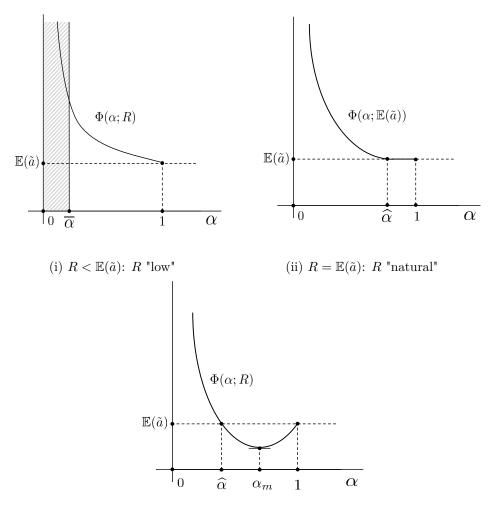
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[2]The bank's equity E should be understood in a broad sense that includes risky debt. In practice regulators look at two capital adequacy ratios, the ratios of Tier 1 and Tier 2 capital to risk-adjusted assets. Tier 1 capital is essentially shareholders' equity, while Tier 2 capital adds to equity the value of long-term corporate bonds. Long-term debt can not be incorporated in our model because of its timing, but the relevant characteristics for regulation, namely that the debt is not guaranteed in case of bankruptcy, could be modeled by introducing risky bonds. Since in the model investors are risk neutral, all securities with the same expected return are equivalent, distinguishing between equity and risky bonds would complicate the model without changing the results. However it is important for the interpretation of some of the results, in particular in Sections 5 and 6, to realize that what we call "equity" covers any risky security issued by the banks.

[3]Admati-Hellwig (2013,p.177) note that "in a major innovation" in 2010, Basel III proposed fixing a minimum capital requirement of 3% of assets, commenting that "if this number looks outrageously low, it is because it *is* outrageously low". With $\alpha = 0.03$, $\left(\frac{1}{\alpha}\right) = 33.3$: the return on equity is more than thirty three times the return on assets.

(E, α) reduces to

$$\max_{(E,\alpha)} \left\{ E \cdot (\Phi(\alpha; R) - R^E) \mid \alpha \geq \overline{\alpha} \right\} \tag{3}$$

Modulo its choice of E (namely its scale) the bank's problem reduces to the optimal choice of its equity ratio α: this decision depends on the behavior of the expected rate of return $\Phi(\alpha; R)$ as a function of α. There are three cases, which are distinguished by the magnitude of the interest rate R on the bank's debt relative to the expected return $\mathbb{E}(\tilde{a})$ on its assets: (i) $R < \mathbb{E}(\tilde{a})$ (ii) $R = \mathbb{E}(\tilde{a})$, (iii) $R > \mathbb{E}(\tilde{a})$, which we call *low, natural* and *high* interest rate cases. The graphs of the expected rate of return on equity $\Phi(\alpha; R)$ for the three cases are shown in Figure 1.

(i) $R < \mathbb{E}(\tilde{a})$: R "low"   (ii) $R = \mathbb{E}(\tilde{a})$: R "natural"

(iii) $R > \mathbb{E}(\tilde{a})$: R "high"

Figure 1: Bank's expected return on equity $\Phi(\alpha; R)$

To understand the geometric form of the function $\Phi(\cdot; R)$ for all $R > 0$ note first that $\Phi(\alpha; R) >$

0 for $0 < \alpha \leq 1$, $\Phi(1; R) = \mathbb{E}(\tilde{a})$ and

$$\frac{\partial \Phi}{\partial \alpha}(\alpha; R) = -\frac{1}{\alpha^2}\psi(\alpha; R) \quad \text{with } \psi(\alpha; R) = \int_{(1-\alpha)R}^{\infty}(a - R)f(a)da$$

As $\alpha \to 0$, $\psi(\alpha; R) \to \int_{R}^{\infty}(a - R)f(a)da > 0$, so $\Phi(\alpha; R)$ is decreasing when α is close to zero. $\frac{\partial \psi}{\partial \alpha}(\alpha; R) = -\alpha R^2 f((1 - \alpha)R)$ so that $\psi(\alpha; R)$ is decreasing in α as long as $f((1 - \alpha)R) > 0$ and constant in α when $(1 - \alpha)R \leq \underline{a}$, in which case $f((1 - \alpha)R) = 0$. For a given R, $\psi(\cdot; R)$ thus attains a minimum for $\alpha = 1$ with value $\psi(1; R) = \mathbb{E}(\tilde{a}) - R$.

(i) If $R < \mathbb{E}(\tilde{a})$, $\psi(\alpha; R) > 0$ for all $\alpha \in [0, 1]$ which implies $\frac{\partial \Phi}{\partial \alpha}(\alpha) < 0$ for all $\alpha \in (0, 1]$ and the graph of Φ has the form shown in Figure 1 (i).

(ii) If $R = \mathbb{E}(\tilde{a})$, $\psi(1; R) = 0 = \psi(\alpha; R)$ for $(1 - \alpha)R \leq \underline{a}$. Thus if $\hat{\alpha}$ is defined $(1 - \hat{\alpha})R = \underline{a}$, $\Phi(\alpha; R)$ is decreasing on $(0, \hat{\alpha})$ and constant on $[\hat{\alpha}, 1]$ and the graph of Φ is as shown in Figure 1 (ii).

(iii) If $R > \mathbb{E}(\tilde{a})$, $\psi(1; R) < 0$ and there exists $\alpha_m > 0$ such that $\psi(\alpha_m; R) = 0$ with $\psi(\alpha; R) > 0$ if $\alpha < \alpha_m$ and $\psi(\alpha; R) < 0$ if $\alpha > \alpha_m$. Thus $\Phi(\alpha; R)$ is decreasing on $(0, \alpha_m)$ and increasing on $(\alpha_m, 1]$. Thus $\Phi(\alpha_m; R) < \mathbb{E}(\tilde{a})$ and the graph of Φ is as shown in Figure 1 (iii).

We can now readily deduce the bank's choice of equity ratio α which maximizes its expected return on equity (and hence its expected profit) subject to the regulatory requirement $\alpha \geq \overline{\alpha}$. In case (i) since Φ is decreasing the bank chooses $\alpha = \overline{\alpha}$, the lowest permissible ratio (i.e. excluding α in the shaded region). In case (ii) if $\overline{\alpha} < \hat{\alpha}$, the bank chooses $\overline{\alpha}$ and if $\hat{\alpha} \leq \overline{\alpha} < 1$ then the bank is indifferent between all $\alpha \in [\overline{\alpha}, 1]$, which corresponds to the Modigliani-Miller theorem since there is no default. In the high interest rate case (iii), if $\overline{\alpha} < \hat{\alpha}$ (where $\hat{\alpha}$ is defined by $\Phi(\hat{\alpha}; R) = \mathbb{E}(\tilde{a})$) then the bank chooses $\alpha = \overline{\alpha}$. If $\overline{\alpha} > \hat{\alpha}$ then the bank does not borrow, setting $\alpha = 1$, financing all investment by equity.

The bank's maximum problem (3) consists of a joint choice of E and α. For this problem to have a solution the minimum capital requirement $\overline{\alpha}$ imposed by the regulator cannot be chosen independently of (R, R^E). In view of the constant returns to scale assumption when the bank's optimal choice of α is $\overline{\alpha}$ (case (i), (ii) and (iii) with $\overline{\alpha} \leq \hat{\alpha}$) there is a non-trivial solution to the choice of E if and only if

$$\Phi(\overline{\alpha}; R) = \frac{1}{\overline{\alpha}} \int_{(1-\overline{\alpha})R}^{\infty}(a - (1 - \overline{\alpha})R)f(a)da = R^E \tag{4}$$

i.e. the bank's expected profit is zero. In case (iii) if $\overline{\alpha} > \hat{\alpha}$ the bank's problem has a solution if and only if $R^E = \Phi(1) = \mathbb{E}(\tilde{a})$.

Our analysis of the bank's choice problem can be summarized in the following proposition.

Proposition 2. (Bank Maximum Problem) *Let $(R, R^E, \overline{\alpha})$ denote the loan rate, rate of return on equity and minimum equity-asset ratio faced by a bank, then*

(i) *if the bank faces a low loan rate $R < \mathbb{E}(\tilde{a})$, or if $R = \mathbb{E}(\tilde{a})$ and $(1 - \overline{\alpha})R > \underline{a}$, then there is a non-trivial solution to the bank's maximum problem if and only if the zero profit condition (4) holds. The solution is such that the capital constraint $E \geq \overline{\alpha} K$ binds. E is indeterminate and D is such that $D = \left(\frac{1-\overline{\alpha}}{\overline{\alpha}}\right)E$;*

(ii) *if $R = \mathbb{E}(\tilde{a})$ and $(1 - \overline{\alpha})R \leq \underline{a}$ then the bank's maximum problem has a non-trivial solution if and only if $R^E = \mathbb{E}(\tilde{a})$ and the bank is indifferent between all equity-asset ratios $\alpha \in [\overline{\alpha}, 1]$;*

(iii) *if the bank faces a high loan rate $(R > \mathbb{E}(\tilde{a}))$ then there is a critical value $\hat{\alpha}$ of its equity ratio defined by*

$$\Phi(\hat{\alpha}; R) = \frac{1}{\hat{\alpha}} \int_{(1-\hat{\alpha})R}^{\infty} (a - (1 - \hat{\alpha})R)f(a)da = \mathbb{E}(\tilde{a}), \quad 0 < \hat{\alpha} < 1 :$$

such that

– *if $\overline{\alpha} \leq \hat{\alpha}$ there is a solution to the bank's maximum problem if $(R, R^E, \overline{\alpha})$ are such that (4) holds: as in (i) the capital constraint $E \geq \overline{\alpha} K$ binds, E is indeterminate and $D = \left(\frac{1-\overline{\alpha}}{\overline{\alpha}}\right)E$;*

– *if $\overline{\alpha} > \hat{\alpha}$ there is a solution if $R^E = \mathbb{E}(\tilde{a})$ which consists of equity only, $\alpha = 1$, $D = 0$, E indeterminate.*

The cost of borrowing for the bank is influenced by two components: the interest rate R that it pays on its debt and the proportion of the time (the probability) that it repays its debt. When the interest rate is low $(R < \mathbb{E}(\tilde{a}))$ the interest cost is less than the expected return on investment and the greater the proportion financed by debt the higher the profit for the shareholders. Thus the capital requirement $\alpha \geq \overline{\alpha}$ is always binding. When the interest rate is the natural rate $(R = \mathbb{E}(\tilde{a}))$, the debt is still "cheap" if the probability of repaying it is less than one, which occurs when the capital requirement $\overline{\alpha}$ is low enough $(\overline{\alpha} < \hat{\alpha})$. The bank still finances as much as possible by debt and the capital requirement is binding. (Proposition 2 (i)).

When the interest rate is the natural rate and the capital requirement limits the leverage of the bank so that its debt will always be repaid $((1 - \overline{\alpha})R \leq \underline{a})$ then there is no default and the cost of debt for the bank is "fair". The bank is then indifferent between debt and equity—whatever the equity ratio α the expected revenue is the expected return on the investment. (Proposition 2 (ii)).

When the bank's interest rate exceeds the natural rate ($R > \mathbb{E}(\tilde{a})$) debt can still be "cheap" if the probability of repaying it is sufficiently small. This occurs when the proportion of investment financed by debt is high i.e. when the capital requirement is small ($\overline{\alpha} \leq \hat{\alpha}$), in which case the bank borrows as much as possible and the capital requirement is binding. When the capital requirement is high ($\overline{\alpha} > \hat{\alpha}$) the probability of default is small and the true cost of debt to the bank exceeds its expected return—the debt is too expensive and the bank. chooses all equity financing. (Proposition 2 (iii)).

3   Banking Equilibrium

We now complete the description of the economy and introduce the concept of a banking equilibrium. The economy consists of three types of agents: depositors, institutional cash pools (or rather the managers which represent them) and investors. Banks channel the funds of these agents into risky productive ventures, and the government determines interest rates, regulates banks, insures the agents who lend to banks and finances its expenditures through taxes. Depositors and cash pools have fundamentally the same objective, they seek a safe haven in which to place their funds; they will thus only lend to banks if they are sure of having their funds returned. Investors are more flexible and are prepared to accept risk.

