Fall 2024 Solutions

1a

Let $\mathbb{R}P^2 \vee \mathbb{R}P^2$ denote the wedge sum of two copies of $\mathbb{R}P^2$. Explicitly picking a basepoint $x_0 \in \mathbb{R}P^2$ we put:

$$\mathbb{R}P^2 \vee \mathbb{R}P^2 := (\mathbb{R}P^2 \times \{x_0\}) \cup (\{x_0\} \times \mathbb{R}P^2) \subset \mathbb{R}P^2 \times \mathbb{R}P^2$$

(a) Compute the fundamental groups of $\mathbb{R}P^2 \vee \mathbb{R}P^2$ and $\mathbb{R}P^2 \times \mathbb{R}P^2$

proof:

By

> Seifert-Van Kampen
>
> **Rotman: Corollary 7.42**
> If K is a simplicial complex, having connected subcomplexes L_1 and L_2 such that $L_1 \cup L_2 = K$ and $L_1 \cap L_2$ is simply connected, then for $v_0 \in \text{Vert}(L_1 \cap L_2)$
>
> $$\pi(K, v_0) \cong \pi(L_1, v_0) * \pi(L_2, v_0)$$
>
> **Hatcher: Theorem 1.20**
> If X is the union of path-connected open sets A_α each containing the basepoint $x_0 \in X$ and if each intersection $A_\alpha \cap A_\beta$ is path-connected, then the homomorphism $\Phi : *_a \pi_1(A_\alpha) \to \pi_1(X)$ is surjective. If in addition each intersection $A_\alpha \cap A_\beta \cap A_\gamma$ is path-connected, then the kernel of Φ is the normal subgroup N generated by all elements of the form $i_{\alpha\beta}(\omega) i_{\beta\alpha}(\omega)^{-1}$ for $\omega \in \pi_1(A_\alpha \cap A_\beta)$ and hence Φ induces an isomorphism $\pi_1(X) \cong *_\alpha \pi_1(A_\alpha)/N$
>
> **Andrews University (simply connected intersection):**
> If $X = A \cup B$ where A, B open, path connected and $A \cap B$ is simply connected then
>
> $$\pi_1(X) \cong \pi_1(A) * \pi_1(B)$$
>
> **Andrews University (general version):**
> If $X = A \cup B$ where A, B open, path connected and $A \cap B$ is path-connected then
>
> $$\pi_1(X) \cong \pi_1(A) *_{\pi_1(A \cap B)} \pi_1(B)$$
>
> #Algebraic_Topology

we have that $\pi_1(\mathbb{R}P^2) \cong \mathbb{Z}/2\mathbb{Z}$

Furthermore by

> Seifert-Van Kampen
>
> **Rotman: Corollary 7.42**
> If K is a simplicial complex, having connected subcomplexes L_1 and L_2 such that $L_1 \cup L_2 = K$ and $L_1 \cap L_2$ is simply connected, then for $v_0 \in \text{Vert}(L_1 \cap L_2)$
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> If $X = A \cup B$ where A, B open, path connected and $A \cap B$ is path-connected then
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> $$\pi_1(X) \cong \pi_1(A) *_{\pi_1(A \cap B)} \pi_1(B)$$
>
> #Algebraic_Topology

we have $\pi_1(\mathbb{R}P^2 \vee \mathbb{R}P^2) \cong \pi_1(\mathbb{R}P^2) * \pi_1(\mathbb{R}P^2) \cong \langle a, b | a^2 = b^2 = 1 \rangle$

By

> Fundamental Group of Products is Product of Fundamental Groups
>
> Rotman 3.7
>
> If (X, x_0) and (Y, y_0) are pointed spaces, then
>
> $$\pi_1(X \times Y) \cong \pi_1(X) \times \pi_1(Y)$$
>
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we have

$$\pi_1(\mathbb{R}P^2 \times \mathbb{R}P^2) \cong \pi_1(\mathbb{R}P^2) \times \pi_1(\mathbb{R}P^2) \cong \mathbb{Z}_2 \times \mathbb{Z}_2$$

