Qualifying Exam in Geometry/Topology Fall 1997.

1. Let ω be a 1-form defined on the sphere $S^2 = \{x \in R^3 | |x| = 1\}$. Assume ω is invariant under rotations, i.e. $\phi^*\omega = \omega$ for any $\phi \in SO(3)$, show $\omega = 0$.

2. Show the set $M = \{x \in R^4 | x_1 x_2 = x_3 x_4, |x| = 1\}$ is a smooth orientable surface.

3. Let M, N be smooth manifolds of dimension n, and $\pi : M \to N$ be a smooth map which is onto and has rank n at each point. Prove or disprove the statements:
   a) π is locally a diffeomorphism;
   b) π is a covering map.

4. Let S^1 be the unit circle in $R^2 = R^2 \times \{0\} \subset R^3$. Compute the fundamental group of $R^3 - S^1$.

5. Compute the homology of $R^3 - S^1$ with coefficients in Z.

6. Let $f : RP^2 \to T^2$ be a continuous map from the projective plane RP^2 to the torus $T^2 = S^1 \times S^1$.
   (a) Show that the induced homomorphism $f_* : \pi_1(RP^2) \to \pi_1(T^2)$ is trivial.
   (b) Show that f is homotopic to a constant map.