Two important hypotheses lie behind the framework: first, the infinitely risk-averse lenders will not lend directly to investors because investors cannot commit to pay back their loans: thus it is not feasible for the risk-neutral agents to insure the risk-averse agents. Second, banks are the only institutions with the know-how to invest in productive projects—neither investors nor the government can directly fund productive projects without going through bank intermediaries. We thus abstract from that part of the productive sector which receives market-based financing by issuing traded bonds or equity. However in Section 6 we introduce a parallel production sector directly financed by investors via capital markets, to check the robustness of the results obtained in the simple model.

Infinitely risk-averse depositors deposit their funds with banks and cash pools lend to banks, despite the fact that they know banks will invest these funds in risky ventures, because the deposits are explicitly insured by the government (FDIC insurance in the US) and the cash pools are either protected by the presence of safe collateral, or if they lend more than the safe collateral, because they believe they are *implicitly* insured i.e. they believe that the government will rescue the banks if their assets prove insufficient to pay back their debt. Such a belief was essentially confirmed in 2008 since, in order to avoid a collapse of the financial system, governments either directly bailed out the failing institutions or, via the Central Bank, purchased the assets serving as collateral for

their debts to increase their resale value. Because bank debt is explicitly or implicitly "insured" there is no possibility of runs on the banks. The cost of a failure of the banking system could however be incorporated into the model as a loss in output when the government has to step in to pay the banks' debts.[4] However none of the results that we obtain depends on this cost, whose introduction would simply reinforce the result of inefficiency of a banking equilibrium and the need for regulation to prevent bankruptcy.

We now describe the characteristics and decisions made by the three groups of agents, depositors, cash pool managers and investors.

Depositors The representative depositor has an endowment of funds w_{d0} (the single good) at date 0 and no endowment at date 1. The depositor places funds in a bank so as to be able to transfer them to date 1 for consumption and to make use of the payment services provided by the bank at that date. The utility the agent derives from the consumption stream $x_d = (x_{d0}, \widetilde{x}_{d1})$ consisting of the consumption x_{d0} at date 0 and the random consumption $\widetilde{x}_{d1}$ at date 1 is given by

$$u_d(x_{d0}) + \min\{\widetilde{x}_{d1}\} + \rho \min\{\widetilde{x}_{d1}\} \tag{5}$$

where u_d is a concave increasing function, $\min\{\widetilde{x}_{d1}\}$ expresses the depositor's infinite risk aversion and $\rho(\min\{\widetilde{x}_{d1}\})$ denotes the convenience yield obtained from the transaction services offered by the banks at date 1: for simplicity we assume that the convenience yield is linear. Payment services exist only for deposits; for example, if the depositor gets the funds $\widetilde{x}_{d1}$ from investing in government bonds then the third term in (5) is zero—no convenience yield is obtained from holding government bonds. If R^d denotes the interest rate paid by banks on deposits, depositing the amount d in a bank generates the consumption stream $x_d = (w_{d0} - d, R^d d)$ from which a depositor derives the utility

$$u_d(w_{d0} - d) + (1 + \rho) R^d d$$

The date 0 utility function u_d models the opportunity cost of depositors and replaces the frequently made assumption that depositors have access to a safe storage technology.

Cash Pools In addition to the (insured) deposits of households, banks have access to a large supply of funds from a variety of institutional investors (corporations, wealth managers, money market funds,...) through what is generally referred to as the wholesale money market. Like depositors these institutional investors insist on the strict safety and liquidity of their funds, the mandate of

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[4]As shown by the recent financial crisis the rescue of the banking sector occurs only after the beginning of a run, which is sufficient to disrupt the functioning of the system. Government intervention attenuates the cost of a systemic failure but does not completely eliminate it.

their managers being: "do not lose" (Pozsar (2015)). This insistence on safety and liquidity made these funds vulnerable to runs which in the recent financial crisis were halted by actions of Central Banks and Treasuries, confirming the perception that these funds are "implicitly" insured by the government. To capture the role of these investors as purveyors of funds to the banking sector we introduce a group of agents that we call *cash pools*. Although cash pool investors might be a better terminology, for clarity in the paper we reserve the term "investor" exclusively for risk-tolerant agents who accept to invest in risky equity. The representative cash pool has a date 0 endowment w_{c0} and, like a depositor, infinite risk aversion with utility function

$$u_c(x_{c0}) + \min\{\widetilde{x}_{c1}\}$$

where u_c is a concave increasing function which models the opportunity cost of their funds. It follows that cash pools will only lend under the form of sure debt. If R^c denotes the interest rate that they receive (from banks or government bonds) the representative cash pool will choose c to maximize

$$u_c(w_{c0} - c) + R^c c$$

If the date 0 utility functions u_d and u_c of depositors and cash pools satisfy the Inada conditions

$$u'_d(x_{d0}) \to \infty \text{ as } x_{d0} \to 0, \quad u'_c(x_{c0}) \to \infty \text{ as } x_{c0} \to 0$$

then the solutions of their maximization problems are characterized by the first-order conditions

$$u'_d(w_{d0} - d) = (1 + \rho)R^d,$$
$$u'_c(w_{c0} - c) = R^c.$$

Investors To keep the number of different types of agents to a minimum we assume that investors play two roles: they represent both the agents who are long-term investors accepting to take risks, and the taxpayers. Investors have an endowment stream $w_i = (w_{i0}, w_{i1})$, where w_{i1} is non risky, and a utility function with risk neutrality at date 1

$$u_i(x_{i0}) + \mathbb{E}(\tilde{x}_{i1})$$

where the date 0 utility u_i is a concave increasing function satisfying the Inada conditions, which, as for the other agents, represents the opportunity cost of their date 0 funds. Investors can place their funds either in the equity of banks or in government bonds or can lend to the banks on the same terms as cash pools. If they buy the equity of a bank they receive the payoff $V(a)$ per unit of equity, where a denotes a realization of the random payoff $\tilde{a}$, and if they invest in cash pools they

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receive R^c per unit. If c_i denotes the funds placed in riskless assets by the representative investor and if e denotes the amount invested in bank equity, then the problem of an investor is to choose (c_i, e) to maximize

$$u_i(w_{i0} - c_i - e) + \mathbb{E}(w_{i1} - t(a) + V(a)e + R^c c_i)$$

where $t(a)$ is the lump-sum tax (or subsidy) from the government at date 1. Define the expected return on equity $R^E = \mathbb{E}(V(\tilde{a}))$ then the first-order conditions characterizing the solution of the investor's maximum problem are

$$u_i'(w_{i0} - c_i - e) = R^E \geq R^c \text{ (with } c_i = 0 \text{ if } R^E > R^c)$$

We do not consider the case $R^c > R^E$ for which $c_i > 0$ and $e = 0$, since banks must have positive equity in equilibrium. If $R^E = R^c$ then we assume investors only invest in equity (i.e. $c_i = 0$) and this is without loss of generality under the assumptions that we will introduce shortly.

There is a unit mass of each of the three types of agents and a unit mass of banks, to which we now turn our attention.

Banks Banks collect the deposits (d), the equity (e) and a part (c_b) of the lending of cash pools (the rest finances the government) and invest the proceeds $K = d + e + c_b$ in risky projects with payoff $\tilde{a}$ per unit of investment at date 1. The random variable $\tilde{a}$ is as described in Section 2 with support on the interval $[\underline{a}, \infty)$ and with continuous density $f(a)$ extended to the interval $[0, \infty)$. The safe part $K\underline{a}$ of their date 1 payoff can be interpreted as the safe component which can be pledged as collateral for borrowing from cash pools (akin to the senior tranche of Asset-Backed Securities). Cash pools will lend an amount in excess of this sure component i.e. $R^c c_b > K\underline{a}$ only if they are sure to recover their funds. In this section we consider two possibilities, with or without *implicit* insurance for the cash pools. If there is implicit insurance, cash pools believe that the government will reimburse their loans if the banks default and this belief is realized. In this case they may accept to lend more than the value of the safe collateral $K\underline{a}$, i.e. they accept risky collateral. We call the insurance "implicit" because there is no explicit contract or insurance premium attached to it. An example of implicit insurance is the belief that the government will bail out too-big-to-fail banks if they are in difficulty. However if the government makes it credible that it will not intervene in case of banks' default, then there is no implicit insurance, and the cash pools will not lend to the banks more than the sure collateral $K\underline{a}$. As for the depositors, we assume that they are explicitly insured[5] (FDIC in the US): for simplicity however we assume that no insurance premium is charged

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[5]Large uninsured deposits enter as "implicitly" insured cash pool loans since these loans are unsecured and uninsured. The implicit insurance of such deposits was made explicit after the financial crisis when temporarily (up to December 2012) all the non-interest-bearing accounts of banks were insured for an unlimited amount.

to the banks. In the Appendix we show that introducing an insurance premium would not change the analysis and would only only strengthen the inefficiency results. Finally banks provide payment services to depositors which cost them μ per unit of spending by a depositor at date 1.

Introducing cash pools into the model serves to capture the change in banking from traditional banking based on deposits to modern banking based on securitization of assets and collateralized borrowing on the wholesale money market. Such loans are safe as long as the collateral retains its value: when collateral is at risk of losing value (low return on bank assets) the Central Bank may intervene on the security markets to enhance their value (and liquidity) and avoid runs on the wholesale money market. Such interventions, and sometimes more direct interventions by the Treasury in times of crisis, are what justify the assumption of implicit insurance of the loans by cash pools in our model.

Banks are required to hold at least a minimal level of equity $E \geq \overline{\alpha} K$ where $0 < \overline{\alpha} \leq 1$: this is in line with current regulation, and as we saw in Section 2, is needed to have a solution to the bank's maximum problem when the interest rate charged on its loans does not adjust to the riskiness of its investment.

Let R^d denote the return promised on deposits, R^c the rate on cash pools and R^E the required rate of return on equity. The bank acts in the best interests of its shareholders and chooses (d, c_b, E, K) to maximize

$$\int_{\hat{a}}^{\infty} (Ka - \mu R^d d - (R^d d + R^c c_b)) f(a) da - R^E E$$

under the constraints $d \geq 0, \ c_b \geq 0, \ E \geq 0,$

$$K = d + c_b + E, \quad K\hat{a} = \mu R^d d + (R^d d + R^c c_b), \ E \geq \overline{\alpha} K.$$

Since deposits and cash pools are perfect substitutes for investment, both sources of funds will be used only if they have the same cost

$$(1 + \mu) R^d = R^c.$$

If we let $D = d + c_b$ and $R = R^c = (1 + \mu) R^d$ then the bank's problem is the problem studied in Section 2. From Proposition 2 the profit of the bank is zero, the scale of its investment is indeterminate and the bank chooses $\alpha = \overline{\alpha}$, unless both the interest rate R and the capital requirement $\overline{\alpha}$ are too high ($R > \mathbb{E}(\tilde{a}), \overline{\alpha} > \hat{\alpha}(R)$), in which case it chooses $\alpha = 1$ (see Figure 1(iii)). Since this latter case is incompatible with equilibrium, we only consider the case where $\alpha = \overline{\alpha}$.

The return to an equity holder is the random variable $V(a)$ defined by

$$V(a) = \begin{cases} \frac{K}{E}(a - \hat{a}), & \text{if } a \geq \hat{a}, \\ 0, & \text{if } a \leq \hat{a}. \end{cases} \quad \text{where } \hat{a} = (1 - \overline{\alpha}) R \tag{6}$$

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Government In broad terms the government in our model combines the role of a fiscal authority which finances government expenditures and the role of a Central Bank (backed by the fiscal authority) which conducts monetary and prudential policies. The government is assumed to have exogenously given expenditure (G) that the fiscal authority finances by issuing bonds B at date 0 for the value $B = G$, imposing taxes on investors at date 1 to pay back the government debt. It also insures banks' deposits, reimbursing depositors when banks go bankrupt. The Central Bank fixes the interest rate R^B on government bonds. and the minimum capital requirement $\overline{\alpha}$ for banks. If the cash pools are reimbursed at date 1 when banks are bankrupt, taxes are increased to cover the cost. The taxes imposed at date 1 in outcome a are thus given by

$$t(a) = \begin{cases} R^B B, & \text{if } a \geq \hat{a}, \\ R^B B + (1+\mu)R^d d + R^c c_b - Ka, & \text{if } a \leq \hat{a}. \end{cases} \tag{7}$$

where we have assumed full recovery of output when there is bankruptcy.[6] We assume that taxes are paid by the investors, who have sufficient resources $w_{i1} > 0$ to pay for them at date 1.