1b

Let $\mathbb{R}P^2 \vee \mathbb{R}P^2$ denote the wedge sum of two copies of $\mathbb{R}P^2$. Explicitly picking a basepoint $x_0 \in \mathbb{R}P^2$ we put:

$$\mathbb{R}P^2 \vee \mathbb{R}P^2 := (\mathbb{R}P^2 \times \{x_0\}) \cup (\{x_0\} \times \mathbb{R}P^2) \subset \mathbb{R}P^2 \times \mathbb{R}P^2$$

Prove that $\mathbb{R}P^2 \vee \mathbb{R}P^2$ is not a

> Retract
>
> #Algebraic_Topology
> **Definition**
> A subspace $X \subset Y$ of a topological space X is called a ***retract*** of Y if there exists
>
> $$r : Y \to X$$
>
> with $r \circ i_X = \text{Id}_X$.

of $\mathbb{R}P^2 \times \mathbb{R}P^2$.

proof:

We have from

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>
> Let $\mathbb{R}P^2 \vee \mathbb{R}P^2$ denote the wedge sum of two copies of $\mathbb{R}P^2$. Explicitly picking a basepoint $x_0 \in \mathbb{R}P^2$ we put:
>
> $$\mathbb{R}P^2 \vee \mathbb{R}P^2 := (\mathbb{R}P^2 \times \{x_0\}) \cup (\{x_0\} \times \mathbb{R}P^2) \subset \mathbb{R}P^2 \times \mathbb{R}P^2$$
>
> (a) Compute the fundamental groups of $\mathbb{R}P^2 \vee \mathbb{R}P^2$ and $\mathbb{R}P^2 \times \mathbb{R}P^2$
>
> proof:
>
> By Seifert-Van Kampen we have that $\pi_1(\mathbb{R}P^2) \cong \mathbb{Z}/2\mathbb{Z}$
>
> Furthermore by Seifert-Van Kampen we have $\pi_1(\mathbb{R}P^2 \vee \mathbb{R}P^2) \cong \pi_1(\mathbb{R}P^2) * \pi_1(\mathbb{R}P^2) \cong \langle a, b | a^2 = b^2 = 1 \rangle$
>
> By Fundamental Group of Products is Product of Fundamental Groups we have
>
> $$\pi_1(\mathbb{R}P^2 \times \mathbb{R}P^2) \cong \pi_1(\mathbb{R}P^2) \times \pi_1(\mathbb{R}P^2) \cong \mathbb{Z}_2 \times \mathbb{Z}_2$$
>
> #Algebraic_Topology_Qual

that $\pi_1(\mathbb{R}P^2 \vee \mathbb{R}P^2) \cong \mathbb{Z}_2 * \mathbb{Z}_2$ and $\pi_1(\mathbb{R}P^2 \times \mathbb{R}P^2) \cong \mathbb{Z}_2 \times \mathbb{Z}_2$

Assume we have a retraction $r : \mathbb{R}P^2 \times \mathbb{R}P^2 \to \mathbb{R}P^2 \vee \mathbb{R}P^2$, then we have an induced map $r_* : \pi_1(\mathbb{R}P^2 \times \mathbb{R}P^2)) \to \pi_1(\mathbb{R}P^2 \vee \mathbb{R}P^2))$ which will also act on identity when composed with the i_* inclusion. Thus r_* should be surjective onto $\mathbb{Z}_2 * \mathbb{Z}_2$, but this is not possible since $\pi_1(\mathbb{R}P^2 \times \mathbb{R}P^2))$ is a finite group, while $\mathbb{Z}_2 * \mathbb{Z}_2$ is infinite. ■

i_* must be injective, but this is not possible

1c

Let $\mathbb{R}P^2 \vee \mathbb{R}P^2$ denote the wedge sum of two copies of $\mathbb{R}P^2$. Explicitly picking a basepoint $x_0 \in \mathbb{R}P^2$ we put:

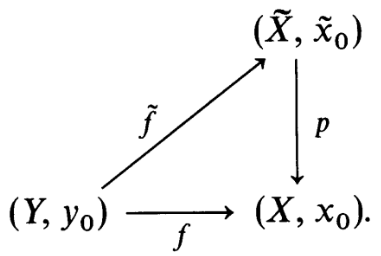
$$\mathbb{R}P^2 \vee \mathbb{R}P^2 := (\mathbb{R}P^2 \times \{x_0\}) \cup (\{x_0\} \times \mathbb{R}P^2) \subset \mathbb{R}P^2 \times \mathbb{R}P^2$$

Prove that any map $\mathbb{R}P^2 \vee \mathbb{R}P^2 \to S^1$ is

> Nullhomotopic
>
> **Rotman:**
> A continuous map $f : X \to Y$ is *nullhomotopic* if there is a constant map $x : X \to Y$ with $f \simeq c$ (that is, f is homotopic to c).
>
> **Hatcher:**
> A map is *nullhomotopic* if it is homotopic to a constant map.
>
> #Algebraic_Topology

.

proof:

> Lifting Criterion
>
> **Rotman:**
>
> **Lemma 10.3.** *Let $(\tilde{X}, p)$ be a covering space of X, let Y be a connected space, and let $f: (Y, y_0) \to (X, x_0)$ be continuous. Given $\tilde{x}_0$ in the fiber over x_0, there is at most one continuous $\tilde{f}: (Y, y_0) \to (\tilde{X}, \tilde{x}_0)$ with $p\tilde{f} = f$.*
>
> 
>
>
> **Theorem 10.13** (Lifting Criterion). *Let Y be connected and locally path connected, and let $f: (Y, y_0) \to (X, x_0)$ be continuous. If $(\tilde{X}, p)$ is a covering space of X, then there exists a unique $\tilde{f}: (Y, y_0) \to (\tilde{X}, \tilde{x}_0)$ (where $\tilde{x}_0 \in p^{-1}(x_0)$) lifting f if and only if $f_* \pi_1(Y, y_0) \subset p_* \pi_1(\tilde{X}, \tilde{x}_0)$.*

1d

Let $\mathbb{R}P^2 \vee \mathbb{R}P^2$ denote the wedge sum of two copies of $\mathbb{R}P^2$. Explicitly picking a basepoint $x_0 \in \mathbb{R}P^2$ we put:

$$\mathbb{R}P^2 \vee \mathbb{R}P^2 := (\mathbb{R}P^2 \times \{x_0\}) \cup (\{x_0\} \times \mathbb{R}P^2) \subset \mathbb{R}P^2 \times \mathbb{R}P^2$$

Give an example of a map $\mathbb{R}P^2 \times \mathbb{R}P^2 \to \mathbb{R}P^2 \vee \mathbb{R}P^2$ which is not

> Nullhomotopic
>
> **Rotman:**
> A continuous map $f : X \to Y$ is *nullhomotopic* if there is a constant map $x : X \to Y$ with $f \simeq c$ (that is, f is homotopic to c).
>
> **Hatcher:**
> A map is *nullhomotopic* if it is homotopic to a constant map.
>
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proof:

Consider the map $(a, b) \to a$, the induced map is not trivial so it is not nullhomotopic by

> Nullhomotopic Map Has Trivial Induced Homomorphism on Fundamental Group
>
> **Corollary 3.13.** *If $\beta: (X, x_0) \to (Y, y_0)$ is (freely) nullhomotopic, then the induced homomorphism $\beta_*: \pi_1(X, x_0) \to \pi_1(Y, y_0)$ is trivial.*[2]

#Algebraic_Topology

3a

Give an example of a path-connected topological space X whose fundamental group is nonzero and isomorphic to $H_1(X)$

proof:

> Sn is path connected
>
> **Rotman: Ex 1.15**
> S^n is path connected for $n \geq 1$
>
> #Algebraic_Topology