An economy $\mathcal{E}(u, \omega, \tilde{a})$ is characterized by the date 0 utility functions (u_{d0}, u_{c0}, u_{i0}) of the different types of agents, their endowments $\omega = ((\omega_{d0}, 0), (\omega_{c0}, 0), (\omega_{i0}, \omega_{i1}))$ and the risky return $\tilde{a}$ on investment. We introduce assumptions on the economy's characteristics which ensure that there exist equilibria with positive debt and equity for banks. In this model with constant returns in technology and linear date 1 preferences, there is a *natural rate of interest* $R = \mathbb{E}(\tilde{a})$ determined by the technology which is the expected return at date 1 from a one unit investment of the good at date 0. This is the benchmark interest rate that we use to express the willingness of agents to supply debt and equity in the economy.

Assumption 1. (a) $u_i'(w_{i0}) < \mathbb{E}(\tilde{a})$; (b) $w_{i1} > (w_{d0}(1+\mu) + w_{c0})\mathbb{E}(\tilde{a})$

Assumption (1)(a) guarantees that investors want to invest in the technology even if the profit of banks is not increased by leverage, while (b) guarantees that investors have sufficient resources at date 1 to reimburse the maximum that can be due to depositors and cash pools.

In keeping with the recent literature on shadow banking which emphasizes the magnitude of the funds on the wholesale money market seeking a safe haven, we assume that (short-term) government bonds[7] do not absorb all funds that cash pools are willing to lend.

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[6]It would be more realistic to introduce a cost of bankruptcy expressed as a loss of output when banks go bankrupt. It will however become clear from the analysis that follows that introducing such a cost of bankruptcy would not change the efficiency results, and would only serve to further increase the inefficiency of banking equilibria in high-debt economies. Thus for simplicity we omit such costs.

[7]We could also have used the term Treasury Bills: long-term government bonds are not modeled.

Assumption 2. $u'_c(w_{c0} - B) < \mathbb{E}(\tilde{a})$

Under this assumption, for all interest rates R^B such that $u'_c(w_{c0} - B) < R^B \leq \mathbb{E}(\tilde{a})$, the cash pools want to lend an amount which exceeds the supply of (short-term) government bonds B.

Deposits differ from government bonds by the payment services they offer, modeled by the convenience yield $\rho R^d d$. To ensure that in equilibrium deposits are positive and preferred by depositors to government bonds we assume

Assumption 3. (a) $u'_d(w_{d0}) < u'_c(w_{c0} - B)$; (b) $\rho > \mu$.

Assumption 3(a) ensures that for interest rates R^B such that cash pools absorb the government bonds B, depositors would want to buy government bonds if they did not have any other choice. Assumption 3(b) ensures that for such interest rates depositors prefer to place their funds as deposits with banks.

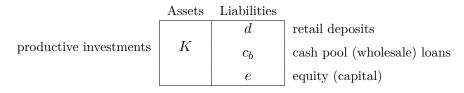Finally we restrict our attention to equilibria such that

$$u'_c(w_{c0} - B) \leq R^B \leq \mathbb{E}(\tilde{a}) \tag{8}$$

that is, we are interested in "low interest rate" equilibria where (short-term) government bonds do not offer a rate of return in excess of the expected rate of return in production.

Banking Equilibrium For this economy a *banking equilibrium* consists of interest rates (R^B, R^c, R^d), capital requirement $\overline{\alpha}$, rate of return on equity R^E, and choices $(d, c, e, E, D, K, \hat{a})$, such that

(i) $R^B = R^c$ (cash pools are indifferent between government bonds and lending to banks);

(ii) $R^d = \frac{R^c}{1+\mu}$ (banks are indifferent between deposits and borrowing from cash pools);

(iii) d is optimal for depositors given R^d

(iv) c is optimal for cash pools given R^c

(v) $D = d + c - B$, E, and $K = D + E$ are optimal for the representative bank faced with interest rates (R^d, R^c), required rate of return on equity R^E and capital constraint $E \geq \overline{\alpha} K$;

(vi) $E = e$ and e is optimal for investors given the rate of return R^E on equity.

The aggregate balance sheet of the banking sector is thus:

	Assets	Liabilities	
		d	retail deposits
productive investments	K	c_b	cash pool (wholesale) loans
		e	equity (capital)

In such an equilibrium, the date 1 consumption of investors in state a is given by: $x_{i1}(a) = w_{i1} - t(a) + V(a)e$, where $V(a)$ is given by (6) and $t(a)$ given by (7). (i) reflects the fact that because of the explicit insurance given by collateral and/or the implicit insurance of the government for less secure forms of debt, buying government bonds and lending to banks are perfect substitutes for cash pools.[8] (ii) reflects the fact that cash pools and deposits are perfect substitutes for investment by banks and thus must have the same cost. The other conditions incorporate the optimization of the depositors, cash pools, investors and banks given the prices that they face and the market clearing conditions. Assumptions 1-3 imply that $d > 0$, $c \geq B$, $e > 0$, and $x_{i1}(a) > 0$ for all $a \geq \underline{a}$. Replacing the optimality requirements by equivalent first-order conditions and incorporating the market clearing conditions, the equations that characterize an equilibrium are

$$u_d'(w_{d0} - d) = R^B \frac{(1 + \rho)}{1 + \mu} \tag{9}$$

$$u_c'(w_{c0} - c) = R^B \tag{10}$$

$$\frac{1}{\alpha} \int_{\hat{a}}^{\infty} (a - \hat{a}) f(a) da = R^E, \quad \hat{a} = (1 - \overline{\alpha}) R^B, \tag{11}$$

$$u_i'\left(w_{i0} - \frac{\overline{\alpha}}{1 - \overline{\alpha}} D\right) = R^E, \quad \text{with } D = d + c - B \tag{12}$$

.

Given the choice of policy $(R^B, \overline{\alpha})$, equations (9)-(12) express the conditions of compatibility that must be satisfied to obtain a banking equilibrium. Under the assumptions on u_d and u_c, the equations (9) and (10) have a unique solution for any $R^B > 0$. Let $d(R^B)$ denote the supply of deposits when depositors are given the interest rate $\frac{R^B}{1+\mu}$ (i.e. the solution to equation (9)), and let $c(R^B)$ denote the supply function of debt by the cash pools (the solution to equation(10)), then

$$D(R^B) \equiv d(R^B) + c(R^B) - B, \tag{13}$$

which denotes the total supply of debt to the banks when the interest rate is R^B, incorporates the solutions to equations (9) and (10). As a result the equilibrium equations reduce to the zero profit condition for the bank (11) and the first-order condition for the investors (12) with $D = D(R^B)$. That is, it must be optimal for the banks to use the supply of debt $D(R^B)$ while respecting their capital requirement $\overline{\alpha}$ and thus, given the return on equity, investors must want to supply the equity $e = \frac{\overline{\alpha}}{1-\overline{\alpha}} D(R^B)$ to the banks. Let

$$s(\alpha, D) = u_i'\left(w_{i0} - \frac{\alpha}{1 - \alpha} D\right) \tag{14}$$

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denote the return on equity (supply price) required by investors to supply the equity $\frac{\alpha}{1-\alpha}D$: thus $s(\alpha, D(R^B))$ is the return they require to supply the equity $\frac{\alpha}{1-\alpha}D(R^B)$ which complements the debt provided by depositors and cash pools. In equilibrium this return must be equal to the return on equity $\Phi(\alpha; R)$ of banks when faced with the capital requirement α and the cost of debt $R = R^B$. Equations (11) and (12) require that these two rates of return be the same

$$s(\alpha, D(R^B)) = \Phi(\alpha; R^B) \tag{15}$$

which implies that the monetary and prudential policy variables R^B and α are not independent. If the interest rate R^B is the choice variable, it determines the supply of debt to be absorbed, and the capital requirement α has to be compatible with the supply of equity by investors. Alternatively, if the focus is on macroprudential policy and the choice variable is the capital requirement α for the banking sector, the interest rate has to adjust to ration the supply of debt so that the capital requirement can be met by the banks while absorbing the supply of debt.

Given the monotonicity properties of the model, for each choice of either R^B or α the other variable is uniquely determined. To see this suppose R^B is fixed in the interval (8). The function $\alpha \rightarrow \Phi(\alpha, R^B)$ is decreasing (strictly if $R^B < \mathbb{E}(\tilde{a})$, weakly if $R^B = \mathbb{E}(\tilde{a})$) while the function $\alpha \rightarrow s(\alpha, R^B)$ is strictly increasing. Thus the function $\alpha \rightarrow s(\alpha, D(R^B)) - \Phi(\alpha, R^B)$ increases from $-\infty$ when $\alpha \rightarrow 0$ to $+\infty$ when $\alpha \rightarrow \frac{\omega_{i0}}{\omega_{i0}+D(R^B)}$. It follows that there is a unique $\alpha(R^B)$ such that

$$s(\alpha(R^B), D(R^B)) - \Phi(\alpha(R^B), R^B) = 0. \tag{16}$$

The equilibrium value of the capital requirement is shown in Figure 2(i) for an interest rate $R^B \leq \mathbb{E}(\tilde{a})$.

The function $R^B \rightarrow \alpha(R^B)$ is decreasing. This can be seen either by differentiating (16) or geometrically by noting that a decrease of R^B to $\tilde{R}^B < R^B$ shifts the Φ curve up (if debt is less expensive the return on equity goes up) and shifts the s curve down, since investors have to provide less equity and consume more at date 0. The intersection of the s and Φ curves occurs for $\alpha(\tilde{R}^B) > \alpha(R^B)$ (see Figure 2(ii))

Thus if we let $\left[R^B_{min} \ \mathbb{E}(\tilde{a})\right]$ denote the interval (8) for R^B, the function $\alpha(R^B)$ has values in the interval $\left[\alpha(\mathbb{E}(\tilde{a})) \ \alpha(R^B_{min})\right]$, is invertible in this interval, and the inverse function is decreasing. These results can be summarized in the following proposition

Proposition 3. (Dependence of Monetary and Prudential Policies) *Let Assumptions 1–3 hold. For any interest rate R^B in the interval (8) there is a unique capital requirement $\overline{\alpha} = \alpha(R^B)$ such that there is a banking equilibrium associated with the policy $(R^B; \overline{\alpha})$. The function $\alpha(R^B)$ is decreasing,*
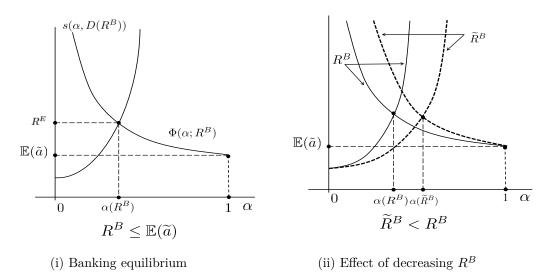
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Figure 2: Banking equilibrium with $R^B \leq \mathbb{E}(\widetilde{a})$.

so that for any $\overline{\alpha} \in \left[\alpha(\mathbb{E}(\tilde{a})) \ \alpha(R^B_{min}) \right]$ there is a unique interest rate $R^B(\overline{\alpha})$ compatible with the capital requirement $\overline{\alpha}$ in equilibrium. A higher $\overline{\alpha}$ implies a lower equilibrium interest rate.