We know by

> Sn Fundamental Group
>
> $$\pi_1(S^n) \cong \begin{cases} \mathbb{Z} & \text{if } n = 1 \\ 0 & \text{otherwise} \end{cases}$$
>
> By n-1 connectedness of Sn
>
> check fundamental group of S^0
>
> #Algebraic_Topology

that the fundamental group of the circle

$$\pi_1(S^1) \cong \mathbb{Z}$$

and from

> Sn Homology Groups
>
> ## Homology Groups
>
> If $n > 0$
>
> $$H_k(S^n) \cong \begin{cases} \mathbb{Z} & \text{if } k = 0, n \\ 0 & \text{otherwise} \end{cases}$$
>
> If $n = 0$
>
> $$H_k(S^n) \cong \begin{cases} \mathbb{Z} \oplus \mathbb{Z} & \text{if } k = 0 \\ 0 & \text{otherwise} \end{cases}$$
>
> ## Reduced Homology Groups
>
> $$\tilde{H}_k(S^n) \cong \begin{cases} \mathbb{Z} & \text{if } k = n \\ 0 & \text{otherwise} \end{cases}$$
>
> ## Calculation
>
> #Algebraic_Topology

that the reduced homology of any sphere is

$$\tilde{H}_p(S^n) = \begin{cases} \mathbb{Z} & \text{if } n = p \\ 0 & \text{otherwise} \end{cases}$$

Thus $H_1(S^1) \cong \mathbb{Z} \cong \pi_1(S^1)$

3b

Let X be a connected CW complex and $A \subset X$ a subcomplex such that A is

> Homotopy Equivalence
>
> **Rotman:**
> A continuous map $f : X \to Y$ is a *homotopy equivalence* if there is a continuous map $g : Y \to X$ with $g \circ f \simeq 1_X$ and $f \circ g \simeq 1_Y$.
> Two spaces X and Y have the same homotopy type if there is a homotopy equivalence $f : X \to Y$
>
> **Hatcher:**
> A map $f : X \to Y$ is called a *homotopy equivalence* if there is a map $g : Y \to X$ such that $fg \simeq 1$ and $gf \simeq 1$. The spaces X and Y are said to be homotopy equivalent or to have the same homotopy type.
>
> #Algebraic_Topology

to S^3 and X/A is

> Homotopy Equivalence
>
> **Rotman:**
> A continuous map $f : X \to Y$ is a *homotopy equivalence* if there is a continuous map $g : Y \to X$ with $g \circ f \simeq 1_X$ and $f \circ g \simeq 1_Y$.
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> **Hatcher:**
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>
> #Algebraic_Topology

to S^5. Compute $H_n(X)$ for all n.

By

> Same Homotopy Type Implies Same Homology
>
> **Rotman Corollary 4.24:**
> If X and Y have the same homotopy type, then $H_n(X) \cong H_n(Y)$ for all $n \geq 0$, where the isomorphism is induced by any homotopy equivalence.
>
> **Hatcher Corollary 2.11:**
> The maps $f_* : H_n(X) \to H_n(Y)$ induced by a homotopy equivalence $f : X \to Y$ are isomorphisms for all n.
>
> #Algebraic_Topology

we have that

$$H_k(X/A) \cong H_k(S^5) \quad \text{and} \quad H_k(A) \cong H_k(S^3)$$

By

> Sn Homology Groups
>
> ## Homology Groups
>
> If $n > 0$
>
> $$H_k(S^n) \cong \begin{cases} \mathbb{Z} & \text{if } k = 0, n \\ 0 & \text{otherwise} \end{cases}$$
>
> If $n = 0$
>
> $$H_k(S^n) \cong \begin{cases} \mathbb{Z} \oplus \mathbb{Z} & \text{if } k = 0 \\ 0 & \text{otherwise} \end{cases}$$
>
> ## Reduced Homology Groups
>
> $$\tilde{H}_k(S^n) \cong \begin{cases} \mathbb{Z} & \text{if } k = n \\ 0 & \text{otherwise} \end{cases}$$

Calculation

#Algebraic_Topology

we have

$$H_k(A) \cong H_k(S^3) \cong \begin{cases} \mathbb{Z} & \text{if } k = 0, 3 \\ 0 & \text{otherwise} \end{cases}$$

By

Sn Homology Groups

Homology Groups

If $n > 0$

$$H_k(S^n) \cong \begin{cases} \mathbb{Z} & \text{if } k = 0, n \\ 0 & \text{otherwise} \end{cases}$$