Types of Banking Equilibria In a banking equilibrium there is a threshold $\hat{a} = (1-\bar{\alpha})R^B$ which defines the minimum per-unit payoff on investment for which banks can pay off the depositors and the cash pools. Depending on the value of $\hat{a}$ the equilibrium can be of one of three types

Equilibria of type 1: $(1 - \overline{\alpha})R^B \leq \underline{a} \iff \hat{a} \leq \underline{a}.$

For this type of equilibrium banks never default: the supply of loans by cash pools and depositors is sufficiently low at the interest rate R^B for the sure part $K\underline{a}$ of the payoff of the bank to cover the requisite reimbursement to cash pools and depositors at date 1. It is then natural that banks finance their debt at the rate R^B since their debt is sure.

Equilibria of type 2: $(1 - \overline{\alpha})R^B > \underline{a} \ (\iff \hat{a} > \underline{a})$ and $R^B c_b \leq K\underline{a}.$

For this type of equilibrium banks default for low realizations of $\widetilde{a}$, but there is enough sure collateral to insure the cash pools, provided their debt has priority over deposits in case of bankruptcy, which is the case if cash pools lend to banks through repo markets. The cash pools are willing to lend at the rate R^B since their debt is secured. Depositors also are willing to lend at rate R^B because of deposit insurance but they need to be reimbursed by the government in case of a bad realization of $\tilde{a}$.

Equilibria of type 3: $R^B c_b > K\underline{a}.$

For this type of equilibrium banks default when $a < \hat{a}$ and the government pays back both depositors and the unsecured component of the loans of cash pools. The cash pools are willing

to lend to the banks at the interest rate R^B provided they feel confident that their funds will be reimbursed either directly via the collateral or indirectly via the implicit insurance of the government. Such equilibria are however "fragile" in that they depend on the lenders' trust in an implicit insurance, and for this reason in practice these equilibria are subject to runs.

For an economy with fixed characteristics, the type of equilibrium which prevails depends on the interest rate R^B. For the same economy, lowering the interest rate may (depending on the magnitude of the change) shift the equilibrium to an equilibrium of a lower type. In particular for an economy satisfying Assumptions 1-3, an interest rate sufficiently close to the low end of the interval (8) generates an equilibrium of type 1 or 2.

Since during the financial crisis, the US government (Fed and Treasury) had to intervene to prevent banks (at least the largest ones) from defaulting on their debts to the wholesale money market, we may interpret the situation prevailing at that time as a type-three equilibrium where part of the wholesale money-market lending was unsecured. This type of equilibrium is justified in the model by the assumption that there is implicit insurance by the government: this assumption is an abstract way of modeling a variety of explanations which have been proposed to justify why the banks were able to borrow so much on the wholesale money market—either that they were believed to be "too-big-to fail", or that lenders had become lulled into a false sense of security so that debt was "information insensitive" (Gorton-Metrick (2010, 2012)), or simply that many lenders did not understand the magnitude of the risks to which the banks were exposed (Gennaioli-Shleifer-Vishny (2012)). What is important for our analysis is that at the interest rate R^B cash pools want to lend more than what can be absorbed by government bonds (B) and the safe debt of banks ($K\underline{a}$).

Many of the recent proposals for improving the safety of the banking system involve regulating the terms on which banks can issue debt and can be interpreted in this model as ways of moving from a type-three equilibrium, where the costs of systemic failure are high, to a type-two or type-one equilibrium where the banking system can withstand a low return on assets without needing government intervention. To achieve this goal, the regulators—Central Banks such as the Federal Reserve, advised by the Financial Stability Board—introduced a variety of measures to significantly increase the capital adequacy requirements[9] of large financial institutions. In the language of our model this means that Central Banks have made $\overline{\alpha}$ their primary policy variable and have increased its value from the very low values characteristic of the pre-2008 crisis period to significantly higher values. As predicted by Proposition 3 this has been accompanied by a decrease in the short-term

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[9]The new requirements are expressed in terms of "Total Loss Absorbing Capacity" (TLAC) and mandate that large systemically important institutions hold a high percentage of long-term unsecured debt which transforms into equity in case of resolution (see Federal Reserve Press Release, October 30, 2015).

interest rate which is now negative in several developed countries.[10]

4 Banking Equilibrium and Pareto Optimality

To gain a better understanding of the qualitative properties of a banking equiibrium it is useful to study its normative properties. We begin by examining the first-order conditions for Pareto optimality and then compare them with the FOCs satisfied at an equilibrium. An interior Pareto optimal allocation consists of consumption streams and investment

$$(x_{d0}, x_{d1}, x_{c0}, x_{c1}, x_{i0}, (x_{i1}(a))_{a \in A}, K) \gg 0$$

which maximize social welfare

$$\beta_d[u_d(x_{d0}) + (1 + \rho)\,x_{d1}] + \beta_c[u_c(x_{c0}) + x_{c1}] + \beta_i[u_i(x_{i0}) + \int_0^\infty x_{i1}(a)f(a)da] \tag{17}$$

subject to the date 0 and date 1 resource constraints

$$x_{d0} + x_{c0} + x_{i0} + K + G = w_0 \equiv w_{d0} + w_{c0} + w_{i0}$$
$$(1 + \mu)x_{d1} + x_{c1} + x_{i1}(a) = w_{i1} + Ka, \quad a \in A \tag{18}$$

where $(\beta_d, \beta_c, \beta_i) \gg 0$ are the relative weights of the agents. We have incorporated into the description of the allocation the property that the date 1 consumption streams of depositors and cash pools must be non-random because of their infinite risk aversion. The variable $x_{i1}(a)$ (consumption of investors at date 1 in state a) can be eliminated by using the second feasibility constraint. It follows that an interior solution only exists when $\beta_c = \beta_i = \frac{1+\rho}{1+\mu}\beta_d$. The necessary and sufficient conditions for an interior Pareto optimum are given by

$$\frac{1 + \mu}{1 + \rho}\,u'_d(x_{d0}) = u'_c(x_{c0}) = u'_i(x_{i0}) = \mathbb{E}(\tilde{a}). \tag{19}$$

and the resource constraints (18). While the Pareto optimal allocations may differ by the date 1 values of the agents' consumption streams, by (19) they all share the same vector of date 0 consumption and investment $(x^*_{d0}, x^*_{c0}, x^*_{i0}, K^*)$. We call $\mathbb{E}(\tilde{a})$ the *natural rate of return* of the economy since it is the expected return on investment and by (19) it is the rate that must be earned by all agents contributing to the financing of the investment at a Pareto optimal allocation. The implicit contributions (d^*, c^*, e^*) of each type of agent to investment and government expenditure in the Pareto optimal allocation are then defined by

$$u'_d(w_{d0} - d^*) = \mathbb{E}(\tilde{a})\frac{1 + \rho}{1 + \mu}, \quad u'_c(w_{c0} - c^*) = \mathbb{E}(\tilde{a}), \quad u'_i(w_{i0} - e^*) = \mathbb{E}(\tilde{a}).$$

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[10]The fact that short-term interest rates are still positive in the US is attributable to the Reverse Repo policy of the Federal Reserve which we discuss later.

We call the proportion of funds

$$\alpha^* = \frac{e^*}{e^* + d^* + c^* - B}$$

contributed by the investors at the Pareto optimal allocation the *natural equity ratio*: α^* is a fundamental attribute of the economy which depends on agents' preferences and endowments, and on the expected return on the technology. This ratio depends in particular on the relative endowments of the risk averse and risk neutral agents, and on their opportunity cost of funds determined by their date 0 utility functions.

Suppose now that we attempt to decentralize a Pareto optimal allocation as a banking equilibrium. In view of (19) both the interest rate and the return on equity must equal the natural rate of return: $R^B = R^E = \mathbb{E}(\tilde{a})$. Whether or not a Pareto optimal banking equilibrium exists thus depends on whether the policy $(R^B, \overline{\alpha}) = (\mathbb{E}(\tilde{a}), \alpha^*)$ is compatible with equilibrium and whether such an equilibrium satisfies $R^E = \mathbb{E}(\tilde{a})$. To see when this happens we need an additional definition which links the proportion of equity to the risk characteristics of the return on investment $\tilde{a}$. Let $\hat{\alpha}_c$ define the minimum equity ratio such that, if the cost of debt is $R = \mathbb{E}(\tilde{a})$, banks never default on their debt. $\hat{\alpha}_c$ is such that the threshold value $\hat{a}$ for which banks can just pay their debt coincides with the minimum return $\underline{a}$ on investment: $\hat{a} = (1 - \hat{\alpha}_c)\mathbb{E}(\tilde{a}) = \underline{a}$. When the cost of debt is equal to the natural interest rate, if $\alpha < \hat{\alpha}_c$ there is a positive probability of bankruptcy in a banking equilibrium and if $\alpha \geq \hat{\alpha}_c$ bankruptcy never occurs.[11] We call

$$\hat{\alpha}_c = 1 - \frac{\underline{a}}{\mathbb{E}(\tilde{a})}$$

the *critical equity ratio* of the economy. $\hat{\alpha}_c$ depends only on the characteristics of the banks' random return $\tilde{a}$ and can be considered as a normalized measure of the downside risk of $\tilde{a}$: it satisfies $0 \leq \hat{\alpha}_c \leq 1$; $\hat{\alpha}_c = 0 \iff \underline{a} = \mathbb{E}(\tilde{a})$ corresponds to zero risk, and $\hat{\alpha}_c = 1 \iff \underline{a} = 0$ corresponds to maximum downside risk.

The *set of all economies* $\mathcal{E}(u, \omega, \tilde{a})$ falls into two categories: those for which the preference-endowment-risk characteristics $(u, \omega, \tilde{a})$ are such that $\alpha^* \geq \hat{\alpha}_c$ and those for which $(u, \omega, \tilde{a})$ are such that $\alpha^* < \hat{\alpha}_c$. Economies in the first categories are called *high equity economies*, and those in the latter category are called *high-debt economies*.

- *High-equity economies:* $\alpha^* \geq \hat{\alpha}_c$. For these economies the optimal supply of equity is relatively

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[11]If banks make a large number of loans, there is a Law of Large Numbers at work for their *idiosyncratic risks*, so that the risks in $\tilde{a}$ should be thought of as the *aggregate risks* to which all banks in the economy are exposed. If, as a rough back-of-the-envelope calculation, we assume that one dollar invested cannot lose more than 20% ($\underline{a} = .8$) and the expected return on investment is 3% ($\mathbb{E}(\tilde{a}) = 1.03$), then the critical capital requirement $\hat{\alpha}_c$ above which there is no bankruptcy is 22%. This is in the ball park of the Total Loss Absorbing Capacity recently proposed by the Federal Reserve for large (GSIB) banks.

high and/or the risk of the technology is relatively low in the Pareto optimal allocations. To simplify we attribute the inequality to a high supply of equity.

- *High-debt economies:* when $\alpha^* < \widehat{\alpha}_c$ the optimal supply of equity by the investors is relatively low and a large proportion of the investment at the Pareto optimal allocations is provided by the depositors and the cash pools, either because they have a relatively large share of the date 0 resources or because they are eager to transfer income to date 1, or both. Since in the market equilibrium these agents provide funds in the form of safe debt, we call economies with $\alpha^* < \widehat{\alpha}_c$ high-debt economies. These are economies in which a high proportion of debt would be optimal, were it not for the fact that with limited liability a high proportion of debt results in bankruptcy for low returns on the banks' investments.

Let us show that high-equity economies have Pareto optimal banking equilibria, but that all banking equilibria of high-debt economies are inefficient. As argued earlier a banking equilibrium can be Pareto optimal only if $R^B = \mathbb{E}(\widetilde{a})$. In view of the properties of the equity return function Φ established in Section 2, when $R = \mathbb{E}(\widetilde{a})$

$$\Phi(\alpha, \mathbb{E}(\widetilde{a})) = \begin{cases} > \mathbb{E}(\widetilde{a}) & \text{if} \quad 0 < \alpha < \widehat{\alpha}_c \\ = \mathbb{E}(\widetilde{a}) & \text{if} \quad \alpha \geq \widehat{\alpha}_c \end{cases}$$

To be Pareto optimal the equilibrium must be such that $\overline{\alpha} = \alpha^*$ and $\Phi(\alpha^*, \mathbb{E}(\widetilde{a})) = \mathbb{E}(\widetilde{a})$. This is possible only if $\alpha^* \geq \widehat{\alpha}_c$, i.e. if the economy is a high-equity economy (see Figure 3(ii)). If $\alpha^* < \widehat{\alpha}_c$, the capital requirement $\overline{\alpha}$ compatible with $R^B = \mathbb{E}(\widetilde{a})$ satisfies $\alpha^* < \overline{\alpha} < \widehat{\alpha}_c$[12] and the rate of return on equity is higher than $\mathbb{E}(\widetilde{a})$ (see Figure 3(i)). Thus no banking equilibrium of high-debt economy can satisfy the FOCs for Pareto optimality. We can summarize these results in the following proposition:

Proposition 4. (Suboptimality of Banking Equilibrium) (i) *In a high-debt economy, no banking equilibrium is Pareto optimal.* (ii) *In a high-equity economy the policy $(R^B, \overline{\alpha}) = (\mathbb{E}(\widetilde{a}), \alpha^*)$ leads to a Pareto optimal banking equilibrium.*

If we view our model of banking equilibrium as an abstract and stylized representation of the pre-crisis banking system in the US, then it should be clear that the relevant case is where $\alpha^* < \widehat{\alpha}_c$. If $\alpha^* \geq \widehat{\alpha}_c$ then only equilibria without bankruptcy (i.e. of type 1) can occur: if the supply of debt and deposits had been small, and most of the banking system had been financed by equity, then there would not have been a banking crisis in 2008. A number of recent papers have highlighted

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[12]If $\alpha^* < \widehat{\alpha}_c$, $s(\alpha^*, \mathbb{E}(\widetilde{a})) - \Phi(\alpha^*, \mathbb{E}(\widetilde{a})) < 0$ and $s(\widehat{\alpha}_c, \mathbb{E}(\widetilde{a})) - \Phi(\widehat{\alpha}_c, \mathbb{E}(\widetilde{a})) = s(\widehat{\alpha}_c, \mathbb{E}(\widetilde{a})) - \mathbb{E}(\widetilde{a}) > 0$. Thus $\alpha^* < \overline{\alpha} < \widehat{\alpha}_c$.

(i) High-debt economy $\alpha^* < \widehat{\alpha}_c$.   (ii) High-equity economy $\alpha^* \geq \widehat{\alpha}_c$.
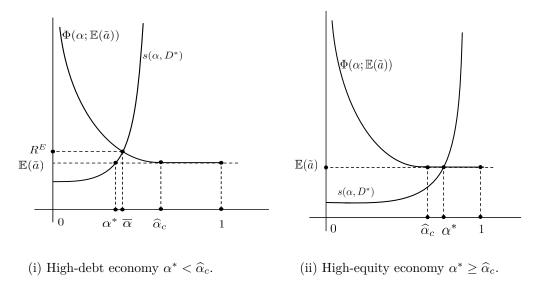
Figure 3: Banking equilibrium with $R^B = \mathbb{E}(\widetilde{a})$.

the importance of the *safe asset phenomenon*: Pozsar (2012), (2014)) stresses the importance of the fact that money market funds or more generally cash pools have very substantial amounts of money that they seek to lend safely and in liquid form. In practice this means that in addition to buying short-term government bonds, cash pools lend to large institutions for short periods, often with collateral—and that the supply of these funds inevitably encourages high leverage by banks, shadow banks and investment funds.[13]

Since the financial crisis much of the focus of bank regulation has been on increasing the safety of the financial system because of the high perceived costs of the crisis in terms of lost output. The current trend in regulation is to require that a much larger share of the financing of banks come from equity and long-term risky corporate bonds. Such regulation, and in addition the regulation of the repo markets, does not however take into account that the buyers of equity and long-term bonds are distinct from the suppliers of funds on the wholesale money market so that prices need to change to induce a change in the supply of debt and equity. Mandating that a smaller share of the financing of the banking sector comes from deposits and safely collateralized short-term debt, and a higher proportion from equity and long-term unsecured bonds (TLAC) will take the economy from the current type-three equilibrium, where the taxpayer has to rescue both depositors and cash pools in the case of bad outcomes, to a type-two or type-one equilibrium where at most depositors may need to be rescued. The economy will indeed be safer, but the interest rate will have to be

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[13]Gorton-Metrick (2010) and Gennaioli-Schleifer-Vishny (2012) argue that the high demand for a safe asset in large part serves to explain the emergence of the shadow banking system.

very low to induce cash pools to cut back on their supply of debt. Moreover the investment will have to be substantially reduced, since a low leverage implies a low return on equity, which in turn implies that investors will only supply a small amount of equity (and/or buy a small amount of risky corporate bonds).

Another approach, mentioned among others by Pozsar (2014) consists in increasing the supply of short-term government bonds (Treasury bills) by tilting the maturity structure of government debt towards the short end. However given the magnitudes involved[14], it does not seem realistic that short-term government debt could absorb all the funds in the wholesale money market.[15] Under the pre-crisis institutional framework there was thus an unavoidable trade-off between safety and efficiency.

In the next section we show that two policies recently adopted by many Central Banks, namely paying interest on banks' reserves, and purchasing financial assets such as corporate bonds, may make it possible to improve on the banking equilibria of this section for economies with a high demand for a safe asset.

5 Interest on Reserves & Asset Purchase System

In a high-debt economy a banking equilibrium cannot achieve a Pareto optimum because it does not permit the rate paid to debt holders and the return to equity holders to simultaneously equal the natural rate of return.[16] Since in a high-debt economy there is a positive probability of bankruptcy when $R^B = \mathbb{E}(\tilde{a})$), banks do not pay their debts in all circumstances so that the return on equity is greater than the natural rate. Taxpayers pay the banks' debt when $a \leq \hat{a}$, and this amounts to a gift from the taxpayers to the equity holders which distorts the first-order conditions of investors. The best way to avoid this distortion is to avoid bankruptcy. If the real costs of bankruptcy in terms of lost output were also taken into account, then the need to avoid bankruptcy to obtain efficiency would be further reinforced.

To avoid bankruptcy, the Central Bank must find a feasible way of increasing the capital requirement from $\overline{\alpha}(\mathbb{E}(\tilde{a}))$, which for a high-debt economy lies in the interval $(\alpha^*, \hat{\alpha}_c)$, to $\hat{\alpha}_c$ or higher. Thus a way must be found of decreasing the debt used by the banks for risky investment or in-

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[14]Pozsar (2014) estimates the amount that institutional cash investors placed in safe short-term liquid instruments in 2013 as approximately \$6 trillion, while the amount of Treasury Bills outstanding was \$1.6 trillion. Data in Gorton-Lewellen-Metrick (2012) and Greenwood-Hanson-Stein (2015, 2016) show the progressively increasing discrepancy between money-like claims and the outstanding quantity of (very) short-term Treasury bills over the last 25 years.

[15]A model of the optimal term-structure of government debt when agents have a preference for short-term debt but there is a rollover risk for the Treasury is studied in Greenwood-Hanson-Stein (2015).

[16]In this model equity investors are assumed to be risk-neutral, which greatly facilitates the general equilibrium analysis. With the more realistic assumption of risk-averse equity investors, the optimal return on equity would incorporate a positive risk premium.

creasing their equity funding, or both—without decreasing the interest rate from the natural rate. At first sight this appears like a difficult, if not an impossible task: after all, it calls for changing debt and equity without changing the rate of interest. We show nevertheless that this objective can be achieved if the Central Bank draws on the two "unconventional" policy instruments of paying interest on reserves and purchasing assets on the capital market. We call the resulting system in which the Central Bank uses these two policy instruments the *Interest on Reserves & Asset Purchase* system, or more briefly, the IRAP system.

We thus introduce two changes to the banking model of the previous section. First, the Central Bank accepts whatever funds the banks wish to place as reserves, on which they are paid the interest rate R^r. Second, the Central Bank makes use of these reserves to purchase risky securities from the private sector, using the payoffs from the securities to pay interest on the reserves. Let us see how the above modifications alter the decisions made by the different actors in the model.

Depositors, Cash Pools and Investors Here there is essentially no change. Depositors continue to place deposits in the banks to get the convenience yield of the payment system they offer. Cash pools buy government bonds and lend the rest of their funds to banks. If their supply of funds at the interest rate R^B exceeds B then they must be indifferent between lending to banks or buying bonds so that equilibrium requires $R^c = R^B$, and for banks to accept deposits we must have $R^d = \frac{R^B}{1+\mu}$. Investors buy bank equity with a required return on equity R^E.

Banks Banks choose debt D, equity E and the amount of reserves M to place at the Central Bank, investing $K = D + E - M$ in risky projects. They take the cost of debt R^B, the interest rate R^r on reserves and the cost of equity R^E as given. The payoff per unit of bank equity is

$$V(a) = \begin{cases} \frac{Ka - R^B D + R^r M}{E}, & \text{if } a \geq \hat{a}, \\ 0, & \text{if } a \leq \hat{a}, \end{cases}$$

where $\hat{a}$ is the bankruptcy threshold defined by

$$K\hat{a} + R^r M = R^B D.$$

Banks maximize the expected payoff to shareholders net of the cost of equity, under the capital requirement $E \geq \overline{\alpha} K$. If $R^r < R^B$ they choose $M = 0$; if $R^r > R^B$ they choose $K = 0$, $D = M = \infty$ and there cannot be an equilibrium. If $R^r = R^B$, banks are indifferent between all combinations (D, M) giving the same value to $\widetilde{D} = D - M$. $R^r = R^B$ is the only case compatible with an equilibrium with positive reserves. When $R^r = R^B$, the problem of choosing $(\widetilde{D}, E, K)$ for a bank is exactly the same as that studied in Section 2 with $R = R^B$.

The possibility of placing funds M on reserve at the Central Bank permits banks to simultaneously accept a large amount of deposits and funds from the cash pools (i.e. a large D) and to satisfy an capital requirement $E \geq \overline{\alpha}K$ with a high $\overline{\alpha}$, even when the supply of equity is relatively low and would not be sufficient to satisfy the requirement $E \geq \overline{\alpha}(E + D)$ in a standard banking equilibrium.

Government   As before the Treasury finances government expenditure by borrowing at the interest R^B chosen by the Central Bank. In addition the Central Bank accepts deposits of banks as reserves on which it pays the interest rate $R^r = R^B$. The reserves M are then used to buy risky securities from the investors. In this model the only risky security is the equity of banks, so that strictly speaking we have to assume the the Central Bank buys shares of the banks on the equity market. Given the current practice of Central Banks, this is not too realistic an assumption. It could be made realistic only in a more developed model which includes both equity and risky corporate bonds, since in practice Central Banks rarely buy equity but do buy corporate bonds. As mentioned earlier introducing such securities would not change the analysis but would complicate the notation, so that when we assume that the Central Bank "buys equity" with the reserves, it has to be understood as a proxy for the Central Bank buying risky bonds from the banking sector which are counted as "bank capital" for regulatory purposes.

Thus we assume that the Central Bank uses M to buy equity at date 0 and receives the payoff $V(a)M$ [17] at date 1, using these dividends to pay back the reserves with interest, any surplus going to the Treasury to reduce taxes. However when $V(a)M = 0$ and the Central bank needs to pay back reserves with interest, taxes are used the finance the Central Bank. The taxes needed to balance the government budget are
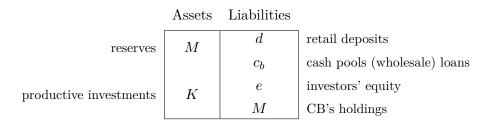
$$t(a) = \begin{cases} R^B B - V(a)M + MR^B, & \text{if } a \geq \hat{a}, \\ R^B B - (Ka - \widetilde{D}R^B) + MR^B, & \text{if } a < \hat{a}, \end{cases} \tag{20}$$

that is, we maintain the assumption that depositors and cash pools are insured by the government either explicitly (for depositors), or by collateral, or implicitly (for cash pools). As in the previous section the Central Bank imposes a minimum capital-asset ratio $\overline{\alpha}$.[18]

It is instructive to summarize the IRAP system by exhibiting the aggregate balance sheet of the banking sector:

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[17]Nothing in the analysis would change if $V(a)$ in (20) were replaced by a different payoff $\widehat{V}(a)$, provided that the new payoff has the same expected value i.e. $\mathbb{E}(\widehat{V}(\tilde{a})) = \mathbb{E}(V(\tilde{a}))$.