If $n = 0$

$$H_k(S^n) \cong \begin{cases} \mathbb{Z} \oplus \mathbb{Z} & \text{if } k = 0 \\ 0 & \text{otherwise} \end{cases}$$

Reduced Homology Groups

$$\tilde{H}_k(S^n) \cong \begin{cases} \mathbb{Z} & \text{if } k = n \\ 0 & \text{otherwise} \end{cases}$$

Calculation

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we have

$$\tilde{H}(X/A) \cong \tilde{H}_k(S^5) \cong \begin{cases} \mathbb{Z} & \text{if } k = 5 \\ 0 & \text{otherwise} \end{cases}$$

By

Relative Homology vs Homology of Quotient

Rotman: Theorem 8.41
Let (X, E) be a CW complex with CW subcomplex (Y, E'). Then the natural map $\nu : X \to X/Y$ induces isomorphisms for every $k \geq 0$

$$\nu_* : H_k(X, Y) \cong H_k(X/Y, *) \cong \tilde{H}_k(X/Y)$$

where $*$ denotes the singleton point $\nu(Y)$ in X/Y

Hatcher: Proposition 2.22
For good pairs (X, A), the quotient map $q : (X, A) \to (X/A, A/A)$ induces isomorphisms $q_* : H_n(X, A) \to H_n(X/A, A/A) \cong \tilde{H}_n(X/A)$ for all n

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we have

$$\tilde{H}_k(X/A) \cong H_k(X, A)$$

By

Exact Sequence of the Pair

Rotman: Theorem 5.8
If A is a subspace of X, there is an exact sequence

$$\cdots \to H_n(A) \to H_n(X) \to H_n(X, A) \xrightarrow{d} H_{n-1}(A) \to \dots$$

> **Hatcher:
>
> **Theorem 2.13.** *If X is a space and A is a nonempty closed subspace that is a deformation retract of some neighborhood in X, then there is an exact sequence*
>
> $$\cdots \longrightarrow \tilde{H}_n(A) \xrightarrow{i_*} \tilde{H}_n(X) \xrightarrow{j_*} \tilde{H}_n(X/A) \xrightarrow{\partial} \tilde{H}_{n-1}(A) \xrightarrow{i_*} \tilde{H}_{n-1}(X) \longrightarrow \cdots$$
> $$\cdots \longrightarrow \tilde{H}_0(X/A) \longrightarrow 0$$
>
> *where i is the inclusion $A \hookrightarrow X$ and j is the quotient map $X \to X/A$.*
>
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we can start diagram chasing by plugging in our above homology values to obtain

$$\cdots \to \cancelto{0}{H_5(A)} \to H_5(X) \to H_5(X/A) \cong \tilde{H}_5(X/A) \cong \cancelto{0}{H_5(X,A)} \dots$$

So by

> Flanking Zeros of Exact Sequence
>
> **Rotman: Exercise 5.1 (iv)**
> If $0 \to A \to 0$ is exact, then $A = 0$
>
> #Algebraic_Topology

we have that $H_5(X) \cong \mathbb{Z}$

Then

$$\cdots \to \cancelto{0}{H_k(X,A)} \to \cancelto{\mathbb{Z}}{H_3(A)} \to H_3(X) \to \cancelto{0}{H_3(X,A)} \dots$$

and by

> Sandwiched Zeros of Exact Sequence
>
> **Rotman: Exercise 5.1 (iii)**
> If $0 \to A \xrightarrow{f} B \to 0$ is exact, then f is an isomorphism. Hence $A \cong B$
>
> #Algebraic_Topology

we have that $H_3(X) \cong \mathbb{Z}$

Lastly

$$\cdots \to \cancelto{0}{H_1(X,A)} \to \cancelto{\mathbb{Z}}{H_0(A)} \to H_0(X) \to H_0(X,A) \cong \cancelto{0}{\tilde{H}_0(X/A)} \dots$$

So again by

> Sandwiched Zeros of Exact Sequence
>
> **Rotman: Exercise 5.1 (iii)**
> If $0 \to A \xrightarrow{f} B \to 0$ is exact, then f is an isomorphism. Hence $A \cong B$
>
> #Algebraic_Topology

we have $H_0(X) \cong \mathbb{Z}$
Everywhere else $H_k(X)$ is trivial.