[18]$\overline{\alpha} = \frac{E}{K}$ is the ratio of equity (capital) to risk-adjusted assets with a zero coefficient on the safe reserves M in the calculation of the banks' assets.

	Assets	Liabilities	
reserves	M	d	retail deposits
		c_b	cash pools (wholesale) loans
productive investments	K	e	investors' equity
		M	CB's holdings

IRAP Banking Equilibrium   An *IRAP equilibrium* consists of an interest rate and capital requirement $(R^B, \overline{\alpha})$, a rate of return R^E, choices $(d, c, e, E, D, K, M, \hat{a})$ for the agents and banks such that depositors, cash pools and investors maximize their utilities and banks maximize expected profit, and all choices of the agents, banks, and government are compatible. The equations characterizing an IRAP equilibrium are

$$
\begin{aligned}
u_d'(w_{d0} - d) &= \frac{R^B(1+\rho)}{1+\mu}, \\
u_c'(w_{c0} - c) &= R^B, \\
\frac{1}{\overline{\alpha}} \int_{\hat{a}}^{\infty} (a - \hat{a}) f(a) da &= R^E, \quad \text{with } \hat{a} = (1 - \overline{\alpha}) R^B, \\
u_i'(w_{i0} - e) &= R^E, \\
E = e + M, \quad \widetilde{D} &= d + c - B - M, \quad K = E + \widetilde{D}, \quad E = \frac{\overline{\alpha}}{1 - \overline{\alpha}} \widetilde{D}.
\end{aligned}
$$

As in the previous section the Central Bank has two policy instruments $(R^B, \overline{\alpha})$. However as shown in Proposition 3, in the standard banking equilibrium the two policies are *dependent* since there is a unique choice $\alpha(R^B)$ compatible with R^B. We now show that the key property of the IRAP system is that the two instruments become *independent*: more precisely, any capital requirement $\overline{\alpha}$ which exceeds $\alpha(R^B)$ is compatible with an IRAP banking equilibrium.

Proposition 5. (Independence of Monetary and Prudential Policies) *Let $\mathcal{E}$ be an economy satisfying Assumptions 1–3. If the interest rate R^B lies in the interval* (8), *then for any capital requirement $\overline{\alpha}$ in the interval*

$$
\alpha(R^B) \leq \overline{\alpha} \leq 1
$$

there exists an IRAP banking equilibrium for the policy $(R^B, \overline{\alpha})$.

Proof. Let $(R^B, \overline{\alpha})$ be a monetary-prudential policy satisfying the assumptions of Proposition 5 and let $D(R^B)$ denote the supply of debt by the depositors and cash pools. The equations of an IRAP equilibrium reduce to

$$
\begin{aligned}
u_i'(\omega_{i0} - (E - M)) &= \Phi(\overline{\alpha}; R^B) & (21) \\
E &= \frac{\overline{\alpha}}{1 - \overline{\alpha}} \Big(D(R^B) - M \Big) & (22)
\end{aligned}
$$

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There is an IRAP equilibrium associated with $(R^B, \overline{\alpha})$ if there exists (E, M) with $E > 0$ and $0 \leq M \leq D(R^B)$, such that (21) and (22) hold. Inserting (22) into (21) leads to the single equation

$$u_i' \left(\omega_{i0} + \frac{M}{1-\overline{\alpha}} - \frac{\overline{\alpha}}{1-\overline{\alpha}} D(R^B) \right) = \Phi(\overline{\alpha}; R^B). \tag{23}$$

for determining M. The function $M \to u_i' \left(\omega_{i0} + \frac{M}{1-\overline{\alpha}} - \frac{\overline{\alpha}}{1-\overline{\alpha}} D(R^B) \right)$ is decreasing in M. If $\overline{\alpha} \geq \alpha(R^B)$ and $M = 0$, $u_i' \left(\omega_{i0} - \frac{\overline{\alpha}}{1-\overline{\alpha}} D(R^B) \right) \geq \Phi(\overline{\alpha}; R^B)$ (see (14), (15), and Figure 2(i)). If $M = D(R^B)$, $u_i' \left(\omega_{i0} + D(R^B) \right) < u_i'(\omega_{i0}) < \mathbb{E}(\tilde{a}) \leq \Phi(\overline{\alpha}; R^B)$, where we have used Assumption 1, and the property that $\Phi(\overline{\alpha}; R^B) \geq \mathbb{E}(\tilde{a})$ when $R^B \leq \mathbb{E}(\tilde{a})$. Thus for $\overline{\alpha} \geq \alpha(R^B)$ there exists $M \in [0, D(R^B))$ such that (23) is satisfied, and there is an IRAP equilibrium associated with the policy $(R^B, \overline{\alpha})$. □

In an IRAP equilibrium, banks can be given a high capital requirement because they can reduce the amount of debt they use to finance investment by placing a part of the funds obtained from depositors and cash pools as reserves at the Central Bank: since the rate they earn on their reserves is the same as the rate they pay on their loans there is no loss in such a reduction.[19] However a high capital requirement implies decreased leverage and a decreased return on equity and this decreases the supply of equity by investors. Under the IRAP system this reduction in the funds obtained from investors is compensated by the increase in capital funding from the Central Bank's asset purchases. For each value of $\overline{\alpha}$ above the ratio $\overline{\alpha}(R^B)$ required for a standard banking equilibrium, there is a value of the reserves M which exactly balances the decrease in debt and the increase in the supply of equity to reach the desired capital requirement. In particular, if $\underline{a} > 0$, the capital requirement can be chosen to be sufficiently high to prevent bankruptcy ($\overline{\alpha} \geq \hat{\alpha}_c$). If in addition the interest rate R^B is fixed at the natural rate $R^B = \mathbb{E}(\tilde{a})$ then the return on equity is equal to $\mathbb{E}(\tilde{a}))$ and all the conditions for Pareto optimality are satisfied.

Corollary 1. (Optimality of IRAP Banking Equilibrium) *Let $\mathcal{E}$ be a high-debt economy satisfying Assumptions 1-3. (i) Any IRAP equilibrium associated with a monetary-prudential policy $(R^B, \overline{\alpha}) = (\mathbb{E}(\tilde{a}), \overline{\alpha})$ with $\overline{\alpha} \geq \hat{\alpha}_c$ is Pareto optimal; (ii) the expected additional taxes for funding the IRAP policy are zero.*

Proof. (i) Consider the policy $(R^B, \overline{\alpha}) = (\mathbb{E}(\tilde{a}), \overline{\alpha})$ with $\overline{\alpha} \geq \hat{\alpha}_c$. Since in a high-debt economy $\alpha(\mathbb{E}(\tilde{a})) < \hat{\alpha}_c$ Proposition 5 applies and there is an IRAP equilibrium associated with $(\mathbb{E}(\tilde{a}), \overline{\alpha}))$. Since there is no bankruptcy when $\overline{\alpha} = \hat{\alpha}_c$, the return on equity satisfies $\Phi(\overline{\alpha}, \mathbb{E}(\tilde{a})) = \mathbb{E}(\tilde{a})$ (see Figure 3) and the first-order conditions for Pareto optimality hold for all types of agents.

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[19]We thus assume that there is no transaction cost for the banks to act as pass-through for transferring the funds of the cash pools to the Central Bank. In practice there are transaction costs (see footnote 20) and this leads the Federal Reserve to directly absorb the cash pool funds via Reverse Repo Agreements, thereby bypassing the banks.

(ii) Since there is no bankruptcy in this equilibrium the formula for the taxes is given by (20) with $\hat{a} = \underline{a}$ (the first expression in (20)). Then $\mathbb{E}(t(\tilde{a})) = R^B B - M\mathbb{E}(V(\tilde{a})) + M\mathbb{E}(\tilde{a}) = R^B B$, since $R^E = \mathbb{E}(V(\tilde{a})) = \mathbb{E}(\tilde{a})$. □

In an IRAP equilibrium the Central Bank essentially uses its balance sheet to transform safe debt into risky debt. Because of the *segmented markets*, such a shift can not be achieved by the private sector on its own. The Central Bank does the transformation by drawing safe debt out of the private sector, and using these funds to purchase risky securities. In the end the risk is born by the taxpayers, but the system is fair. When the realized return on the technology is higher than average the Central Bank makes a gain which serves to decrease taxes: in expected value the gain exactly compensates the loss when the Central Bank pays more on reserves than it earns on its risky assets. The IRAP system is better for the taxpayer than an insurance system with a low insurance premium, where the taxpayers foot the bill when there is an adverse aggregate shock, but receive little or no compensation when the aggregate shock is favorable.

One advantage of the IRAP system is that it only draws on policy instruments which have become part of the Central Bank's policy toolkit. In the US the Federal Reserve began paying interest on reserves (IR) in 2008, and has since come to adopt IR as a standard instrument of monetary policy.[20] This policy is supplemented by a policy of accepting funds from qualified Money Market Funds (MMFs) in the form of Reverse Repo transactions: the Fed uses the securities that it has purchased in prior QE episodes as collateral to borrow from MMFs in as large amounts as the MMFs want to lend, up to the total value of the securities serving as collateral (currently 3 trillion dollars). The goal of these policies is clearly to absorb the excess supply of funds which tend to depress the short-term interest rate when they cannot find an alternative safe haven in the banking system.

The asset purchase part of the model is highly stylized since the only risky security that we have introduced is the equity of banks. As we mentioned earlier nothing would change if we also introduced risky (corporate) bonds issued by banks and purchased by investors and the Central Bank. In

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[20]The Federal Reserve was permitted by Congress in 2008 to pay interest on reserves to enable the Fed to put a floor under the short-term interest rate. The excess supply of short-term funds at that time had made it difficult to raise and/or control the short-term interest rate. Only banks however are permitted to earn interest on reserves and in practice banks are often unwilling to act as pass-through for wholesale funds (see Williamson (2015) and Duffie-Krishnamurthy (2016)). As a result the Central Bank resorted to the device of Reverse Repo to enable (registered) money market funds to lend directly to the Central Bank to more effectively put a floor on the short rate—i.e. to prevent the demand by MMFs for short-term government bonds from driving the short rate to become negative. However the interest rate paid by the Fed on Reverse Repo Agreements is lower than the interest on reserves (currently 25 basis points versus 50 basis points) which limits the amount of funds using this facility. Greenwood-Hanson-Stein (2016) suggest equalizing the two rates to increase the transfer of cash pool funds from the banking sector to the Central Bank.

practice, however, there is a whole parallel system of financial markets—the stock and bond markets for the corporations in the non-banking sector, government bonds and asset-backed securities—in which investors can also place their funds. Typically the Federal Reserve has restricted its purchases of risky securities to long-term government bonds, mortgage-backed securities issued by the Government Sponsored Agencies and to a lesser extent to corporate bonds of private sector firms. To show the robustness of the optimality result of this section, we need to show that the asset purchases by the Central Bank lead to an increase in the demand for the capital market securities of banks even if the Central Bank buys securities in the non-banking sector. In the next section, we show that the results of Proposition 5 and its Corollary do indeed extend to an economy in which investors and the Central Bank can invest both in a banking and a non-banking sector.

6 Banking Equilibrium with More General Security Structure

We extend both the concept of a Banking Equilibrium and an IRAP Equilibrium by adding a second sector financed directly by the capital markets rather than the banks. Since we maintain the assumption that investors are risk neutral, stocks and risky bonds have the same expected return. We thus assume that the securities traded on the capital markets consist of the stocks of banks and the stocks of non-intermediated (NI) firms. As in the previous sections the intermediated firms financed by the banks are only implicitly modeled, appearing indirectly via the payoff of the banks' investment. To distinguish between actions or payoffs associated with banking and non-banking sectors, we use the subscript 1 for the banking sector and the subscript 2 for the non-bank financed sector. We assume that there is a mass one continuum of NI firms each of which uses an amount of funds K_2 financed by investors to obtain the random output $\tilde{a}\,g(K_2)$ at date 1, where $\tilde{a}$ is the same random shock as that for banks, reflecting the fact that $\tilde{a}$ is an aggregate shock which affects all production in the economy. The function $g(\cdot)$ is assumed to be differentiable, strictly increasing and strictly concave: to avoid boundary solutions we assume $g'(K_2) \to \infty$ as $K_2 \to 0$. The characteristics of, and decisions made by, depositors and cash pools are unchanged: the behavior of investors is however changed by the presence of the NI sector.

Investors To map the model to an equilibrium model with financial markets we assume that the investors have initial ownership shares $(\delta_{i1}, \delta_{i2})$ of banks and the NI firms and can change their ownership to a new portfolio $(\theta_{i1}, \theta_{i2})$: we assume $(\delta_{i1}, \delta_{i2}) = (1, 1)$ so investors are the full initial owners of banks and NI firms. Let (q_1, q_2) denote the stock market prices of the banks and the NI firms and let $(V_1(\tilde{a}), V_2(\tilde{a}))$ denote their date 1 random payoffs[21] : note that $V_1(\tilde{a}) \geq 0$ by

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[21]$V_1(a)$ is the payoff per share, while $V(a)$ in the previous section was defined as the payoff per unit of equity.

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limited liability and $V_2(\tilde{a}) \geq 0$ since $g(\cdot)$ is productive and NI firms are financed exclusively by stocks. The representative investor chooses the portfolio of shares $\theta_i = (\theta_{i1}, \theta_{i2})$ so as to maximize $u_i(x_{i0}) + \mathbb{E}(x_{i1})$ subject to the date 0 and date 1 budget equations

$$x_{i0} = w_{i0} + q_1 + q_2 - q_1\theta_{i1} - q_2\theta_{i2} - E_1 - K_2, \tag{24}$$

$$x_{i1}(a) = w_{i1} + V_1(a)\theta_{i1} + V_2(a)\theta_{i2} - t(a), \quad a \in A, \tag{25}$$

where $t(a)$ is the lump sum tax (defined below) imposed by the government and (E_1, K_2) are chosen by the banks and firms' managers[22]. The FOCs for this maximum problem are

$$u_i'(x_{i0})q_1 = \mathbb{E}(V_1(\tilde{a})), \quad u_i'(x_{i0})q_2 = \mathbb{E}(V_2(\tilde{a})).$$

Banks In the standard Banking Equilibrium the payoff of a bank to its shareholders is given by

$$V_1(a) = \begin{cases} K_1 a - ((1+\mu)R^d d + R^c c_b), & \text{if } a \geq \hat{a}, \\ 0, & \text{if } a < \hat{a}, \end{cases} \tag{26}$$

where $K_1 = d + c_b + E_1$ and $\hat{a}$ is defined by $K_1\hat{a} = (1+\mu)R^d d + R^c c_b$. The representative bank chooses (d, c_b, E_1) so as to maximize the present value of the profit of its initial shareholders

$$\frac{\mathbb{E}(V_1(\tilde{a}))}{R^E} - E_1$$

subject to the constraint $E_1 \geq \overline{\alpha}K_1$ imposed by the Central Bank. The bank's problem is thus identical to the problem studied in Sections 2 and 3. In the IRAP Equilibrium, the bank can in addition choose to place an amount M of reserves at the Central Bank on which it will receive the payment $R^B M$ at date 1.

NI firms The representative NI firm chooses K_2 to maximize the present value of its profit

$$\frac{\mathbb{E}(V_2(\tilde{a}))}{R^E} - K_2 \quad \text{with } V_2(a) = a\,g(K_2) \tag{27}$$

Since $g'(K_2) \to \infty$ as $K_2 \to 0$, the optimizing K_2 exists and is defined by the FOC

$$g'(K_2)\mathbb{E}(\tilde{a}) = R^E.$$

Government In the standard Banking Equilibrium the behavior of the government is the same as that in Section 3 and the taxes $t(a)$ are given by (7). In the IRAP Equilibrium the Central Bank

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[22]In equilibrium $q_1 \geq E_1$ and $q_2 \geq K_2$ so that the initial shareholders who finance the investment (E_1, K_2) do not want to get rid of their ownership shares to avoid financing investment.

uses the reserves M to buy θ_{g1} shares of banks and θ_{g2} shares of NI firms on the stock market so that its asset purchases satisfy

$$M = q_1\theta_{g1} + q_2\theta_{g2}$$

while the taxes needed to balance the government's budget are given by

$$t(a) = \begin{cases} R^B B + R^B M - \theta_{g1}V_1(a) - \theta_{g2}V_2(a), & \text{if } a \geq \max\{\underline{a}, \hat{a}\}, \\ R^B B + R^B(d + c_b) - K_1 a - \theta_{g2}V_2(a), & \text{otherwise.} \end{cases} \tag{28}$$

Banking Equilibrium with NI sector A (standard) *Banking Equilibrium with NI sector* consists of interest rates (R^B, R^c, R^d), a rate of return on equity R^E, stock prices (q_1, q_2), capital requirement $\overline{\alpha}$, and choices $((d, c, \theta_{i1}, \theta_{i2}), (E_1, D, K_1), K_2)$ such that (i) - (iv) of a banking equilibrium in Section 3 holds and

(v') $D = d + c - B$, E_1, and $K_1 = D + E_1$ are optimal for banks given (R^d, R^c, R^E), and $E_1 \geq \overline{\alpha} K_1$;

(vi') K_2 is optimal for NI firms given R^E;

(vii') $(\theta_{i1}, \theta_{i2})$ is optimal for investors given stock prices (q_1, q_2);

(viii') $R^E = u_i'(x_{i0})$ with $(x_{i0}, x_{i1}(a))$ given by (24)-(25);

(ix') $\theta_{i1} = 1$, $\theta_{i2} = 1$.

Incorporating the optimality of the choices (d, c) of depositors and cash pools into the function $D(R^B) = d(R^B) + c(R^B) - B$, finding a Banking Equilibrium reduces to finding $(R^B, \overline{\alpha}, E_1, K_2, R^E)$ satisfying the equations

$$\Phi(\overline{\alpha}; R^B) = R^E \tag{29}$$

$$g'(K_2)\,\mathbb{E}(\tilde{a}) = R^E \tag{30}$$

$$u_i'(w_{i0} - E_1 - K_2) = R^E \tag{31}$$

$$E_1 = \frac{\overline{\alpha}}{1 - \overline{\alpha}}D(R^B). \tag{32}$$

where R^B lies in the interval (8) of Section 3. The equilibrium security prices are then given by

$$q_1 = E_1 \qquad q_2 = \frac{g(K_2)\mathbb{E}(\tilde{a})}{R^E}. \tag{33}$$

Define $h(K_2) = g'(K_2)\,\mathbb{E}(\tilde{a})$: then by (29) and (30) in equilibrium $h(K_2) = \Phi(\overline{\alpha}; R^B)$ where h is decreasing. Thus h can be inverted and $K_2(\overline{\alpha}, R^B) = h^{-1}(\Phi(\overline{\alpha}; R^B))$, where $K_2(\overline{\alpha}, R^B)$ is

increasing in $\overline{\alpha}$ and R^B.[23] Using (31) and (32) leads to the equation

$$u_i'\left(\omega_{i0} - K_2(\overline{\alpha}, R^B) - \frac{\overline{\alpha}}{1-\overline{\alpha}}D(R^B)\right) = \Phi(\overline{\alpha}; R^B). \tag{34}$$

which determines the monetary-prudential policies compatible with equilibrium. (34) is the generalization of (15) in Section 3 to an economy with an NI sector. The function $\widehat{s}(\alpha, R^B) \equiv u_i'\left(\omega_{i0} - K_2(\alpha, R^B) - \frac{\alpha}{1-\alpha}D(R^B)\right)$ has the same monotonicity properties as the function $s(\alpha, D(R^B))$ defined by (14), so that the result of Proposition 3 on the dependence of monetary and prudential policies generalizes: for any interest rate R^B in the interval $[R^B_{min} \;\; \mathbb{E}(\widetilde{a})]$, there is a unique capital requirement $\overline{\alpha} = \alpha(R^B)$ which is compatible with equilibrium. Alternatively, if the Central bank fixes a prudential policy $\overline{\alpha}$ in the interval $[\alpha(\mathbb{E}(\widetilde{a})) \;\; \alpha(R^B_{min})]$, then there is a unique interest rate $R^B(\overline{\alpha})$ which is compatible with equilibrium. Thus in the more general setting of a standard Banking Equilibrium with an NI sector it is also true that the Central Bank's monetary and prudential policies can not be chosen independently.

The fundamental definitions of the natural and critical equity ratios of an economy can also be extended to an economy with an NI sector. Let $(x_{d0}^*, x_{c0}^*, x_{i0}^*, K_1^*, K_2^*)$ denotes the date 0 component of a Pareto optimal allocation which maximizes the social welfare function (17) subject to the resource constraints

$$x_{d0} + x_{c0} + x_{i0} + K_1 + K_2 + G = w_{d0} + w_{c0} + w_{i0}$$

$$(1 + \mu)x_{d1} + x_{c1} + x_{i1}(a) = w_{i1} + K_1 a + g(K_2)a, \quad a \in A$$

Then $(x_{d0}^*, x_{c0}^*, x_{i0}^*, K_1^*, K_2^*)$ are characterized by the FOCs (19) and $g'(K_2^*) = 1$. If we then define (d^*, c^*) by

$$x_{d0}^* = w_{d0} - d^*, \quad x_{c0}^* = w_{c0} - c^*,$$

and (E_1^*, D^*) by

$$x_{i0}^* = w_{i0} - E_1^* - K_2^*, \quad D^* = d^* + c^* - B$$

then the natural equity ratio for the banking sector[24] is defined by

$$\alpha^* = \frac{E_1^*}{K_1^*} = \frac{E_1^*}{E_1^* + D^*},$$

As in Section 4 the critical equity ratio $\widehat{\alpha}_c$, below which banks are exposed to bankruptcy for low realizations of $\widetilde{a}$, and above which bankruptcy never occurs when the interest rate is the natural

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[23]Increasing either $\overline{\alpha}$ or R^B decreases the return on equity $\Phi(\overline{\alpha}; R^B)$ of banks and, by no-arbitrage, decreases it for any firm. By concavity of g, a decrease in the return to investment is associated with an increase in investment.

[24]The NI firms are assumed to be fully financed by equity (or risky corporate bonds) and thus have an equity ratio $E_2^*/K_2^* = 1$. In our model there is no government rescue of NI firms, even though private investors may loose money when the random variable $\widetilde{a}$ takes low values.

rate $\mathbb{E}(\tilde{a})$, is given by

$$(1 - \widehat{\alpha}_c)\mathbb{E}(\tilde{a}) = \underline{a} \qquad \Longleftrightarrow \qquad \widehat{\alpha}_c = 1 - \frac{\underline{a}}{\mathbb{E}(\tilde{a})}$$

An economy with an NI sector for which the characteristics $(u, \omega, g, \tilde{a})$ are such that $\alpha^* < \widehat{\alpha}_c$ is called a *high-debt economy*, and if $\alpha^* \geq \widehat{\alpha}_c$ it is called a *high-equity economy*. As before we focus on high-debt economies—namely economies in which there is a high demand for a safe asset. In such an economy a standard Banking Equilibrium is never Pareto optimal. In particular if $R^B = \mathbb{E}(\tilde{a})$ then, by an argument similar to the proof of Proposition 4, the equilibrium capital requirement $\alpha(\mathbb{E}(\tilde{a}))$ (the solution of equation (34)) is such that $\alpha^* < \alpha(\mathbb{E}(\tilde{a})) < \widehat{\alpha}_c$, so that in equilibrium the return to equity satisfies $R^E > \mathbb{E}(\tilde{a})$, and the required FOC for investors at the Pareto optimal allocation is not satisfied.