■

3c

Give an example of a topological space X whose reduced homology groups are $\tilde{H}_5(X) \cong \mathbb{Z}, \tilde{H}_2(X) = \mathbb{Z}/2\mathbb{Z}, \tilde{H}_n(X)$ for $n \neq 2, 5$

proof:
We have from

> RP2 Homology Groups
>
> ## Homology Groups

$$H_n(\mathbb{R}P^2) = \begin{cases} \mathbb{Z}, & \text{n=0} \\ \mathbb{Z}/2\mathbb{Z}, & \text{n=1} \\ 0, & \text{otherwise} \end{cases}$$

Reduced Homology Groups

$$\tilde{H}_n(\mathbb{R}P^2) = \begin{cases} \mathbb{Z}/2\mathbb{Z} & \text{n=1} \\ 0 & \text{otherwise} \end{cases}$$

Calculation

#Algebraic_Topology

$$\tilde{H}_n(\mathbb{R}P^2) = \begin{cases} \mathbb{Z}/2\mathbb{Z} & \text{n=1} \\ 0 & \text{otherwise} \end{cases}$$

And from

> Sn Homology Groups
>
> # Homology Groups
>
> If $n > 0$
>
> $$H_k(S^n) \cong \begin{cases} \mathbb{Z} & \text{if } k = 0, n \\ 0 & \text{otherwise} \end{cases}$$
>
> If $n = 0$
>
> $$H_k(S^n) \cong \begin{cases} \mathbb{Z} \oplus \mathbb{Z} & \text{if } k = 0 \\ 0 & \text{otherwise} \end{cases}$$
>
> # Reduced Homology Groups
>
> $$\tilde{H}_k(S^n) \cong \begin{cases} \mathbb{Z} & \text{if } k = n \\ 0 & \text{otherwise} \end{cases}$$
>
> # Calculation
>
> #Algebraic_Topology

$$\tilde{H}_k(S^5) \cong \begin{cases} \mathbb{Z} & \text{if } k = 5 \\ 0 & \text{otherwise} \end{cases}$$

Then from

> Reduced Homology of Suspension
>
> **Hatcher Ch.2 Ex.32:**
> For SX the suspension of X, $\tilde{H}_n(X) \cong \tilde{H}_{n+1}(SX)$ for all n
>
> #Algebraic_Topology

$$\tilde{H}_n(X) \cong \tilde{H}_{n+1}(SX)$$

So

$$\tilde{H}_n(S(\mathbb{R}P^2)) \cong \begin{cases} \mathbb{Z}/2\mathbb{Z} & \text{if } n = 2 \\ 0 & \text{otherwise} \end{cases}$$

Lastly if we choose good pairs for our space $X = S^5 \vee S(\mathbb{R}P^2)$ then from

> Reduced Homology of Wedge Sum

Hatcher Ch.2 Ex. 31
If the basepoints of X and Y that are identified in $X \vee Y$ are Deformation Retracts of neighborhoods $U \subset X$ and $V \subset Y$ then

$$\tilde{H}_n(X \vee Y) \cong \tilde{H}_n(X) \oplus \tilde{H}_n(Y)$$

Julian Take-home:
if (X, x) and (Y, y) are both good pairs, then

$$\tilde{H}_n(X \vee Y) \simeq \tilde{H}_n(X) \oplus \tilde{H}_n(Y)$$

Proof

Can be shown by mayer-vietoris

#Algebraic_Topology

we obtain

$$\tilde{H}_n(X) = \begin{cases} \mathbb{Z}/2\mathbb{Z} & \text{if } n = 2 \\ \mathbb{Z} & \text{if } n = 5 \\ 0 & \text{otherwise} \end{cases}$$

■

Problem 4

Let G be a topological group. This means G is a group which is also equipped with a topology such that the multiplication map $G \times G \to G, (g, h) \mapsto g \cdot h$ and inversion map $G \to G, g \mapsto g^{-1}$ are both continuous. Assuming that G is connected, prove that the fundamental group of G is abelian.

?