The presence of the NI sector gives further insight into the misallocation of investment in a Banking Equilibrium, when the Central Bank sets the interest rate at the natural rate. In a high-debt economy, if $R^B = \mathbb{E}(\tilde{a})$, the equilibrium values of (E_1, K_1, K_2) satisfy $u_i'(\omega_{i0} - E_1 - K_2) = R^E > \mathbb{E}(\tilde{a})$ and $\mathbb{E}(\tilde{a})g'(K_2) = R^E$, so that $g'(K_2) > 1$, which implies that $K_2 < K_2^*$. To obtain the high return on equity obtained by the leveraged banks, the capital market financed sector has to curtail its investment below the optimal K_2^*. However $x_{i0} < x_{i0}^*$ implies $E_1 + K_2 > E_1^* + K_2^*$, so that $E_1 > E_1^*$: the high return on bank equity induces excessive investment in the bank-financed sector. This seems to correspond to the pre-2008 crisis experience where there was over-investment in real estate, fueled by bank financing of mortgages and construction loans.

Let us now show how the increased flexibility of the IRAP system permits Pareto optimality to be reestablished in a high-debt economy. Under this system the Central Bank gains two additional policy tools, paying interest on reserves and making asset purchases on the financial markets. These two additional tools permit the capital requirement to become an independent instrument so that an appropriate prudential policy can be chosen which leads to Pareto optimality.

Suppose the Central Bank sets the interest rate equal to the natural rate, $R^B = \mathbb{E}(\tilde{a})$ and imposes a capital requirement $\overline{\alpha}$ sufficiently high to avoid bankruptcy. If the interest on reserves is equal to the rate they pay on their debt, banks can, without loss, decrease the debt used to finance investment to $\widetilde{D} = D(R^B) - M$ by placing an amount M as reserves at the Central Bank. The Central Bank can then use the funds M to purchase a portfolio $(\theta_{g1}, \theta_{g2})$ of risky securities

$$M = \theta_{g1}q_1 + \theta_{g2}q_2. \tag{35}$$

It is important to note that the analysis that follows does not depend on the *composition* of this portfolio, and depends only on the total amount M spent by the Central Bank on purchasing risky

securities. Since in equilibrium the portfolios of the investors and the Central bank must satisfy

$$1 - \theta_{i1} = \theta_{g1}, \quad 1 - \theta_{i2} = \theta_{g2}$$

the funds M spent by the Central Bank are essentially transferred through trade on the financial markets to the investors whose date 0 budget equation becomes

$$
\begin{aligned}
x_{i\,0} &= w_{i\,0} + q_1(1 - \theta_{i1}) + q_2(1 - \theta_{i2}) - E_1 - K_2 \\
&= w_{i\,0} + M - E_1 - K_2
\end{aligned}
$$

If the optimal decisions of the depositors and cash pools are summarized by the function $D(R^B)$, the equations of an IRAP equilibrium with NI sector reduce to (29)-(30), with (31)-(32) being replaced by

$$u_i'(w_{i\,0} + M - E_1 - K_2) = R^E \tag{36}$$

$$E_1 = \frac{\overline{\alpha}}{1 - \overline{\alpha}}\Big(D(\mathbb{E}(\widetilde{a})) - M\Big). \tag{37}$$

and the equilibrium prices of the securities are again given by (33).

If $(R^B, \overline{\alpha}) = (\mathbb{E}(\widetilde{a}), \overline{\alpha})$ with $\overline{\alpha} \geq \widehat{\alpha}_c$, then $\Phi(\overline{\alpha}; R^B) = \mathbb{E}(\widetilde{a})$ and $D(\mathbb{E}(\widetilde{a})) = D^*$. Thus there exist reserves M which lead to a Pareto optimal equilibrium if there is a solution to the equation

$$u_i'\Big(w_{i\,0} + \frac{M}{1 - \overline{\alpha}} - \frac{\overline{\alpha}}{1 - \overline{\alpha}}D^* - K_2^*\Big) = \mathbb{E}(\widetilde{a}) \tag{38}$$

satisfying $0 \leq M \leq D^*$. Since $u_i'(w_{i\,0} - \frac{\alpha^*}{1-\alpha^*}D^* - K_2^*) = \mathbb{E}(\widetilde{a})$ the solution is given by

$$-\frac{\alpha^* D^*}{1 - \alpha^*} = \frac{M}{1 - \overline{\alpha}} - \frac{\overline{\alpha}}{1 - \overline{\alpha}}D^* \iff \frac{M}{D^*} = \frac{\overline{\alpha} - \alpha^*}{1 - \alpha^*}.$$

Since in a high-debt economy $\alpha^* < \widehat{\alpha}_c \leq \overline{\alpha}$, it folows that $0 < M < D^*$ holds. The amount of funds that need to be placed as reserves depends on the gap $\overline{\alpha} - \alpha^*$ but, if these reserves are used for "Quantitative Easing", the composition of the Central Bank's portfolio does not matter for the optimality result.

The optimality result does however depend on the assumption that the financial markets on which the Central Bank buys securities and the market for the capital securities of banks are not segmented. This seems to be a realistic assumption when the Central Bank buys corporate bonds, perhaps less when it buys government bonds or the bonds of government-sponsored agencies, which typically attract more risk averse investors than the stock and corporate bond markets. In practice the efficacy of a QE policy as a tool for increasing the supply of capital to banks will depend on the degree to which investors as a group seek to substitute between the different securities in their portfolios.

7 Conclusion

We have studied a stylized equilibrium model which simultaneously incorporates the banking sector and the monetary, fiscal and macroprudential policies of the government. The reason why the government is an integral part of the model is that we view banks as intermediaries with whom depositors—both retail and institutional—place funds that they insist be completely safe and immediately accessible. Given that banks use these funds to lend to businesses that invest in projects subject to unavoidable economy-wide shocks, experience has proved that the resulting system can only work if it is backed by the government. However such explicit or implicit guarantees of bank deposits and liquid debt give debt an advantage over equity which, in an unregulated environment, banks tend to exploit to create excess leverage, leading to a high probability of bankruptcy and the occurrence of crises. Thus prudential regulation is needed to limit bank leverage.

The focus of the paper is then on the dual mandate of the Central Bank, to determine monetary policy, i.e. the terms on which credit is to be made available to the private sector, and to determine prudential policy, i.e. to regulate the capital ratios of banks. We showed that when the sole instrument of monetary policy is the short-term interest rate then banks' capital requirements cannot be chosen independently. In essence the interest rate determines the supply of debt and the return to equity, and hence the debt-equity ratios of banks. In an economy where there is a large supply of short-term debt seeking a safe haven and a relatively low supply of capital by investors willing to accept risks, the capital-asset ratio of banks has to be low. If the Central Bank wants to increase this ratio to enhance the safety of the banking system, the interest rate will have to decrease to ration the debt which tries to migrate from the banking sector to the short-term government bond market.

To permit monetary and prudential policies to become independent policy instruments the government needs to enrich the toolkit at its disposal. In a high-debt economy there are two steps that the Central Bank needs to undertake:

(i) absorb the excess debt which creates high leverage in the banking system;

(ii) re-inject these funds into the banking system in the form of capital.

Step (i) can be achieved by a policy of paying interest on reserves (IR) placed at the Central Bank, a policy introduced by the Federal Reserve in 2008. In our model without transaction costs, this still permits banks to accept a lot of deposits and wholesale funds but, when faced with high capital requirements, they place part of these funds as reserves at the Central Bank instead of using them for risky investment. In practice this has to be complemented by Reverse Repo operations which permit non-bank institutions to lend directly to the Central Bank rather than passing their funds

through banks as in our model. In this way the Central Bank absorbs the excess debt by creating a sufficient supply of a "safe asset" which no longer needs to be supplied entirely by the private sector.

The need to absorb short-term debt and to create a safe asset to be able to control the short-term interest rate has been well understood by the Federal Reserve in the US. Step (ii) has been much less emphasized but is necessary if the intermediation between savers and entrepreneurs/borrowers—the raison d'etre of banks—is to occur. The funds placed as reserves in the Central Bank must be made available for investment, but without creating high leverage and the associated bankruptcy risks. We showed that this is possible if the Central Bank uses the reserves to purchase risky securities (AP) on the capital markets. This increases the supply of capital for banks, which compensates for the decreased use of debt for investment, and makes a higher capital/asset ratio possible without curtailing investment.

Although we do not model the output costs associated with a failure of the banking system, in our model a banking equilibrium in which bankruptcy occurs is necessarily inefficient: the anticipated subsidies of the taxpayers to banks when a rescue is needed distort the incentives of the banks' shareholders, leading to an inefficient choice of investment. We showed that such an inefficiency is unavoidable in a high-debt economy when the Central Bank has only one independent policy tool, either the interest rate or the capital requirement of banks. With the additional tools of the IRAP system bankruptcy can be avoided since high capital requirements become feasible and, if the interest rate is set at the natural rate, the resulting equilibrium is efficient. This suggests that these tools—asset purchases and interest on reserves—should be considered as permanent tools of a Central Bank rather than instruments to be used only at a time of crisis.

There may be opposition to using these instruments on a regular basis—especially the purchase of risky securities—since it exposes the Central Bank, and thus ultimately the taxpayers, to risks. Under the IRAP system the Central Bank uses its balance sheet to convert risky investment into safe debt, so that the excess leverage on the banks' balance sheets is transformed into a leveraged balance sheet for the Central Bank. There are however two reasons why the resulting system is better than the conventional system. Under the standard system the risk is born in the form of government subsidized insurance of the banking sector, and taxpayers see only the drawback, the need to shore up the system when banks are failing. Under the IRAP system they benefit from good times when the Central Bank has a high return on its assets, and this compensates in expected value for the cost of the additional taxes when the assets have a poor return. Second the IRAP system permits investment to be efficient, while it is inevitably inefficient under the standard system since the interest rate is either too low, or the leverage rate of banks is too high, or both.

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Appendix

Inefficiency of Banking Equilibrium with a positive Deposit Insurance Premium Suppose that the economy is as described in Section 3 but that the government requires that banks pay an insurance premium for the deposit insurance. We assume that the insurance premium is proportional to the value of the debt[25] and that it is paid at date 1 when banks are not bankrupt. Thus if $a \geq \hat{a}$, the representative bank pays $\pi R^B D$ to the government, where π is the premium per dollar of debt. This decreases the taxes and compensates the taxpayers for the additional taxes $Ka - R^B D$ that need to be levied to rescue the banks when $\underline{a} \leq a < \hat{a}$. Let us show that even

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[25]This is in line with the way FDIC insurance is assessed.

if the insurance is fair and the real costs of bankruptcy are not taken into account, the banking equilibrium can not be Pareto optimal.

The representative bank now maximizes

$$\int_{\hat{a}}^{\infty} (Ka - \mu R^d d - (1 + \pi)(R^d d + R^c c_b)) f(a) da - R^E E$$

subject to the constraints

$$K = d + c_b + E, \quad K\hat{a} = \mu R^d d + (1 + \pi)(R^d d + R^c c_b], \ E \geq \overline{\alpha} K.$$

Since deposits and cash pools are perfect substitutes for investment, both source of funds will be used only if they have the same cost

$$(1 + \mu + \pi) R^d = (1 + \pi) R^c.$$

Since cash pools must be indifferent between lending to the government or to the banks, it must be that $R^c = R^B$, which implies that $R^d = \dfrac{1 + \pi}{1 + \mu + \pi} R^B$, and the cost of funds for the banks is $R = (1 + \pi) R^B$.

Because cash pools can lend to the government, the banks can not charge them for the insurance premium and the depositors benefit from the arbitrage possibility of the cash pools by receiving a higher interest rate than in a no-insurance equilibrium $\left(\frac{1+\pi}{1+\mu+\pi} > \frac{1}{1+\mu} \right)$. In an equilibrium with insurance premium the first-order conditions for the depositors and cash pools are

$$u'_d(x_{d0}) = R^B \frac{(1 + \rho)(1 + \pi)}{1 + \pi + \mu} \qquad u'_c(x_{c0}) = R^B$$

and, if $\pi > 0$, it is impossible to find a rate R^B such that the FOCs (19) for Pareto optimality hold simultaneously for depositors and cash pools. Thus it is impossible for banking equilibrium with a positive insurance premium to be Pareto optimal. Furthermore if the economy is a high-debt economy with bankruptcy in equilibrium, and if bankruptcy involves real cost of lost output, then the presence of an insurance premium increase the probability and thus the expected cost of bankruptcy, thereby further augmenting the inefficiency of the banking equilibrium.